Proceedings of the ILC Physics Working Group
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Edited by
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The 8th General Meeting of the ILC Physics Working Group, January 21, 2009
Preface

The ILC physics working group is a mixture of experimentalists and theorists mainly working in Japan. It has its origin in the previous LC physics study group and has been reformed with the initiative of a JSPS Creative Scientific Research project: “Research and Development of a Novel Detector System for the International Linear Collider”. The working group is, however, formally independent of the JSPS project and is open to everybody who is interested in ILC physics. The primary task of the working group is to reexamine the ILC physics in the context of the expected LHC outcome and to further strengthen the physics case for the ILC project. The topics covered in the working group activities range from key measurements such as those of the Higgs self-coupling and the top Yukawa coupling to uncover the secrets of the electroweak symmetry breaking to various new physics scenarios like supersymmetry, large extra dimensions, and other models of terascale physics.

The working group has held ten General Meetings in the period of May 2007 to June 2009 to discuss the topics mentioned above. This report summarizes the progress made in this period and sets a milestone for future developments in ILC physics.

Editors, Conveners of the Working Group
## Contents

1. Study of the Higgs Direct Reconstruction in $ZH \rightarrow q\bar{q}H$ for ILC  
   Hiroaki Ono  
   4

2. Measurement of Higgs Branching Ratio at ILC  
   Kohei Yoshida  
   11

3. Analysis of Higgs Self-coupling with $ZHH$ at ILC  
   Yosuke Takubo  
   17

4. Study of Higgs Self-coupling at ILC  
   Junping Tian, Keisuke Fujii, and Yuanning Gao  
   22

5. Feasibility study of the forward-backward asymmetry of the $e^+e^- \rightarrow t\bar{t}$ process in all-hadronic decay modes at $\sqrt{s} = 500$ GeV with the ILD detector  
   Katsumasa Ikematsu, Akiya Miyamoto, and Keisuke Fujii  
   28

6. A study of top-quark Yukawa coupling measurement in $e^+e^- \rightarrow t\bar{t}H$ at $\sqrt{s} = 500$ GeV  
   Ryo Yonamine, Katsumasa Ikematsu, Satoru Uozumi, and Keisuke Fujii  
   40

7. Higgs boson pair production at the Photon Linear Collider in the two Higgs doublet model  
   Daisuke Harada, Eri Asakawa, Shinya Kanemura, Yasuhiro Okada, and Koji Tsumura  
   50

8. Measuring Higgs boson associated Lepton Flavour Violation in electron-photon collisions at the ILC  
   Shinya Kanemura and Koji Tsumura  
   58

9. Feasibility study of Higgs pair creation in $\gamma\gamma$ collider  
   Nozomi Maeda, Keisuke Fujii, Katsumasa Ikematsu, Shinya Kanemura, Yoshimasa Kurihara, and Tohru Takahashi  
   64

10. Analysis of Tau-pair process in the ILD reference detector model  
    Taikan Suehara  
    69

11. Chargino and Neutralino Separation with the ILD Experiment  
    Taikan Suehara and Jenny List  
    83

12. Hidden particle production at the ILC  
    Hideo Itoh, Katsumasa Ikematsu, Keisuke Fujii, Hitoshi Hano, Nobuchika Okada, and Tamaki Yoshioka  
    98

13. ILC phenomenology in a TeV scale radiative seesaw model for neutrino mass, dark matter and baryon asymmetry  
    Mayumi Aoki, Shinya Kanemura, Osamu Seto  
    114

14. Precision Measurements of the model parameters in the Littlest Higgs model with T-parity  
    Masaki Asano, Eri Asakawa, Keisuke Fujii, Tomonori Kusano, Shigeki Matsumoto, Rei Sasaki, Yosuke Takubo, and Hitoshi Yamamoto  
    122

15. Measurement of Heavy Gauge Bosons in the Little Higgs Model with T-parity at ILC  
    Yosuke Takubo, Eri Asakawa, Masaki Asano, Keisuke Fujii, Tomonori Kusano, Shigeki Matsumoto, Rei Sasaki, and Hitoshi Yamamoto  
    127

3
Study of the Higgs Direct Reconstruction in $ZH \rightarrow q\bar{q}H$ for ILC

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Precise measurement of the Higgs boson properties is an important issue of the International Linear Collider (ILC) experiment. We studied the accuracy of the Higgs mass reconstruction in the $ZH \rightarrow q\bar{q}H$ multi-jet process with the Higgs mass of $M_H = 120$ GeV at $\sqrt{s} = 250$ GeV with the ILD detector model. In this study, we obtained the reconstructed Higgs mass of $M_H = 120.79 \pm 0.089$ GeV and 5.3% measurement accuracy of the cross-section for $ZH \rightarrow q\bar{q}b\bar{b}$ with the integrated luminosity of $L = 250$ fb$^{-1}$ data samples.

1 Introduction

International Linear Collider (ILC) [1] is a future $e^+e^-$ collider experiment for the precise measurement and the validation of the Standard Model (SM) physics, especially for the measurement of the Higgs boson property, even the discovery of the Higgs boson will be realized in Large Hadron Collider (LHC) experiment. In the SM, light Higgs boson mass ($M_H$) is predicted around the 114.4 GeV $\leq M_H \leq 160$ GeV from the study in LEP [2] and Tevatron [3] experiment. The largest production cross-section for SM Higgs boson is obtained through the Higgs-strahlung ($e^+e^- \rightarrow Z^* \rightarrow ZH$) process which associated with the $Z$ boson and the $Z$ mainly decays to $q\bar{q}$ pair, as shown in Fig. 1, around the $ZH$ production threshold energy shown in Fig. 2 (a).

Since Higgs boson mainly decays to $b\bar{b}$ pair at the Higgs mass below 140 GeV region as shown in Fig. 2 (b), the final state of the $ZH \rightarrow q\bar{q}H$ process forms the four-jet. In ILC experiment, the most of interesting physics processes including $ZH$ process form the multi-jets final state from the decay of gage bosons ($W, Z$) and heavy flavor quarks ($b, c$), thus ILC detectors are required to have the good jet energy resolution for the precise measurement. There are three detector concepts, SiD, ILD and 4th for the ILC detector, and ILD is the merged concepts of the previous GLD [4] (Asian group) and LDC [5] (European group) models for the Letter of Intent (LOI) submission [6]. In order to achieve the best jet energy resolution, ILD adopt the Particle Flow Algorithm (PFA) suited detector design. Since the PFA performance is degraded by the cluster overlapping and the double-counting of the particles

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energy in the calorimeter, particles separation in the calorimeter is an important key for better PFA performance.

The figure-of-merit of the PFA performance from each detector parameter relating to the particles separation in the calorimeter is described as 
$$F.O.M. = \frac{BR^2}{\sqrt{\sigma^2 + R_{M}^2}}$$

where $B$ is a magnetic field, $R$ is a detector radius, $\sigma$ is a segmentation of the calorimeter and $R_M$ is an effective Moliere radius of the calorimeter. In order to maximize the F.O.M., ILD detector adopts the large radius tracker and high granularity calorimeter with 3.5 T magnetic field.

In this analysis, we study the direct reconstruction of the Higgs boson mass with the full detector simulation for $ZH \rightarrow qqH, H \rightarrow b\bar{b}$ four-jet mode with the ILD detector model.

## 2 Simulation tools

For full detector simulation study, we use the ILD detector model based Monte Carlo (MC) full simulation package called Mokka, which is based on the MC simulation package Geant4 [7]. Generated MC hits are reconstructed and smeared in the reconstruction package called MarineReco which includes the PFA package called PandoraPFA [8]. Since $\sqrt{s} = 250$ GeV reconstructed and skimmed signal and background samples called DST files are generated for the LOI physics analysis in ILD group, we use these DST data samples saved in the linear collider common data format called LCIO. For the DST data sample analysis, we use the useful analysis package library called Anlib for the event shape analysis and jets reconstruction, and analysis process is handled through the Root [9] based analysis framework called JSF [10]. For the comparison of the PFA performance between realistic PFA and perfect-clustering PFA, we also use the GLD detector model MC full simulator called Jupiter [11] with the generating the signal and background events by PYTHIA, and reconstruction package called Satellites [12] based on Root, both of them are also controlled.
in the JSF framework. From the comparison of the $ZH \rightarrow q\bar{q}H$ in GLD detector model, shown in Fig 2, PandoraPFA reconstruction performance (a) achieve the comparable performance with perfect-clustering PFA (b) in terms of the reconstructed Higgs mass distribution width of $\sigma$ which corresponds to the jet energy resolution even only the $ZZ \rightarrow q\bar{q}q\bar{q}$ background is considered. Therefore, we shift to the full SM background analysis with common DST data.

![Graphs showing reconstructed Higgs mass distribution](image)

Figure 3: Comparison of the reconstructed Higgs mass distribution for $ZH \rightarrow q\bar{q}b\bar{b}$ only with $ZZ$ background in GLD detector model, with the different PFA clustering of (a) realistic PandoraPFA and (b) perfect clustering PFA.

### 3 Analysis Procedure of $ZH \rightarrow q\bar{q}H$ mode

#### 3.1 MC samples

The SM Higgs boson is mainly produced through the Higgs-strahlung $e^+e^- \rightarrow ZH$ process around the production threshold center-of-mass energy ($\sqrt{s} \sim 230$ GeV). Since the main decay mode at $M_H < 2M_W$, Higgs boson mainly decays to $b\bar{b}$ pair, thus largest production cross-section is obtained from the $ZH \rightarrow q\bar{q}b\bar{b}$ process, which forms four-jet final state and both $Z$ and $H$ can be reconstructed directly. Fig. 4 shows the typical event display of the $ZH \rightarrow q\bar{q}H$ in JSF. In this analysis, we assume the center-of-mass energy as the $ZH$ production threshold of $\sqrt{s} = 250$ GeV and the light Higgs mass of $M_H = 120$ GeV. Each DST data samples is scaled to the integrated luminosity of $\mathcal{L} = 250$ fb$^{-1}$ and the beam polarization to $P(e^+, e^-) = (30\%, -80\%)$. The main backgrounds for $ZH \rightarrow q\bar{q}b\bar{b}$ are considered as following

![Typical event display](image)

Figure 4: Typical event display of the $ZH \rightarrow q\bar{q}H$ four-jet final state.
processes: \( ZH \rightarrow Z^* / \gamma \rightarrow q\bar{q}, e^+e^- \rightarrow WW/ZZ \rightarrow q\bar{q}'q''\bar{q}'' \) or \( q\bar{q}'q''\bar{q}'' \), \( e^+e^- \rightarrow WW \rightarrow \nu\bar{q}q' \) and \( e^+e^- \rightarrow ZZ \rightarrow \ell\ell\ell\ell \). Generated signal and background MC samples which scaled to be \( \mathcal{L} = 250 \text{ fb}^{-1} \) are summarized in Table 1.

