# Characteristics of Ground Motion

Power Spectrum $P(f)$ $[m^2/Hz]$

$f < 0.1$ Hz diffusive "ATL"

$P(f) = k / f^2$ , $k = 1 \sim 100\ nm^2\ Hz$

$f > 0.1$ Hz elastic "waves"

$P(f) \propto 1 / f^4$

| | |
|---|---|
| ocean swell | 0.2Hz |
| cultural noises | 1~100Hz |
| crustal resonances | 3Hz |
| μ earthquake | |

# ATL Rule

$$\sigma^2 = <\Delta y>^2 = A\,T\,L$$

$\sigma^2$ : RMS of displacement ( $\Delta y$ ) between two points with a distance L after a time T.

A is site-dependent;$10^{-16}$~$10^{-21}$ m/sec.

# ATL at Esashi

National Astronomical
Observatory Mizusawa

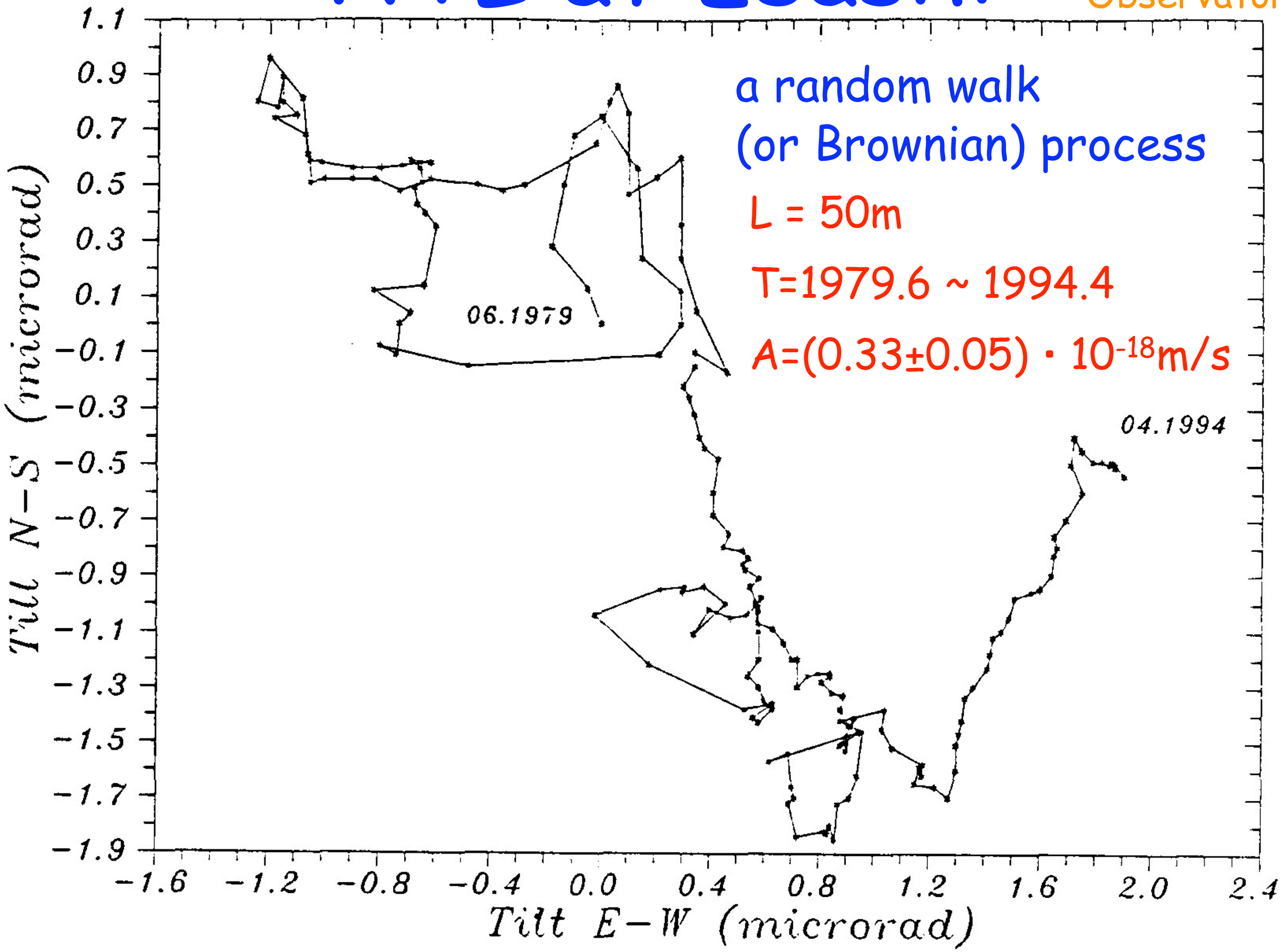
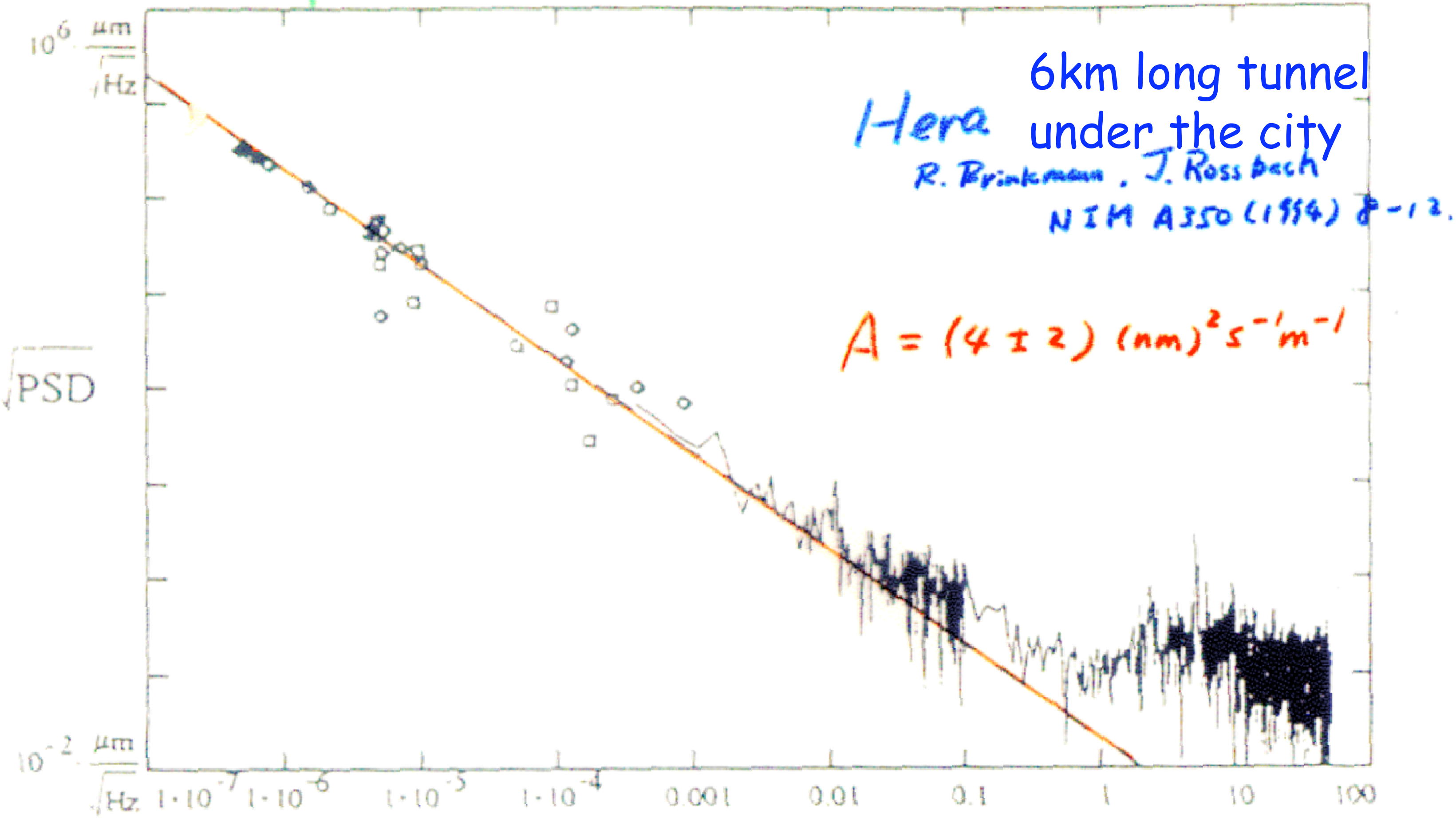