<table>
<thead>
<tr>
<th>MC samples (( \mathcal{L} = 250 \text{ fb}^{-1} ))</th>
<th>( ZH \rightarrow qqH ) (sig)</th>
<th>( qqqq )</th>
<th>( \nu\bar{q}q' )</th>
<th>( \ell\ell\ell\ell )</th>
<th>( qq )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of generated events</td>
<td>51763</td>
<td>814163</td>
<td>302807</td>
<td>98127</td>
<td>2529928</td>
</tr>
</tbody>
</table>

Table 1: Generated signal and background MC data samples scaled with \( \mathcal{L} = 250 \text{ fb}^{-1} \).

In order to correct the escape energy from the heavy quark decay including neutrinos, kinematic five constraint (5C) fit is applied, which consists of the four constraints (4C) of momentum balance \( \sum P_{x,y,z} = 0 \) and jets energy balance \( \sum E_i - \sqrt{s} = 0 \) of the four-jet and one \( Z \) mass constraints for \( Z \) candidate di-jet. For the kinematic fitting, jet energies \( (E_j) \) and jet angles \( (\theta, \phi) \) of each jet are used as measured variables. Finally, reconstructed Higgs mass distribution is fitted with the Gaussian convoluted with Gaussian function for the signal and exponential function for the contribution from background events which remain after the Higgs boson selections.

### 3.b Jet Reconstruction

Since the final state of the \( ZH \rightarrow q\bar{q}H \) mode forms four-jet, after the PandoraPFA clustering, forced four-jet clustering based on Durham jet-clustering algorithm has applied. In order to select the best jet pair combination from the four-jet, following \( \chi^2 \) value is evaluated,

\[
\chi^2 = \left( \frac{M_{12} - M_Z}{\sigma_{M_Z}} \right)^2 + \left( \frac{\text{MissM}_{34} - M_Z}{\sigma_{\text{MissM}_H}} \right)^2
\]

(3.1)

where \( M_{12} \) is \( Z \) candidate di-jet mass, \( \text{MissM}_{34} \) is a missing mass of the remaining Higgs candidate di-jet, \( M_Z \) is the \( Z \) boson mass (91.2 GeV), and \( \sigma_{M_Z} \) and \( \sigma_{\text{MissM}_H} \) are sigma of distribution of the reconstructed \( Z \) boson mass and the missing mass of the Higgs candidate jets, respectively. In order to select the best jets pair combination, \( \chi^2 < 10 \) is required for the reconstructed jets pair.

### 3.c Event selection

After the \( \chi^2 \) cut to select the best jets pair combination, following event selections are applied for background rejection:

(a) visible energy : \( 200 \leq E_{\text{vis}} \leq 270 \text{ GeV} \);
(b) Longitudinal momentum of the \( Z \) : \( |P_T| < 70 \text{ GeV} \) to reduce \( ZZ \) background;
(c) Higgs production angle : \( |\cos \theta_H| < 0.85 \) to reduce the \( ZZ \) background;
(d) thrust angle : \( \text{thrust} < 0.9 \);
(e) Number of particles: \( N_{\text{particle}} > 40 \) to suppress the \( \ell\ell\ell\ell \) background;
(f) Maximum and minimum jet energy fraction: \( E_{\text{min}}/E_{\text{max}} > 0.25 \);
(g) Maximum momentum of jet: \( P_{j_{\text{max}}} < 100 \text{ GeV} \);
(h) Y Plus : $Y_{Plus} > 0.0001$;
(i) Y Minus : $Y_{Minus} > 0.001$;
(j) Minimum angle of Z-H jets : $20 < \theta_{ZH_{min}} < 135$;
(k) Maximum angle of Z-H jets : $110 < \theta_{ZH_{max}}$;
(l) $b$-tagging : $P_{btag} > 0.5$ from LCFIVTX package.

The distribution and its cut positions for each selection variable are shown in Fig. 1. Since the $W/Z$ generated in the $WW/ZZ$ background event are relatively boosted compare to the $Z$ generated in $ZH$ signal event, longitudinal momentum of $Z$ ($P_T^Z$) and maximum momentum in jets ($P_{j_{max}}$) are higher in $WW/ZZ$ background event than in signal event. None jet-like background events are reduced by the number of particles ($N_{PFO}$) cut. Y Plus and Y Minus values are threshold Y-values used in the jet clustering topology which reconstructed from four-jet to five-jet or three-jet, respectively. Minimum and maximum angles between $Z$ and $H$ candidate jets are also used for the separation by the event shape difference between $ZH$ event and backgrounds.

Figure 5: Distribution of each selection variable and its cut positions to select $ZH \rightarrow qqbb$ event.

Finally, we apply the vertex tagging selection for the neural net output of the $b$-likeness analyzed in the vertexing package called LCFIVTX in fullsoft. The reduction summary in each event selection is listed in the Table 2.
Selections $ZH \rightarrow q\bar{q}H$ (Sig) $qq\bar{q}q$ $\nu\bar{q}q\bar{q}$ $\ell\ell\ell\ell$ $qq$

| Selections | $ZH \rightarrow q\bar{q}H$ (Sig) | $qq\bar{q}q$ | $\nu\bar{q}q\bar{q}$ | $\ell\ell\ell\ell$ | $qq$
|------------|---------------------------------|-------------|-----------------|-----------|-
| no cuts    | 51745                           | 814162      | 302807          | 98127     | 2529928
| $\chi^2$  | 36748 (71.02 %)                 | 688703 (84.59 %) | 19043 (6.29 %) | 25375 (25.86 %) | 541852 (21.42 %)
| $|P_{Z}\ell|$ | 34952 (67.55 %)                 | 479403 (58.71 %) | 12370 (4.11 %) | 5335 (5.67 %) | 287324 (11.62 %)
| $\cos \theta_{q}H$ | 30451 (58.85 %)                 | 397270 (48.79 %) | 12457 (4.11 %) | 5335 (5.44 %) | 287324 (11.36 %)
| thrust     | 29916 (57.81 %)                 | 389514 (47.87 %) | 8312 (2.75 %) | 1422 (1.45 %) | 103283 (4.08 %)
| $N_{\text{particles}}$ | 29820 (57.63 %)                 | 389514 (47.84 %) | 4353 (1.44 %) | 0 (0.00 %) | 87022 (3.44 %)
| $E_{\text{vis}}$ | 34924 (67.49 %)                 | 477994 (58.71 %) | 12457 (4.11 %) | 5335 (5.44 %) | 287324 (11.36 %)
| $N_{\text{particles}}/E_{\text{vis}}$ | 27643 (53.81 %) | 297580 (36.55 %) | 1603 (0.53 %) | 0 (0.00 %) | 40880 (1.62 %)
| $p_{T,\ell\ell\ell\ell}$ | 27622 (53.38 %) | 289490 (35.56 %) | 1500 (0.50 %) | 0 (0.00 %) | 31382 (1.24 %)
| $Y_{\ell\ell\ell\ell}$ | 27607 (53.35 %) | 289490 (35.55 %) | 1500 (0.50 %) | 0 (0.00 %) | 31382 (1.24 %)
| $Y_{\ell\ell\ell\ell}$ | 27639 (53.35 %) | 289490 (35.55 %) | 1500 (0.50 %) | 0 (0.00 %) | 31382 (1.24 %)
| $\#_{\ell\ell\ell\ell}$ | 27643 (53.35 %) | 289490 (35.55 %) | 1500 (0.50 %) | 0 (0.00 %) | 31382 (1.24 %)
| $b$-tagging | 5972 (11.54 %) | 4732 (0.58 %) | 0 (0.00 %) | 0 (0.00 %) | 458 (0.02 %)

Table 2: Backgrounds reduction summary in each selection for $ZH \rightarrow q\bar{q}b\bar{b}$.

From the reduction summary of Table 2, $\ell\ell\ell\ell$ four-leptonic background can be suppressed completely by number of particles cut ($N_{PFOs} < 40$) and the remaining backgrounds are $qq\bar{q}q$ and $qq$ which including $b$-quarks event after applying the $b$-tagging.

4 Results

Reconstructed Higgs mass distribution after the selection of $ZH \rightarrow q\bar{q}b\bar{b}$ is fitted with the function of Gaussian convoluted Gaussian with the exponential function assuming the background, as shown in Fig. 6. Fitted results of the reconstructed $ZH \rightarrow q\bar{q}b\bar{b}$ Higgs mass distribution are summarized in the Table 3. From the fitted results, Higgs mass ($M_H = 120$ GeV at MC) is reconstructed as $M_H = 120.79 \pm 0.089$ GeV and the measurement accuracy of cross-section to $ZH \rightarrow q\bar{q}b\bar{b}$ is obtained as $\delta \sigma/\sigma = 5.3\%$.