Fig.1 Secular tilting motion at Esashi station in 1979-1994 (data taken from [16]).

# ATL at HERA




Hz

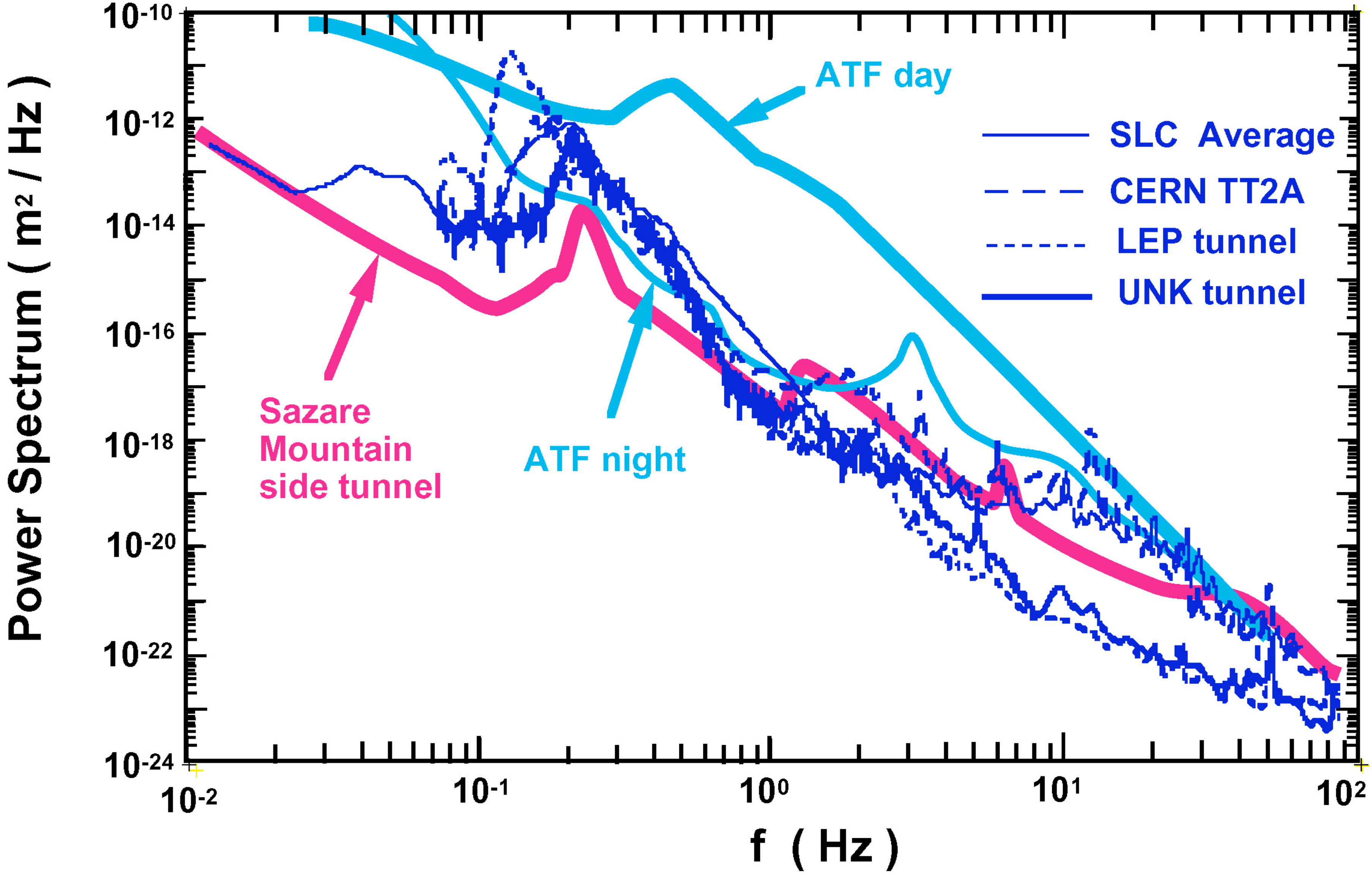
— Fourier spectrum of one BPM reading at HERAe
◇ HERA rms electron orbit motion after certain time intervals
□ HERA rms proton orbit motion after certain time intervals
— spectrum density scaling as expected by ATL rule

$L_{FODO}^{e} = 23.5\ m$

$L_{FODO}^{p} = 47\ m$

$L \equiv L_{FODO}$

# GM Measurements



# GM Model

1. 2 dimentional power spectrum

$$P(\omega,k) = \lim_{T\to\infty} \lim_{L\to\infty} 1/T \; 1/L \int_{-T/2}^{T/2}\int_{-L/2}^{L/2} x(t,s)e^{-i\omega t}e^{-iks} \, dt \, ds$$

$$P(\omega,k) = A/\omega^2/k^2 \left(1-\cos(kB/A/\omega^2)\right) + \Sigma D_i(\omega)U_i(\omega,k)$$

where, $U_i(\omega,k)=2/\sqrt{k^2_{cut} - k^2}$, $k^2_{cut} = \omega/v_i$, $v_i$: phase velociy

$$D_i(\omega)=a_i/(1+[d_i(\omega-\omega_i)/\omega_i]^4), \quad f_i=2\pi\omega_i$$

2. Displacement

$$\sigma^2 = \langle x(t,s)^2 \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(\omega,k) \, d\omega \, dk /(2\pi)^2$$

3. Relative Displacement

$$\sigma^2(T,L) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(\omega,k) \, 2(1-\cos\omega T)2(1-\cos kL) \, d\omega \, dk /(2\pi)^2$$

$$\square^2 (T,L) = \int_{-\infty}^{\infty} \square(\square,L)\, 2(1-\cos\square T)\, d\square\ /(2\square)$$

$$\square(\square,L) = \int P(\square,k) 2(1-\cos kL)\, dk/(2p)$$

$$= AL/\ \square^2 \quad \text{if } 0 < \square < \square_o$$

$$= B/\ \square^4 \quad \text{if } \square_o < \square < \infty$$

where, $\square_o = (B/AL)^{1/2}$

$$\square^2 (T,L) = ATL \quad \text{at } 0 < \square < \square_o$$

$$\square(\square,L) = 2\, P(\square)\, \{ 1 - \mathrm{Re}[N_{12}(\square,L)]\} = 2\, P(\square)\, R\, (\square,L)$$

where, $\mathrm{Re}(N_{12}(\square,L))$ is correlation;