5 Conclusion

Simulation study of the direct reconstruction of the Higgs boson in $ZH \rightarrow q\bar{q}b\bar{b}$ four-jet mode with the Higgs mass of 120 GeV at the $\sqrt{s} = 250$ GeV and the integrated luminosity of $L = 250 fb^{-1}$ has performed for the ILD detector model considering with

<table>
<thead>
<tr>
<th>Higgs mass ($M_H = 120$ GeV at MC)</th>
<th>$M_H = 120.79$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement accuracy of $M_H$</td>
<td>$\delta M_H = 89$ (MeV)</td>
</tr>
<tr>
<td>Measurement accuracy of $\sigma(ZH \rightarrow q\bar{q}b\bar{b})$</td>
<td>$\delta \sigma/\sigma = 5.3%$</td>
</tr>
</tbody>
</table>

Table 3: Fitted results for the reconstructed Higgs mass distribution.
the $qqqq$, $\nu\ell\nu\ell\ell\ell$, $qq$ background processes. From the study, measurement accuracy of the reconstructed Higgs mass is estimated as 87 MeV and the measurement accuracy of the cross-section of $ZH \rightarrow q\bar{q}b\bar{b}$ mode is obtained as $\delta\sigma/\sigma = 5.3\%$.

Acknowledgment

I would like to thank to everyone who join the ILC physics WG subgroup [14] for useful discussion of this work and to ILD optimization group members who maintain the softwares and MC samples. This study is supported in part by the Creative Scientific Research Grant No. 18GS0202 of the Japan Society for Promotion of Science and promotion.

References

Measurement of Higgs Branching Ratio at ILC

Kohei Yoshida

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Measurement of Higgs branching ratio is necessary to investigate Higgs coupling to particle masses. Especially, it is the most important program to measure the branching ratio of $H \rightarrow b\bar{b}$ and $H \rightarrow c\bar{c}$ at the international linear collider (ILC). We have studied the measurement accuracy of Higgs branching ratio at ILC with $\sqrt{s} = 250$ GeV by using $ZH \rightarrow \nu\bar{\nu}H$ events. We obtained the Higgs branching ratio with 1.1% and 13.7% accuracy for $H \rightarrow b\bar{b}$ and $H \rightarrow c\bar{c}$, respectively.

1 Introduction

In the Higgs mechanism, Higgs coupling is proportional to a particle mass. For that reason, it is important to measure the Higgs coupling to particle masses, i.e. Higgs branching ratio, is important to confirm Higgs mechanism and distinguish the Standard Model extensions. Especially, it is the most important program to measure the branching ratio of $H \rightarrow b\bar{b}$ and $H \rightarrow c\bar{c}$ at ILC [1] with the excellent performance of the flavor tagging.

We have studied the measurement accuracy of Higgs branching ratio at ILC by using $ZH \rightarrow \nu\bar{\nu}H$ events. In this paper, we report the measurement accuracy of Higgs branching ratio of $H \rightarrow b\bar{b}$ and $H \rightarrow c\bar{c}$.

2 Simulation tools

In this study, we used common generator samples in the ILC community for $ZH$ events and standard model backgrounds, which were prepared with WITHERD at SLAC [2]. In this study, the Higgs mass was assumed to be 120 GeV. We used the center of mass energy of $\sqrt{s} = 250$ GeV and the integrated luminosity of 250 fb$^{-1}$. Here, the beam energy spread was assumed as 0.3% for the electron and positron beam. The beam polarization was set to 80% left-handed for the electron beam and 30% right-handed for the positron beam.

The signal and background events were simulated by the full simulator, Mokka[3], where the detector model is ILD_00 was implemented as the detector model [4]. Hadronization was done by Pythia6.409, in which the Higgs branching ratio is defined as shown in Table 2 for the Higgs mass of 120 GeV. After the detector simulation, the reconstruction was performed by Marlin[7].

3 Event selection

In this study, the final states of four fermions are considered as background events, where they are classified into 6 groups, $\nu\nuqq$, $qqqq$, $\nu\ellqq$, $\ell\ellqq$, $\nu\nu\ell\ell$ and $\ell\ell\ell\ell$. The signal and
Table 1: The Higgs branching ratio defined in Pythia6.409.

<table>
<thead>
<tr>
<th>Branching ratio</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$b\bar{b}$</td>
<td>65.7%</td>
</tr>
<tr>
<td>$W^+W^-$</td>
<td>15.0%</td>
</tr>
<tr>
<td>$\tau^+\tau^-$</td>
<td>8.0%</td>
</tr>
<tr>
<td>$gg$</td>
<td>5.5%</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

Figure 1: Distribution of the reconstructed di-jet mass for signal (left) and background (right).

background events are summarized in Table 2. All events are reconstructed as 2-jet events by Durham jet algorithm [6]. By using the reconstructed 2 jets, the di-jet mass ($M_{jj}$) was reconstructed as shown in Fig. 1. Since the background events dominate in the Higgs mass region, the selection cuts were investigated.

At first, we studied the distribution of missing mass ($M_{\text{miss}}$). Since a Z boson decays into the neutrino pair in $ZH \rightarrow \nu\bar{\nu}H$ events, the missing mass should be consistent with Z boson mass (91.2 GeV). We, therefore, selected the events with $80 \text{ GeV} < M_{\text{miss}} < 140 \text{ GeV}$. Applying this cut, $\ell\ell\ell\ell$, $\ell\ellqq$, and $qqqq$ events were suppressed. Then, we required that the reconstructed di-jet particles have the transverse momentum ($p_T$) from 20 to 70 GeV and longitudinal momentum ($p_L$) below 60 GeV. We selected the number of charged tracks ($N_{\text{tracks}}$) above 10 to remove $W^+W^- \rightarrow l^+\nu l^-\bar{\nu}$ events.

After the selection cuts so far, $\tau\nu_qqq$ events become the main background. The maximum track momentum in each events ($p_{\text{max}}$) were investigated since the charged tracks from $\tau$ have relatively higher momentum than those from $b$-jets. We selected the events with $p_{\text{max}} < 30 \text{ GeV}$. $Y_+$ is the threshold $y$-value to reconstruct 2-jet as 3-jets. Since the final state of $ZH \rightarrow \nu\bar{\nu}q\bar{q}$ and $\tau\nu_qqq$ is 2 and 3 bodies, respectively, $Y_+$ for $ZH \rightarrow \nu\bar{\nu}q\bar{q}$ events has smaller value than $\tau\nu_qqq$ events. On the other hand, $Y_-$, the $y$-value to reconstruct 2-jet as 1-jets, has larger value for $ZH \rightarrow \nu\bar{\nu}q\bar{q}$ events than $\nu\bar{\nu}qq$ and $l\nuqq$ because $\beta$ of $W$ and
Figure 2: Distribution of the reconstructed di-jet mass after the selection cuts for signal (left) and background (right).

<table>
<thead>
<tr>
<th></th>
<th>cross section (fb)</th>
<th>No. of events</th>
<th>No. of events after all cuts</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ZH \rightarrow \nu\bar{\nu}H$</td>
<td>77.4</td>
<td>19,360</td>
<td>7,384</td>
<td>38.14</td>
</tr>
<tr>
<td>$ZH \rightarrow \nu\bar{\nu}b\bar{b}$</td>
<td>52.2</td>
<td>13,062</td>
<td>6,434</td>
<td>49.26</td>
</tr>
<tr>
<td>$ZH \rightarrow \nu\bar{\nu}c\bar{c}$</td>
<td>2.83</td>
<td>707</td>
<td>318</td>
<td>44.98</td>
</tr>
<tr>
<td>$\nu_{\tau}qq$</td>
<td>5843.2</td>
<td>1,460,797</td>
<td>851</td>
<td>0.06</td>
</tr>
<tr>
<td>$\nu_{\tau}\mu q$</td>
<td>5999.3</td>
<td>1,327,332</td>
<td>2,288</td>
<td>0.17</td>
</tr>
<tr>
<td>$\nu_{\tau}\tau q$</td>
<td>5084.2</td>
<td>1,326,066</td>
<td>24,979</td>
<td>1.88</td>
</tr>
<tr>
<td>$\nu_{\tau}qq$</td>
<td>599.9</td>
<td>149,979</td>
<td>21,653</td>
<td>14.44</td>
</tr>
<tr>
<td>Other</td>
<td>25291</td>
<td>6,322,758</td>
<td>355</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 2: The number of events for signal and background, and the selection efficiencies after the selection cuts.

$Z$ bosons from decay of $WW$ and $ZZ$ events is larger than Higgs from $ZH \rightarrow \nu\bar{\nu}qq$. We, therefore, selected $Y^+ < 0.02$ and $0.2 < Y^- < 0.8$.

Finally, the signal region was set to be $100 \text{ GeV} < M_{jj} < 130 \text{ GeV}$. After all the selection cuts, $\nu\nuqq$ events from $WW$ and $ZZ$ events were reduced as shown in Fig. 2. The number of signal and background events and the selection efficiencies after the selection cut was summarized in Table 2.

### 4 Measurement of Higgs branching ratio

To measure the Higgs branching ratio of $H \rightarrow b\bar{b}$ and $H \rightarrow c\bar{c}$, the template fitting was performed [7]. For the template fitting, 3-dimensional histogram for the $b$-, $c$-, and $bc$-likeness was used, which are obtained as output values from LCFIVertex package [8]. In LCFIVertex, neural-net training was done by using $Z \rightarrow qq$ events at $Z$-pole (91.2 GeV) to derive $b$- and $c$-likeness. $bc$-likeness is $c$-likeness whose neural-net training is done by using
only $Z \rightarrow bb$ events as background. The each flavor-likeness for two jets are combined as,

$$X\text{-likeness} = \frac{X_1 \cdot X_2}{X_1 \cdot X_2 + (1-X_1)(1-X_2)}$$  \hspace{1cm} (4.2)

where $X = b, c$ or $bc$. $X_1$ and $X_2$ are the flavor-likeness of the first and second jet, respectively.

The template sample is separated into $H\rightarrow b\bar{b}$, $H\rightarrow c\bar{c}$, $H\rightarrow other$, and Standard Model background events. Figure 3 shows the 2-dimensional template histogram for $b$-likeness and $c$-likeness. In $H \rightarrow other$ sample, $H \rightarrow gg$ and $H \rightarrow W^-W^+$ events are dominant. Since the both distributions are identical, they are treated in one template sample.