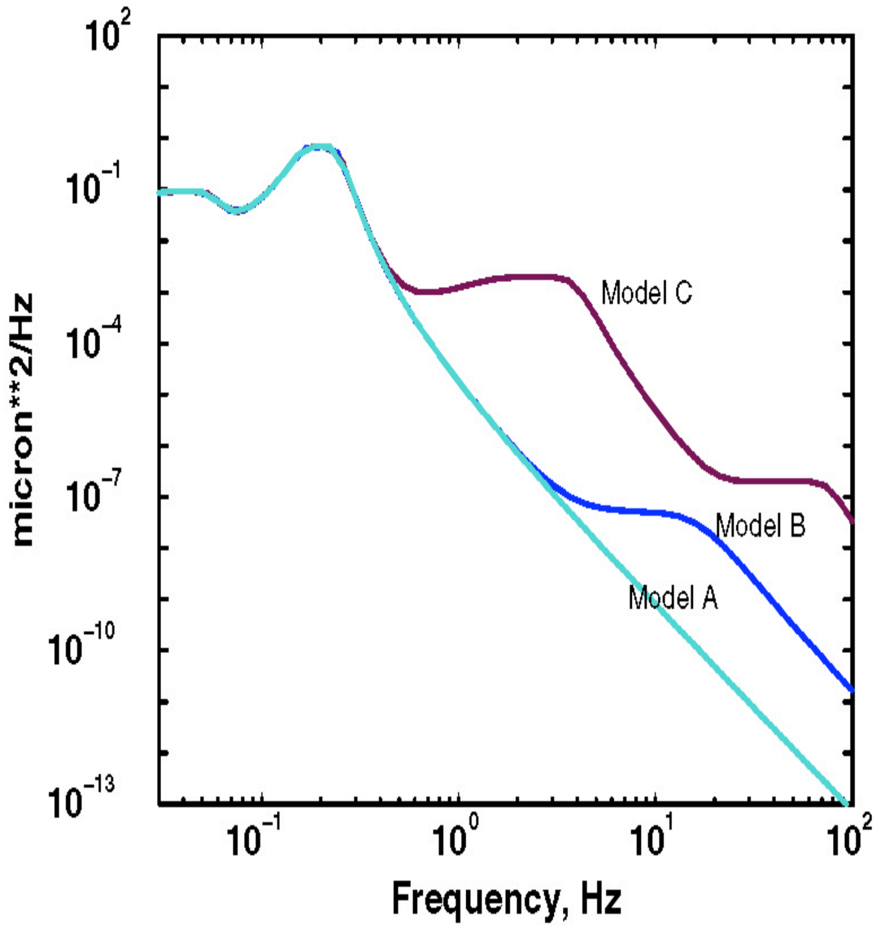
e.g. $N_{12}(\square,L) = J_o(\square\ L/v_i)$ for waves; $v_i\, k_{cut} = \square$