In the template fitting, the fitting parameters ($r_{bb}$, $r_{cc}$, $r_{oth}$, and $r_{bkg}$) were adjusted to minimize the following $\chi^2$ function:

$$\chi^2 = \sum_{i=1}^{n_i} \sum_{j=1}^{n_j} \sum_{k=1}^{n_k} \left( \frac{N_{ijk}^{data} - \sum_s r_s \left( \frac{N_s^{ZH}}{N_{i,j,k}} \right) N_{i,j,k}^{s} - r_{bkg} N_{i,j,k}^{bkg}}{N_{i,j,k}^{all}} \right)^2,$$  \hspace{1cm} (4.3)

where $s$ shows $b\bar{b}$, $c\bar{c}$ and $other$. $r_{bb}$, $r_{cc}$, $r_{oth}$ are the fraction of $H \rightarrow b\bar{b}$, $H \rightarrow c\bar{c}$, $H \rightarrow other$ in $ZH$ events after the selection cut, where we set $r_{other} = 1 - r_{cc} - r_{bb}$. $r_{bkg}$ is the normalization factor of the Standard Model background. $N_{ijk}^{s}$ are the number of expected events in $(i, j, k)$ bin of the 3-dimensional histogram.

To estimate the reconstruction accuracy of $r_{bb}$ and $r_{cc}$, the fitting was done for 1,000 times by using Toy-MC. Figure 4 shows the distributions of $r_{bb}$ and $r_{cc}$ obtained by the fitting. $r_{bb}$ and $r_{cc}$ were determined to be $0.87 \pm 0.01$ and $0.046 \pm 0.009$, respectively. These mean values are consistent with the true $r_{bb}$ (0.87) and $r_{cc}$ (0.046). From the result, if the
The cross section of $e^+e^- \rightarrow ZH$ can be determined with other measurements like a measurement of the Higgs recoil mass [9] and the selection efficiencies of $ZH \rightarrow \nu\bar{\nu}bb$ and $ZH \rightarrow \nu\bar{\nu}cc$ are known, Higgs branching ratio of $H \rightarrow bb$ and $H \rightarrow cc$ can be measured with accuracy of 1.1% and 13.7%, respectively.

To evaluate the influence of Standard Model background on determination of the Higgs branching ratio, we performed the template fitting, fixing $r_{bb}$ and $r_{cc}$ to 1. $r_{bb}$ and $r_{cc}$ were determined to be 0.87 ± 0.01 and 0.046 ± 0.006, respectively. It corresponds to the measurement accuracy of 1.1% and 13.6% for $r_{bb}$ and $r_{cc}$, respectively. From this result, it was found that the fluctuation of the background normalization has only negligible effects on the measurement of Higgs branching ratio.

Without any other measurement, we can measure the relative branching ratio between $H \rightarrow bb$ and $H \rightarrow cc$ by analysis of only $ZH \rightarrow \nu\bar{\nu}H$ events as follows:

$$\frac{BR(H \rightarrow cc)}{BR(H \rightarrow bb)} = \frac{r_{cc}/\epsilon_{cc}}{r_{bb}/\epsilon_{bb}},$$

where $\epsilon_{bb}$ and $\epsilon_{cc}$ are the selection efficiencies of $H \rightarrow bb$ and $H \rightarrow cc$ events as shown in Table 2. The relative branching ratio of 0.054 ± 0.007 was obtained for the template fitting with free and fixed $r_{bbk}$, which corresponds to 13.3% accuracy. The measurement accuracy for Higgs branching ratio is summarized in Table 3.

## 5 Conclusion

Measurement of Higgs branching ratio is necessary to investigate Higgs coupling to particle masses. Especially, it is the most important program to measure the branching ratio of $H \rightarrow bb$ and $H \rightarrow cc$ at ILC. We have studied the measurement accuracy of Higgs branching ratio at ILC with $\sqrt{s} = 250$ GeV by using $ZH \rightarrow \nu\bar{\nu}H$ events. For Higgs mass of 120 GeV and the integrated luminosity of 250 fb$^{-1}$, we obtained the measurement accuracy of 1.1% and 13.7% for $H \rightarrow bb$ and $H \rightarrow cc$, respectively, assuming that the cross section of $ZH$ is determined by other measurements. Finally, the relative branching ratio between $H \rightarrow bb$ and $H \rightarrow cc$ was obtained with 13.3% accuracy.

<table>
<thead>
<tr>
<th>$H \rightarrow bb$</th>
<th>$r_{bbk}$ free</th>
<th>$r_{bbk} = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BR(H \rightarrow bb)$</td>
<td>1.1%</td>
<td>1.1%</td>
</tr>
<tr>
<td>$BR(H \rightarrow cc)$</td>
<td>13.7%</td>
<td>13.6%</td>
</tr>
<tr>
<td>$BR(H \rightarrow cc/H \rightarrow bb)$</td>
<td>13.3%</td>
<td>13.3%</td>
</tr>
</tbody>
</table>

Table 3: The measurement accuracy of Higgs branching ratio. For measurement accuracy of $BR(H \rightarrow bb)$ and $BR(H \rightarrow cc)$, it is assumed that the cross section of $ZH$ is determined by other measurements.
6 Acknowledgments

The authors would like to thank all the members of the ILC physics subgroup [16] for useful discussions and ILD optimization working group. This study is supported in part by the Creative Scientific Research Grant No. 18GS0202 of the Japan Society for Promotion of Science.

References

Analysis of Higgs Self-coupling with ZHH at ILC

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Measurement of the cross-section of \(\text{e}^{+}\text{e}^{-} \rightarrow ZHH\) offers the information of the trilinear Higgs self-coupling, which is important to confirm the mechanism of the electro-weak symmetry breaking. Since there is huge background in the signal region, background rejection is key point to identify ZHH events. In this paper, we study the possibility to observe the ZHH events at ILC by using \(ZHH \rightarrow \nu\bar{\nu}HH/q\bar{q}HH\) events.

1 Introduction

In the standard model, particle masses are generated through the Higgs mechanism. This mechanism relies on a Higgs potential, \(V(\Phi) = \lambda(\Phi^2 - \frac{1}{2}v^2)^2\), where \(\phi\) is an iso-doublet scalar field, and \(v\) is the vacuum expectation value of its neutral component (\(v \approx 246\) GeV). Determination of the Higgs boson mass, which satisfies \(m_H^2 = 2\lambda v^2\) at tree level in the standard model, will provide an indirect information about the Higgs potential and its self-coupling, \(\lambda_{HHH}\). The measurement of the trilinear self-coupling, \(\lambda_{HHH} = 6\lambda v\), offers an independent determination of the Higgs potential shape and the most decisive test of the mechanism of the electro-weak symmetry breaking.

\(\lambda_{HHH}\) can be extracted from the measurement of the cross-section for the Higgs-strahlung process \((\sigma_{ZH HH}^{ZHH}), \text{e}^{+}\text{e}^{-} \rightarrow ZHH\). For a Higgs mass of 120 GeV, the \(W\) fusion process is negligible at \(\sqrt{s} = 500\) GeV. Figure 1 shows the relevant Feynman diagrams for this process. The information of \(\lambda_{HHH}\) is included in the diagram of Fig. 1(a), and the relation between the cross-section of \(ZH HH\) and \(\lambda_{HHH}\) is characterized by \(\frac{\Delta \lambda_{HHH}}{\lambda_{HHH}} \sim 1.75\frac{\Delta \sigma_{ZH HH}}{\sigma_{ZH HH}}\), where \(\Delta \lambda_{HHH}\) and \(\Delta \sigma_{ZH HH}\) are measurement accuracy of \(\lambda_{HHH}\) and \(\sigma_{ZH HH}\), respectively \([1]\). For that reason, precise measurement of the cross-section for the \(ZH HH\) production is essential to determination of the strength of the trilinear Higgs self-coupling.

We have studied the feasibility for observation of \(ZH HH\) events at the ILC. For the analysis, we assumed a Higgs mass of 120 GeV, \(\sqrt{s} = 500\) GeV, and an integrated luminosity of 2\(\text{ab}^{-1}\). The final states of the \(ZH HH\) production can be categorized into 3 types, depending on the decay modes of \(Z\): \(ZH HH \rightarrow q\bar{q}HH\) (135.2 \(\text{ab}^{-1}\)), \(ZH HH \rightarrow \nu\bar{\nu}HH\) (38.8 \(\text{ab}^{-1}\)), and \(ZH HH \rightarrow \ell\bar{\ell}HH\) (19.8 \(\text{ab}^{-1}\)), where the cross-sections were calculated without the beam polarization, initial-state radiation, and beamstrahlung. In this paper, we report status of the analysis with \(ZH HH \rightarrow \nu\bar{\nu}HH/q\bar{q}HH\) events.

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2 Simulation tools

We have used MadGraph [10] to generate $ZHH \rightarrow \nu \bar{\nu}HH/q\bar{q}HH$ and $t\bar{t}b\bar{t}b$ events, where top quarks in $t\bar{t}b\bar{t}b$ events are decayed by using DECAY package in MadGraph. $ZZ \rightarrow b\bar{b}b\bar{b}$, $tt$, and $ZH$ events have been generated by Physsim [11]. In this study, the beam polarization, initial-state radiation, and beamstrahlung have not been included in the event generations. We also have ignored the finite crossing angle between the electron and positron beams. In both event generations, helicity amplitudes were calculated using the HELAS library [12], which allows us to deal with the effect of gauge boson polarizations properly. Phase space integration and the generation of parton 4-momenta have been performed by BASES/SPRING [10]. Parton showering and hadronization have been carried out by using PYTHIA6.4 [13], where final-state tau leptons are decayed by TAUOLA [14] in order to handle their polarizations correctly.

The generated Monte Carlo events have been passed to a detector simulator called JSFQuickSimulator, which implements the GLD geometry and other detector-performance related parameters [15]. Figure 2 shows a typical event display of $ZHH \rightarrow \nu \bar{\nu}HH$.

3 Analysis

3.a $ZHH \rightarrow \nu \bar{\nu}HH$

For the Higgs mass of 120 GeV, the Higgs boson mainly decays into $bb$ (76% branching ratio in MadGraph). Therefore, we concentrated on $ZHH \rightarrow \nu \bar{\nu}b\bar{b}b\bar{b}$ from $\nu \bar{\nu}HH$ events. As background events, we considered $ZZ \rightarrow b\bar{b}b\bar{b}$ (9.05 fb), $tt$ (583.6 fb), $ZH$ (62.1 fb), and $t\bar{t}b\bar{t}b$ (1.2 fb). They have much larger cross-sections than $ZHH$, necessitating powerful background rejection.

The clusters in the calorimeters are combined to form a jet if the two clusters satisfy $y_{ij} < y_{\text{cut}}$, where $y_{ij}$ is $y$-value of the two clusters. All events are forced to have four jets by adjusting $y_{\text{cut}}$. Then, mass of the Higgs boson was reconstructed to identify $\nu \bar{\nu}HH$ events by minimizing $\chi^2$ value defined as

$$\chi^2 = \frac{(\text{rec}~M_{H1} - \text{true}~M_{H})^2}{\sigma_{H1}^2} + \frac{(\text{rec}~M_{H2} - \text{true}~M_{H})^2}{\sigma_{H2}^2},$$

(3.5)

where $\text{rec}~M_{H1,2}$, $\text{true}~M_{H1,2}$, and $\sigma_{H1,2}$ are the reconstructed Higgs mass, the true Higgs mass (120 GeV), and the Higgs mass resolution, respectively. $\sigma_{H1,2}$ was evaluated for each reconstructed Higgs boson by using $31%/\sqrt{E_{\text{jet}}}$, where $E_{\text{jet}}$ is the jet energy. Figure 3 shows the distri-
bution of the sum of the two reconstructed Higgs boson masses for $\nu\bar{\nu}HH$ and background events. With no selection cuts, the signal is swamped in huge number of background events.

To identify the signal events from the backgrounds, we applied the following selection cuts. We required $\chi^2 < 20$ and $95 \text{ GeV} < M_{H_{1,2}} < 125 \text{ GeV}$ to select events, for which the Higgs bosons could be well reconstructed. Since Higgs mainly decay into a b-quark pair, the reconstructed mass distribution have a tail in lower mass region due to missing energy by neutrinos from decay processes of the b-quark. For that reason, the mass cut is applied asymmetrically against the Higgs mass. Then, since a $Z$ boson is missing in $\nu\bar{\nu}HH$ events, we set the selection cut on the missing mass ($\text{miss } M$): $90 \text{ GeV} < \text{miss } M < 170 \text{ GeV}$.

The angular distribution of the particles reconstructed as the Higgs bosons has a peak at $\cos \theta = \pm 1$ for $ZZ$ events whereas the distribution becomes more uniform in $\nu\bar{\nu}HH$ events. We applied the angular cut of $|\cos \theta_{H_{1,2}}| < 0.9$ to reject these $ZZ$ events.

The 4-jet events from $ZH$ events have small missing transverse momentum ($\text{miss } P_T$), which contaminate in the signal region. For that reason, we required $\text{miss } P_T$ above 50 GeV.

After the selection cuts so far, the dominant background was $tt$ events. The leptonic decay mode of $W$ from $t \rightarrow bW$ can be rejected by indentifying isolated charged leptons. We define the energy deposit within 20 degree around a track as $E_{20}$. The isolated lepton track was defined to be a track with $10 \text{ GeV} < E_{20} < 211 E_{\text{trk}} - 1.8 \text{ GeV}$, where $E_{\text{trk}}$ is energy of the lepton track. We required the number of isolated lepton tracks ($N_{\text{lepton}}$) to be zero.

Finally, the flavor tagging was applied. We identified a jet as a $b$-jet, when it has 2 tracks with 3-sigma separation from the interaction point. Figure 4 shows the distribution of the number of jets tagged as $b$-jets after the selection cuts ($N_{b-\text{tag}}$). Since the Higgs boson decays into $bb$ with a 76% branching ratio, $\nu\bar{\nu}HH$ events have a peak at $N_{b-\text{tag}} = 4$, whereas $tt$ events have a peak at 2. To reject the $tt$ events effectively, we selected events with $N_{b-\text{tag}} = 4$.

Figure 5 shows the distribution of the sum of the two reconstructed Higgs masses for $ZHH \rightarrow \nu\bar{\nu}HH$ after all the selection cuts. We summarize the reduction rate by each selection cut in Table 1. Finally, we obtained 7.3 events for $\nu\bar{\nu}HH$ and 69.2 events for backgrounds. This result corresponds to a signal significance of 0.8 ($= 7.3 / \sqrt{7.3 + 69.2}$). For observation of the $ZHH$ production, further background rejection, especially $tt$ events, is necessary.
For the analysis of $qqHH$, all the events are reconstructed as 6-jet events, adjusting the $y$-value. Here, we considered $tt$ and $tbtb$ events as background events. The masses of the Higgs and Z boson were reconstructed by minimizing $\chi^2$ value defined as

$$\chi^2 = \frac{(\text{rec} M_{H1} - \text{true} M_H)^2}{\sigma^2_{H1}} + \frac{(\text{rec} M_{H2} - \text{true} M_H)^2}{\sigma^2_{H2}} + \frac{(\text{rec} M_Z - \text{true} M_Z)^2}{\sigma^2_Z},$$

(3.6)

where $\text{rec} M_{H1,2}$, $\text{rec} M_Z$, $\text{true} M_{H1,2}$, and $\text{true} M_Z$ are the reconstructed Higgs and Z mass and the true Higgs and Z mass, respectively. $\sigma_{H1,2}$ and $\sigma_Z$ are the Higgs and Z mass resolution, respectively, which are defined in Sec 3.a.

We required $\chi^2 < 20$, $90 \text{ GeV} < M_{H1,2} < 150 \text{ GeV}$, and $60 \text{ GeV} < M_Z < 120 \text{ GeV}$ to select events, for which the Higgs and Z bosons could be well reconstructed. Then, the isolated lepton track was searched to indentify the lepton tracks from decay of top quarks in $tt$ and $tbtb$ events. We required the number of isolated lepton tracks ($N_{\text{lepton}}$) to be zero. Since the missing energy of the signal is smaller than $tt$ and $tbtb$ events, $\text{miss} E < 70 \text{ GeV}$ was required. Finally, we applied the b-tagging whose requirement is the same as the analysis for $\nu\bar{\nu}HH$ events. Here, we required that all the jets are b-jets, $N_{b-tag} = 6$.

After all the cut, we obtained 4.6 events for $qqHH$ and 0.6 events for the background. That corresponds to the signal significance of 2.0 ($= 4.6/\sqrt{4.6 + 0.6}$). The number of the events at each selection cut is summarized in Table 2.

### Summary

20
$ZHH \to \nu\bar{\nu}HH/qqHH$ processes were studied to investigate the possibility of the trilinear Higgs self-coupling at the ILC. In this study, we assumed the Higgs boson mass of 120 GeV, $\sqrt{s} = 500$ GeV, and the integrated luminosity of 2 $ab^{-1}$. After the selection cuts, the signal significance of 0.8 and 2.0 was obtained for $\nu\bar{\nu}HH$ and $qqHH$ events, respectively. To extract the information of $\lambda_{HHH}$, we must improve the flavor tagging to reject background events effectively.

5 Acknowledgments

The authors would like to thank all the members of the ILC physics subgroup [16] for useful discussions. This study is supported in part by the Creative Scientific Research Grant No. 18GS0202 of the Japan Society for Promotion of Science, and Dean’s Grant for Exploratory Research in Graduate School of Science of Tohoku University.

References

Study of Higgs Self-coupling at ILC

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In this Analysis we investigated the possibility of the measurement of Higgs self-coupling at ILC through the process $e^+ + e^- \rightarrow ZHH$ using fast simulation data. So far two combinations of decay modes: $Z \rightarrow q\bar{q}, H \rightarrow b\bar{b}, H \rightarrow WW^*$ and $Z \rightarrow l\bar{l}, H \rightarrow b\bar{b}, H \rightarrow b\bar{b}$ were studied. Our preliminary results show that it is very challenging to suppress the huge standard model backgrounds effectively.

1 Introduction

It is well accepted that the discovery of a Higgs-like boson is not enough to fully understand the mechanism of electro-weak symmetry breaking (EWSB) and mass generation. The Higgs self-coupling can be a non-trivial probe of the Higgs potential and probably the most decisive test of the EWSB mechanism. In the standard model framework, the Higgs potential $V(\Phi) = \lambda (\Phi^2 - \frac{1}{2}v^2)^2$, where $\Phi$ is an isodoublet scalar field and $v \approx 246$ GeV is the vacuum expectation value of its neutral component, is uniquely determined by the self-coupling $\lambda$. Obviously, determination of the Higgs mass, which satisfies $m_H^2 = 2\lambda v^2$ at tree level, can provide an indirect information about the self-coupling. The measurement of the trilinear self-coupling $\lambda_{HHH} = 6\lambda v$ offers direct independent determination of the Higgs potential shape, which is the topic of this analysis.

The trilinear Higgs self-coupling can be measured at ILC through the two leading processes: double Higgs-strahlung [1, 2] and WW fusion [2, 3, 4, 5, 6], which are shown in Fig.1. The former is expected to dominate around the center of mass energy of 500 GeV and the latter to take it over at higher energy. In this analysis we focus on the double Higgs-strahlung process $e^+ + e^- \rightarrow ZHH$ for the Higgs mass of $M_H = 120$ GeV and the center of mass energy of $\sqrt{s} = 500$ GeV with the integrated luminosity $2 \text{ ab}^{-1}$.