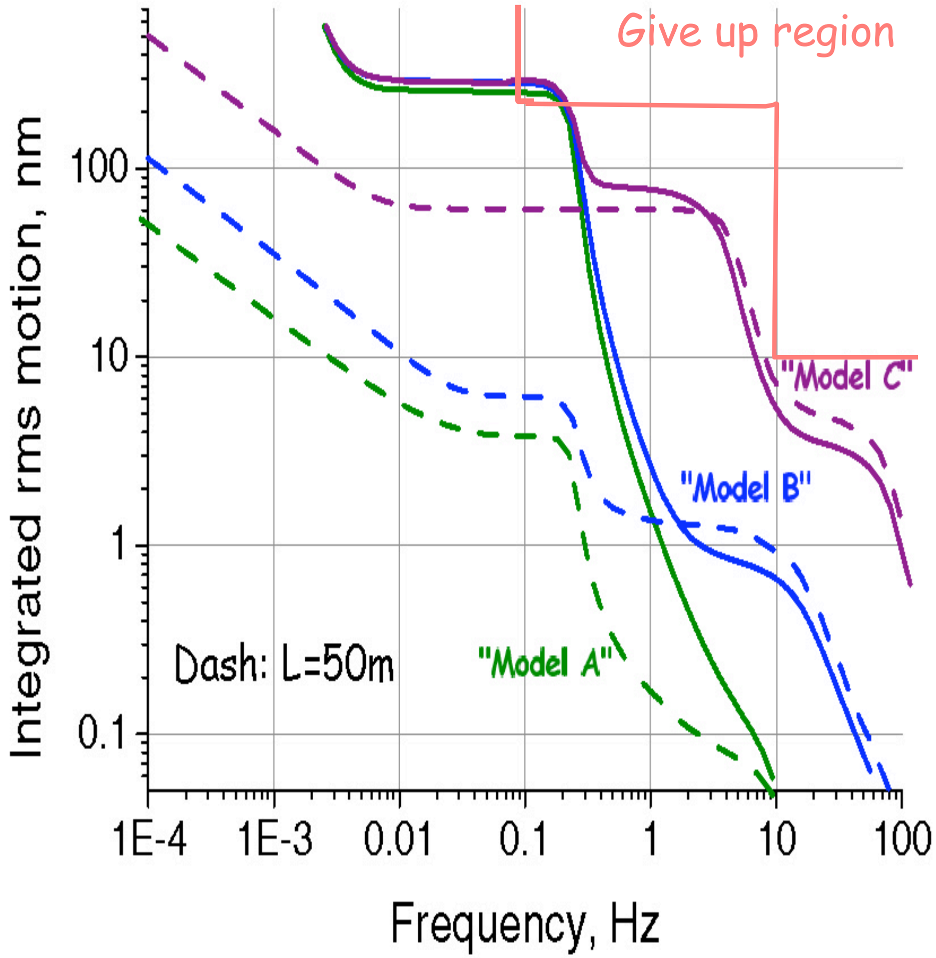
# Parameters of P(□,k) in ILC-TRC

| Parameter | Units | Model A | Model B | Model C |
|---|---|---|---|---|
| A of ATL | m/s | $1.0 \cdot 10^{-19}$ | $0.5 \cdot 10^{-18}$ | $1.0 \cdot 10^{-17}$ |
| B of ATL | $m^2/s^3$ | $0.5 \cdot 10^{-18}$ | $1.0 \cdot 10^{-18}$ | $5.0 \cdot 10^{-18}$ |
| $f_1$ | Hz | 0.001 | 0.001 | 0.14 |
| $a_1$ | $m^2/Hz$ | $1.0 \cdot 10^{-9}$ | $1.0 \cdot 10^{-9}$ | $1.0 \cdot 10^{-11}$ |
| $d_1$ | | 1 | 1 | 5 |
| $v_1$ | m/s | $3 \cdot 10^3$ | $v_{app}(f)$ | $v_{app}(f)$ |
| $f_2$ | Hz | 0.2 | 0.2 | 2.5 |
| $a_2$ | $m^2/Hz$ | $3.5 \cdot 10^{-13}$ | $3.5 \cdot 10^{-13}$ | $1.0 \cdot 10^{-15}$ |
| $d_2$ | | 3.5 | 3.5 | 1.5 |
| $v_2$ | m/s | $3 \cdot 10^3$ | $v_{app}(f)$ | $v_{app}(f)$ |
| $f_3$ | Hz | 5.0 | 4.5 | 50.0 |
| $a_3$ | $m^2/Hz$ | $1.0 \cdot 10^{-21}$ | $2.5 \cdot 10^{-20}$ | $1.0 \cdot 10^{-19}$ |
| $d_3$ | | 1.3 | 0.35 | 1.5 |
| $v_3$ | m/s | $3 \cdot 10^3$ | $v_{app}(f)$ | $v_{app}(f)$ |

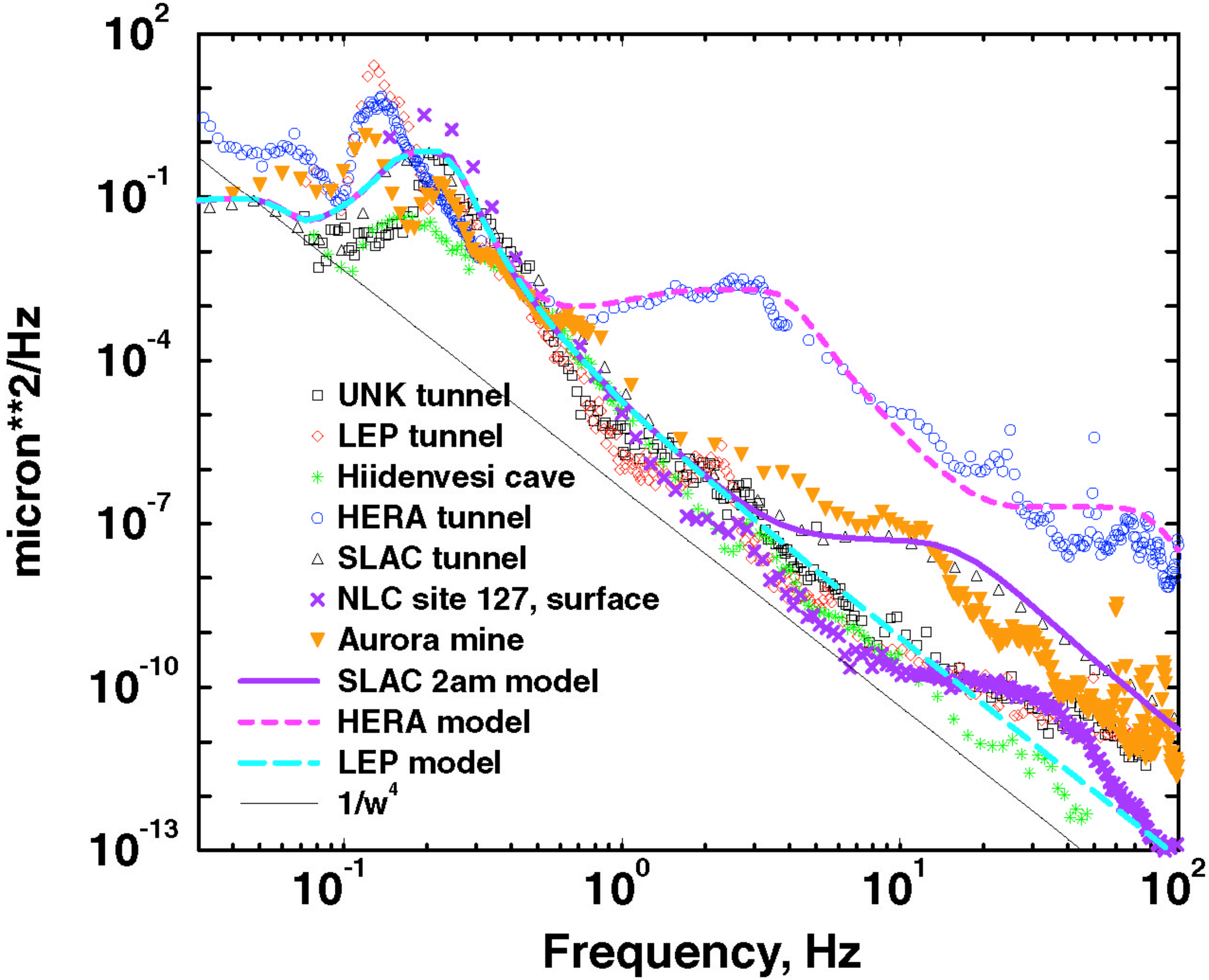
$$V_{app}(f) = 450 + 1900 \exp(-f/2) \quad m/s$$