Depending on the different decay modes of $Z$ and $H$, there are different methods to identify the signal events. Table I shows several most promising combinations of decay modes for $e^+ + e^- \rightarrow ZHH$ and their branching ratios. Modes 1 and 3 are studied in Ref. [7]. We study the other two modes in this analysis.

2 Simulation

The simulations of signal events ($e^+ + e^- \rightarrow ZHH$) and possible background events ($e^+ + e^- \rightarrow t\bar{t}, ZZZ, W^+W^-Z, ZZ, ZH$) were done by Physsim [11]. In Physsim the helicity amplitudes are calculated by the HELAS library [12]. The phase space integration and the
Figure 1: Leading processes involving trilinear Higgs self-coupling: (Left) Double Higgsstrahlung; (Right) WW fusion.

Table 1: Most promising modes for $e^+ + e^- \rightarrow ZHH$

<table>
<thead>
<tr>
<th>Decay Mode</th>
<th>$Z \rightarrow$</th>
<th>$H_1 \rightarrow$</th>
<th>$H_2 \rightarrow$</th>
<th>Branching Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$q\bar{q}$</td>
<td>$bb$</td>
<td>$bb$</td>
<td>34%</td>
</tr>
<tr>
<td>2</td>
<td>$q\bar{q}$</td>
<td>$bb$</td>
<td>$WW^*$</td>
<td>14%</td>
</tr>
<tr>
<td>3</td>
<td>$\nu\bar{\nu}$</td>
<td>$bb$</td>
<td>$bb$</td>
<td>9.8%</td>
</tr>
<tr>
<td>4</td>
<td>$l\bar{l}$</td>
<td>$bb$</td>
<td>$bb$</td>
<td>4.9%</td>
</tr>
</tbody>
</table>

four momenta generation are performed by BASES/SPRING [10]. Parton showering and hadronization are carried out by PYTHIA6.4 [13], where final-state $\tau$ leptons are decayed by TAUOLA [14] in order to handle their polarizations correctly. The detector simulation was done by JSFQuickSimulator, which implements the GLD geometry and other detector-performance related parameters [13].

It is worth mention of that the simulations were performed without the beam polarization but with the initial-state radiation, beam width and beamstrahlung. Then the cross sections used here are shown in Table II. An integrated luminosity of 2 ab$^{-1}$ is assumed in this analysis.

Table 2: Cross sections of the related processes

<table>
<thead>
<tr>
<th>Process</th>
<th>Cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+ + e^- \rightarrow ZHH$</td>
<td>152 ab</td>
</tr>
<tr>
<td>$e^+ + e^- \rightarrow H$</td>
<td>530 fb</td>
</tr>
<tr>
<td>$e^+ + e^- \rightarrow ZZZ$</td>
<td>800 ab</td>
</tr>
<tr>
<td>$e^+ + e^- \rightarrow W^+W^-Z$</td>
<td>36 fb</td>
</tr>
<tr>
<td>$e^+ + e^- \rightarrow ZZ$</td>
<td>515 fb</td>
</tr>
<tr>
<td>$e^+ + e^- \rightarrow ZH$</td>
<td>70 fb</td>
</tr>
</tbody>
</table>
3 Analysis

3.a $e^+ + e^- \rightarrow ZHH \rightarrow (q\bar{q})(b\bar{b})(WW^*)$

The full hadronic decays of $W$ and $W^*$ were investigated. In this mode the final state of a candidate signal event contains of 8 jets, two of which are $b$ jets. To select the signal events, first we find all the good tracks and require the number of tracks be greater than 20. We then try to combine tracks with a small $Y$ value to a current jet cluster, where the $Y$ value between two momenta $p_1, p_2$ is defined as

$$Y(p_1, p_2) = \frac{M^2(p_1, p_2)}{E_{vis}}$$

with $M(p_1, p_2)$ being the invariant mass of $p_1, p_2$ and $E_{vis}$ the total visible energy. We continue the jet clustering until there are 7 jets left, because the two jets coming from $W^*$ are very close to each other which means the $Y$-value between them is very small, thereby being likely to be clustered as one jet. At this point we calculate the $Y$ values for all the pairs from these 7 jets and choose the minimum denoted by $Y_{cut}$. The $Y_{cut}$ distributions of signal events and background events (here we consider the $t\bar{t}$ events as background) are shown in Fig.2. The 7 jets are combined by minimizing the $\chi^2$ which is defined as

$$\chi^2 = \frac{(M(b, \bar{b}) - M_H)^2}{\sigma^2_{H_1}} + \frac{(M(W, W^*) - M_H)^2}{\sigma^2_{H_2}} + \frac{(M(q, \bar{q}) - M_Z)^2}{\sigma^2_{Z}} + \frac{(M(q, q') - M_W)^2}{\sigma^2_{W}}$$

where $M(q, q')$ is the reconstructed invariant mass of jet $q$ and jet $q'$, $M_H$, $M_Z$ and $M_W$ are the mass of $H$, $Z$ and $W$, respectively, and $\sigma_{H_1}, \sigma_{H_2}, \sigma_Z$ and $\sigma_W$ are their corresponding mass resolutions.

In order to further suppress the background, we require that $\chi^2 < 20, 90 \text{GeV} < M(H_1) < 130 \text{GeV}, 110 \text{GeV} < M(H_2) < 150 \text{GeV}, 70 \text{GeV} < M(Z) < 110 \text{GeV}, Y_{cut} > 0.0076$, where the asymmetry of two Higgs mass requirement is due to their different decay modes. The preliminary result of this cut-based analysis is shown in Table III. Though we can still add other cuts like $b$ tagging requirement, the signal events will become too few to be observed. It seems very challenging to reject the huge $t\bar{t}$ background in this mode.

We are going to investigate the semi-lepton decays of $W$ and $W^*$.

<table>
<thead>
<tr>
<th>Process</th>
<th>$ZHH \rightarrow (q\bar{q})(b\bar{b})(WW^*)$</th>
<th>$t\bar{t}$</th>
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<td>1062000</td>
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<tr>
<td>pre-selection</td>
<td>12.6</td>
<td>483949</td>
</tr>
<tr>
<td>$\chi^2 &lt; 20$</td>
<td>5.2</td>
<td>65144</td>
</tr>
<tr>
<td>$90 \text{GeV} &lt; M_H &lt; 130 \text{GeV}$</td>
<td>5.1</td>
<td>63157</td>
</tr>
<tr>
<td>$110 \text{GeV} &lt; M_{H_2} &lt; 150 \text{GeV}$</td>
<td>3.6</td>
<td>36670</td>
</tr>
<tr>
<td>$90 \text{GeV} &lt; M_Z &lt; 110 \text{GeV}$</td>
<td>3.5</td>
<td>34359</td>
</tr>
<tr>
<td>$Y_{cut} &gt; 0.005$</td>
<td>2.3</td>
<td>8454</td>
</tr>
<tr>
<td>$Y_{cut} &gt; 0.0076$</td>
<td>1.1</td>
<td>2644</td>
</tr>
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</table>
3.b $e^+ + e^- \rightarrow ZHH \rightarrow (l\bar{l})(b\bar{b})(b\bar{b})$

In this mode a candidate signal event contains two leptons and four $b$ jets, where we only consider the $Z$ boson decaying into $e^+ e^-$ and $\mu^+ \mu^-$. The two isolated charged lepton tracks are required to have an energy greater than 20 GeV and the energy deposited in the cone of $20^\circ$ around each lepton track be less than 20 GeV. We then force the other tracks to four jets and combine the four jets by minimizing the $\chi^2$ defined by

$$
\chi^2 = \frac{(M(b, \bar{b}) - M_H)^2}{\sigma_H^2} + \frac{(M(b, \bar{b}) - M_H)^2}{\sigma_H^2} + \frac{(M(l, \bar{l}) - M_Z)^2}{\sigma_Z^2}.
$$

Table IV shows that 15.4 signal events survived the pre-selection but with thousands times more background events left. In order to reject the background effectively, while keeping a reasonable signal efficiency, we used the neural net method MLP in the TMVA package [14] which gives some useful classifiers. Here we mainly consider the five kinds of backgrounds that are shown in Table IV. First we separately do the neural net analysis between the signal and each of the five kinds of backgrounds. For each event we can get five classifiers to separate signal and backgrounds. Figure 3 histograms the classifiers obtained by the MLP method for the signal and $t\bar{t}$ samples.

We then add some more cuts on the five classifiers denoted by $mva_{tt}$, $mva_{zzz}$, $mva_{wwz}$, $mva_{zz}$ and $mva_{zh}$ as shown in Table IV. We further impose cuts on the $Z$ mass, $Y_{cut}$, and require $b$ tagging, which is based on the number of tracks with $2.5\sigma$ separation from the interaction point. Our preliminary result is listed in Table IV. The final cut is applied with the neural net for the signal and the $ZZZ$ background after all of the above cuts. We end up with 3...
Figure 3: The classifier obtained by neural net training for signal and $t\bar{t}$ background.

signal events with 0.82 ZZZ events left, while four backgrounds are eliminated. The result shows that the ZHH events can be observed in this mode with the significance $\sqrt{S+B} \sim 1.5\sigma$.

4 Summary

The two modes, $e^+ e^- \rightarrow ZHH \rightarrow (q\bar{q})(b\bar{b})(WW^*)$ and $e^+ e^- \rightarrow ZHH \rightarrow (l\bar{l})(b\bar{b})(b\bar{b})$, were investigated for the purpose of the measurement of the trilinear Higgs self-coupling at ILC for $M_H = 120$ GeV, $\sqrt{s} = 500$ GeV and the integrated luminosity of 2 ab$^{-1}$. The former mode is very difficult to use for the signal observation, while the latter mode can be useful to observe the self-coupling.

5 Acknowledgments

We would like to thanks all the members of the ILC physics subgroup [15] for useful discussions. This study is supported in part by KEK, Center of High Energy Physics, Tsinghua University and the JSPS Core University Program.