# GM Model Specta



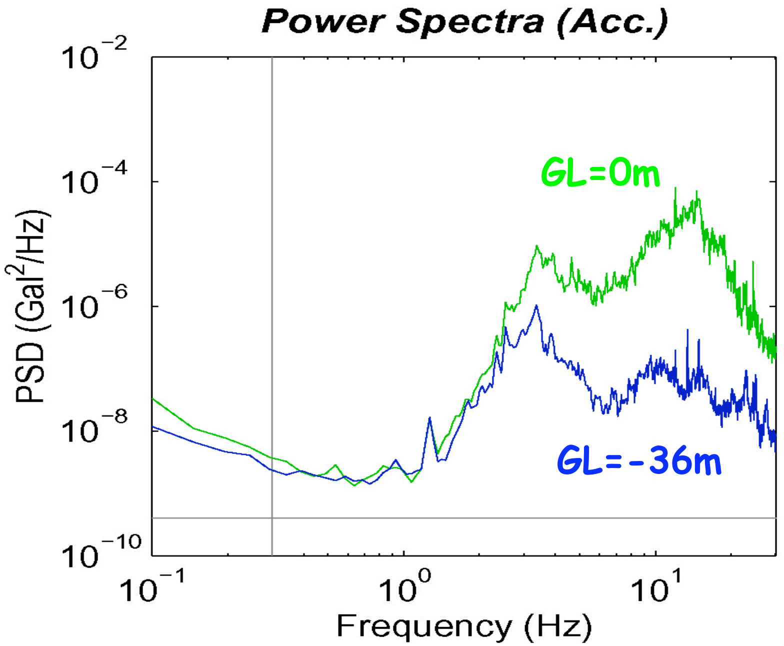



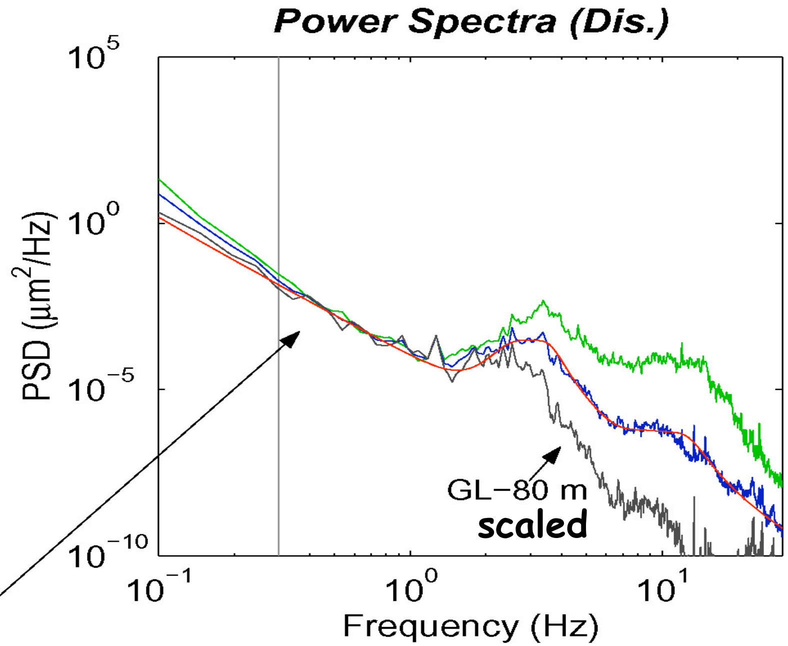
# GM Power Spectra and Models



# GM Model at Tsukuba

N. Uchida, Tsukuba univ.




*Red Line:* **fitted to GL-36m data**

```
P = a1 ./ ( ((f−f1)/f1*d1).^4 +1 )
  + a2 ./ ( ((f−f2)/f2*d2).^4 +1 )
  + a3 ./ ( ((f−f3)/f3*d3).^4 +1 );
```

| | | |
|---|---|---|
| a1 = 1e4 | f1 = 0.01 | d1 = 1 |
| a2 = 3e−4 | f2 = 3 | d2 = 4 |
| a3 = 5e−5 | f3 = 10 | d3 = 3 |

$a_{1,2,3} = 1 \cdot 10^{-8}\ , 3 \cdot 10^{-16}, 5 \cdot 10^{-17}\ \ m^2/Hz$