References

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Feasibility study of the forward-backward asymmetry of the $e^+e^- \rightarrow t\bar{t}$ process in all-hadronic decay modes at $\sqrt{s} = 500$ GeV with the ILD detector

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We have studied the measurement accuracy of the forward-backward asymmetry of the $e^+e^- \rightarrow t\bar{t}$ process in the 6-jet mode at $\sqrt{s} = 500$ GeV with the ILD detector. In the analysis the vertex charges of $b$-jets were used to distinguish $t$ from $\bar{t}$ in each event. The distribution of the cosine of the reconstructed polar angle of so identified $t$ or $\bar{t}$ showed a clear forward-backward asymmetry. After the correction for charge misidentification the forward-backward asymmetry was determined to be $A_{tFB} = 0.334 \pm 0.0079$ for 500 fb$^{-1}$ with the beam polarization combination of $P(e^+, e^-) = (+30\%, -80\%)$, demonstrating a very good statistical accuracy ($\sim 2\%$) even in the 6-jet mode.

1 Introduction

The forward-backward asymmetry of the $e^+e^- \rightarrow t\bar{t}$ process is sensitive to the $ttZ$ and $tt\gamma$ couplings and serves as a probe for new physics, which may appear as anomaly in these couplings. Needless to say we have to distinguish $t$ from $\bar{t}$ in each event in order to measure the forward-backward asymmetry, $A_{tFB}$. In the lepton+4-jet mode, it is straightforward because the lepton charge tells the charge of the leptonically decayed $W$, and hence identifies its parent to be either $t$ or $\bar{t}$. In the 6-jet mode, however, we need some other way to separate $t$ and $\bar{t}$, which is non-trivial. Nevertheless it is worth investigating the feasibility of $A_{tFB}$ measurement in the 6-jet mode, since it has a major branching fraction of 46%. In addition, the kinematical fit works better in the 6-jet mode than in the lepton+4-jet mode, where a large energy is taken away by the neutrino from the leptonically decayed $W$. The 6-jet mode might, hence, be advantageous when the influence of beamstrahlung is significant.

In this paper, we use the vertex charges of $b$ jets to distinguish $t$ from $\bar{t}$ in each event. The measurement of the vertex charges requires a high performance detector system as well as a sophisticated reconstruction algorithm. The forward-backward asymmetry in the 6-jet mode is therefore a very good measure for the overall performance of a detector system, hence being included as one of the ILC LoI benchmarks [1]. The benchmark conditions are the center of mass energy of $\sqrt{s} = 500$ GeV, an integrated luminosity of 500 fb$^{-1}$, and a beam polarization combination of $P(e^+, e^-) = (+30\%, -80\%)$.

The ILD [2] is equipped with an unprecedentedly excellent vertex detector, which allows efficient $b$-jet charge identification with the vertex charge. The vertex charges of $b$ jets were

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reconstructed by the LCFIVertex algorithm [3].

This paper is organized as follows. After we briefly describe our analysis framework in section 2, we move on to the vertex charge reconstruction and show how well we can identify each jet as $b$ or $\bar{b}$ in section 3. We then apply this to $t/\bar{t}$ identification for the determination of the production angle distribution ($dN/d\cos\theta_t$) in section 4. After the correction for charge misidentification we derive the $A_{FB}$ and discuss the result in comparison with the Monte Carlo truth in section 5. Section 6 summarizes our analysis and concludes this paper.

2 Analysis framework

In general, Monte Carlo (MC) simulation consists of the following steps: event generation, detector simulation, event reconstruction, and data analysis.

All of the MC event samples, both the signal and the backgrounds, used in this study were produced in the StdHep [5] format by a SLAC team [4] as common inputs to LoI studies, using WHIZARD 1.4 [6] for generating parton 4-momenta and PYTHIA 6.2 [7] for parton-showering and hadronization. The beam energy spread and the beamstrahlung were properly taken into account using the spectrum generated with Guinea-Pig [8] for the default ILC design parameters at $\sqrt{s} = 500$ GeV.

The final-state particles output in the StdHep format from the event generation step were passed to a Geant4-based detector simulator called Mokka [9] and swum through the ILD detector to create exact hits in trackers and calorimeters.

These exact hits were smeared or digitized, if necessary, depending on the detector elements in the first part of MarlinReco [10] [11]. The pattern recognition was done for the smeared tracker hits, separately in the TPC and the silicon detectors and so found track segments were then linked together and fed into a Kalman-filter-based track fitter in the second part of MarlinReco. From the fitted tracks and the calorimeter hits, individual particles were reconstructed as particle flow objects (PFO) with a sophisticated particle flow algorithm called PandoraPFA [12] in the third part of MarlinReco.

These PFOs were forced to cluster into 6 jets for the signal and all background events with Durham jet clustering algorithm [13] in the fourth part of MarlinReco.

The next step is heavy flavour tagging with LCFIVertex [3]. The LCFIVertex package consists of two parts. The first part is to search for secondary and tertiary vertices inside each jet and locate them, thereby determining the decay length, the transverse mass, and the momentum at each of these vertices. In the second part these quantities are translated into the impact parameter joint probability and the highest two impact parameter significances, which are used as inputs into a neural net (NN) trained with jet samples having 0 and 1 or more secondary vertices. Each reconstructed jet is then assigned with the three NN outputs, corresponding to $b$-, $c$-, and $bc$-tags.

Once a bottom-flavoured jet is identified we can determine whether it is $b$ or $\bar{b}$ by measuring the vertex charge. We will discuss this in detail in section 3.

The reconstruction of $e^+e^- \rightarrow t\bar{t}$ events in 6-jet final states has been studied extensively in the context of the top quark mass measurement. The top quark mass in the 6-jet mode is one of the benchmark observables for ILC LoIs and the ILD study has been reported in reference [2]. Similar to the top quark mass measurement, the measurement of the forward-backward asymmetry requires a correct jet-parton association. In this paper we employed the same reconstruction algorithm as those used for the top quark mass measurement. For the reconstruction, therefore, we refers the readers to the above reference.
3 Vertex charge reconstruction and its performance for a single b-jet

In principle, we can tell which three jets are from top and the rest from anti-top in the 6-jet final state by identifying either a $b/\bar{b}$-jet or a $c/\bar{c}$-jet from $W^+/W^-$ decay as shown in Fig. 1. In this analysis, however, we used only $b/\bar{b}$-jets for the top/anti-top separation.

Figure 1: Schematic diagram showing fully hadronic decay channel of $e^+e^- \rightarrow t\bar{t}$ process

A bottom quark hadronizes into a $B$-hadron, which flies over a finite distance thanks to its large $c\tau$, making a secondary vertex significantly away from the primary vertex and hence identifiable by the vertex detector. We define the vertex charge as the sum of the charges of the charged tracks associated with the secondary vertex. If the charged tracks are reconstructed and associated perfectly, the vertex charge is equal to the charge of the primary $B$-hadron, from which the charge sign of the $b/\bar{b}$ is uniquely determined.

In practice, however, there is no perfect vertexing, and the resultant distribution of the vertex charges of charged $B$-hadrons will have a finite width and hence their charges might sometimes be mis-identified. By the same token, the charges of neutral $B$-hadrons might be mis-identified, causing confusion in bottom charge sign identification.

Figure 2 shows the distribution of the MC level vertex charge, which is defined by the charge of a $B$-meson ($B^+$ or $B^0$) involved in the jet tagged as a $b$-jet.

Inspection of the figures tells us that we can separate $B^+$s from $B^-$s by selecting jets with negative or positive vertex charges with some contamination from $B^0$s.

Figure 3 shows the distribution of the reconstructed vertex charges of the jets which are $b$-tagged. In this distribution the $b$-quark ($\bar{b}$-quark) contribution is shown by hatched blue (red).

The charge sign identification efficiency for a single $b$-jet using the vertex charge is 28% with a purity of 75%. Notice that only 40% of the $b$-jets hadronize into charged $B$-hadrons and hence maximum efficiency one can hope for is 40% in this method.
Figure 2: Reconstructed vertex charge distributions for (a) charged $B$-mesons and (b) neutral $B$-mesons as separated using MC truth information (PDG particle ID code).

Figure 3: Reconstructed vertex charge distributions for $b$-quarks (hatched red) and $\bar{b}$-quarks (hatched blue), as identified using MC truth information, shown together with that of all the reconstructed $b$-jets (solid gray).
4 \( t \) and \( \bar{t} \) identification and determination of the production angle distribution \( (dN/d \cos \theta_t) \)

Let's call two reconstructed top systems \( t_1 \) and \( t_2 \) and \( b \)-tagged jets associated to them \( b_1 \) and \( b_2 \), respectively. The identification of \( t \) and \( \bar{t} \) is performed by using the vertex charges of \( b_1 \) and \( b_2 \) as follows. We define \( c_i \) (\( i = 1 \) and 2) as the vertex charge of \( b_i \) and \( C \equiv c_1 - c_2 \) as the event charge. If \( C \) is 0, the event is thrown away as we cannot tell \( t \) from \( \bar{t} \). If \( C \) is positive, \( t_1 \) is \( t \), while if \( C \) is negative, \( t_1 \) is \( \bar{t} \). The typical distribution of the event charge \( C \) is shown in Fig. 4.

![Event charge: C](image)

**Figure 4: Distribution of event charge: \( C = c_1 - c_2 \).**

Figures 5-(a) through -(i) show the distributions of the production angle of \( t_1 \) for 9 possible combinations of the signs of the vertex charges: \( c_1 \) and \( c_2 \), for the samples where both \( W \) bosons decayed into light quarks.

We can see a clear forward-backward asymmetry in the case of \( C \neq 0 \) (Figs. 5-(a) to -(f)). Notice also that in the first row (-(a) to -(c)) \( t_1 \) is \( t \) while in the second row (-(d) to -(f)) \( t_1 \) is \( \bar{t} \), and hence showing opposite asymmetries. On the other hand, in the case of \( C = 0 \), for which the charge signs of both \( b \)-jets were undetermined or they were inconsistent (both \( b_1 \) and \( b_2 \) had the same sign), there is no forward-backward asymmetry visible. In each figure the contribution from the events with wrong charge sign is hatched red, showing an opposite forward-backward asymmetry. Hatched blue is that from the combinatorial background in which the reconstructed \( b \)-jet candidates were not actually \( b \)-jets, showing no forward-backward asymmetry.

The combinatorial background depends on the flavour into which \( W \) bosons decay. Figures 6-(a) through -(i) are the same figures as Figs. 5-(a) through -(i), but plotted for the samples where one of the two \( W \) bosons decayed into a \( c/\bar{c} \)-quark. Figures 7-(a) through -(i) are similar figures for the samples where both \( W \) bosons decayed into \( c/\bar{c} \)-quarks. We
can see clearly that the combinatorial background grows with the number of $c$-jets in the final states, since the probability of mis-identifying charm as bottom decreases.

If $C$ is positive, $t_1$ is $t$, while if $C$ is negative, $t_1$ is $\bar{t}$. If we can assume that $t_1$ and $t_2$ are back-to-back, the production angle of $t$ is obtained from the angle of $t_1$ as

$$\cos \theta_t \equiv \sigma_C \cdot | \cos \theta_{t_1} |,$$

where $\sigma_C$ is the sign of $C$. In order to test this assumption we compared the distributions for $t_1$ with those of $t_2$ after reflection (hatched green) in Figs. 5, 6, and 7-(d) to -f). This comparison confirmed the assumption\(^1\), allowing us to use Eq. (4.7) to combine all the figures with $C \neq 0$. The selection efficiency of this cut is 69%.

The differential cross section for $e^+ e^- \rightarrow t\bar{t}$ and consequently its forward-backward asymmetry ($A_{FB}^t$) depend on the center-of-mass energy of the $t\bar{t}$ system, which, in turn, depends on the amount of initial state radiation and beamstrahlung. In order to make $A_{FB}^t$ well-defined, we hence rejected events with $\sqrt{s}$ significantly less than 500 GeV, by requiring the acollinearity between $t_1$ and $t_2$ to be less than $8^\circ$. This final cut discarded 24% of the events so far survived. The overall selection efficiency is 20% for fully hadronic $t\bar{t}$ events.

The resultant production angle distribution is shown in Fig. 8. Of the final sample 83% have correctly identified signs of top quark charge.

5 Determination of $A_{FB}^t$

The production angle distribution Fig. 8 is distorted by the charge mis-identification. The distortion can be corrected by using the following formulae:

$$\left\{ \begin{array}{l}
\frac{dN_{\text{obs}}(\theta)}{dN_{\text{obs}}(\pi - \theta)} = p(\theta) \cdot \eta(\theta) \cdot \mathcal{L} \cdot \sigma(\theta) + \bar{p}(\pi - \theta) \cdot \eta(\pi - \theta) \cdot \mathcal{L} \cdot \sigma(\pi - \theta) + dB(\theta) \\
\frac{dN_{\text{obs}}(\pi - \theta)}{dN_{\text{obs}}(\theta)} = p(\pi - \theta) \cdot \eta(\pi - \theta) \cdot \mathcal{L} \cdot \sigma(\pi - \theta) + \bar{p}(\theta) \cdot \eta(\theta) \cdot \mathcal{L} \cdot \sigma(\theta) + dB(\pi - \theta)
\end{array} \right.$$  

(5.8)

where $\eta(\theta)$ is the acceptance, $p(\theta)$ ($\bar{p}(\theta)$) is the probability of correctly (wrongly) assigning the charge sign at production angle $\theta$, and $\mathcal{L}$ is the integrated luminosity.

$p(\theta)$ and $\bar{p}(\theta)$ are plotted in Fig. 9 as the black and red lines, respectively.

The figure shows no $\theta$-dependency, allowing us to set $p(\theta) = p$ and $\bar{p}(\theta) = \bar{p}$. Solving for $d\sigma(\theta)$, we thus obtain the following formula for the differential cross section:

$$d\sigma(\theta) = \frac{p \cdot (dN_{\text{obs}}(\theta) - dB(\theta)) - \bar{p} \cdot (dN_{\text{obs}}(\pi - \theta) - dB(\pi - \theta))}{(p^2 - \bar{p}^2)\eta(\theta) \cdot \mathcal{L}}.$$

(5.9)

Figure 10 shows the production angle distribution after the correction for the charge mis-identification.

The acceptance function $\eta(\theta)$ is in general $\theta$-dependent. In our case, however, it turned out at that the $\theta$-dependence was negligible\(^1\). The acceptance hence cancels out in the calculation of the forward-backward asymmetry, resulting in

$$A_{FB}^t = \frac{\int_{0 < \theta < \pi/2} dN(\theta) - \int_{\pi/2 < \theta < \pi} dN(\theta)}{\int_{0 < \theta < \pi/2} dN(\theta) + \int_{\pi/2 < \theta < \pi} dN(\theta)}.$$

(5.10)

\(^1\)Strictly speaking, Eq. (4.7) does not hold on an event-by-event basis because of initial state radiation and beamstrahlung. On average, however, $t_1$ and $t_2$ are back-to-back, allowing us to merge the six cases with $C \neq 0$.

\(^1\)Notice that for the reconstructed $t$ or $\bar{t}$, being a 3-jet system, there is no acceptance hole even at $\theta = 0$ or $\pi$.  

33
Figure 5: Distributions of the reconstructed polar angle of the identified top quark in fully-hadronic $t\bar{t}$ events, which are tagged as $(Q_{b_1}, Q_{b_2}) = (a) (+, -), (b) (+, 0), (c) (0, -), (d) (-, +), (e) (-, 0), (f) (0, +), (g) (0, 0), (h) (+, +), and (i) (-, -) for the sample in which both $W$ bosons decayed into light-quark pairs ($bbuddu$ sample).
Figure 6: Distributions of the reconstructed polar angle of the identified top quark in fully-hadronic $t\bar{t}$ events, which are tagged as $(Q_{b_1}, Q_{b_2}) = (a) (+, -), (b) (+, 0), (c) (0, -), (d) (-, +), (e) (-, 0), (f) (0, +), (g) (0, 0), (h) (+, +), and (i) (-, -) for the samples in which one of the two $W$ bosons decayed into a $c/\bar{c}$-quark ($bbudsc/bbecsdu$ samples).
Figure 7: Distributions of the reconstructed polar angle of the identified top quark in fully-hadronic $t\bar{t}$ events, which are tagged as $(Q_{b_1}, Q_{b_2}) = (a) (+, -), (b) (+, 0), (c) (0, -), (d) (-, +), (e) (-, 0), (f) (0, +), (g) (0, 0), (h) (+, +), and (i) (-, -) for the sample in which both $W$ bosons decayed into $c/\bar{c}$-quarks ($bc\bar{c}sc$ sample).
Figure 8: Distribution of the reconstructed polar angle of the identified top quark in fully-hadronic $t\bar{t}$ events. The contributions from events with the wrong charge (red) and the case where the $b$-quark is mis-identified are also shown (blue).

Figure 9: $\cos \theta_{t1}$ dependence of the charge-sign identification (black) and mis-identification (red) probabilities.
where $dN(\theta) \equiv \eta \cdot L \cdot d\sigma(\theta)$.

From Fig. 10 we finally obtain

$$A_{FB}^t = 0.334 \pm 0.0079 \text{ (stat.)},$$

where the center of mass energy of $\sqrt{s} = 500$ GeV, an integrated luminosity of 500 fb$^{-1}$, and a beam polarization combination of $P(e^+, e^-) = (+30\%, -80\%)$.

6 Summary

We have studied the measurement accuracy of the forward-backward asymmetry with the ILD detector for the $e^+e^- \rightarrow t\bar{t}$ process in the 6-jet mode at $\sqrt{s} = 500$ GeV. In the analysis the vertex charges of $b$-jets were used to identify $t$ and $\bar{t}$. The efficiency to tag the vertex charge of a $b$-jet was 28% with the purity of 75%. Having two $b$-jets in each event, the probability to identify $t$ and $\bar{t}$ in the event was 71% with the probability of wrong charge assignment of 12%. The measured angular distribution was corrected for wrong $t/\bar{t}$ charge assignments. From the number of events in forward and backward hemispheres after the correction, we obtained $A_{FB}^t = 0.334 \pm 0.0079$, where quoted error is statistical only.

Acknowledgments

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References


A study of top-quark Yukawa coupling measurement in $e^+e^- \rightarrow t\bar{t}H$ at $\sqrt{s} = 500$ GeV

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We report on the feasibility of measuring the top Yukawa coupling in the process: $e^+e^- \rightarrow t\bar{t}H$. This measurement is crucial to test the mass generation mechanism for matter particles. Since the cross section for this process attains its maximum around $\sqrt{s} = 700$ GeV, most of the past studies were done assuming this energy region. It has been pointed out, however, that the QCD threshold correction enhances the cross section significantly and might enable its measurement at $\sqrt{s} = 500$ GeV, which will be accessible already in the first phase of the ILC project. We have implemented this threshold enhancement into our $t\bar{t}H$ event generator and carried out Monte Carlo simulations. Our results show that $t\bar{t}H$ events can be observed with a significance of $4.1\sigma$ with no beam polarization and $5.4\sigma$ with the $e^-$ and $e^+$ beam polarization combination: ($-0.8$, $+0.3$).

1 Introduction

The standard model of elementary particle physics is based on two pillars: one is the gauge principle and the other is the electroweak symmetry breaking and mass generation mechanism. The first pillar, the gauge principle, has been tested by precision electroweak measurements. On the other hand, the second pillar has not yet been tested. In order to confirm this second pillar we have to measure the Higgs self-coupling and the top Yukawa coupling.

In this study, we investigate the feasibility of measuring the top Yukawa coupling at 500 GeV with the process: $e^+e^- \rightarrow t\bar{t}H$. Since the top quark is the heaviest among all the matter particles, the measurement of its Yukawa coupling will be the most decisive test of the mass generation mechanism for matter particles. Since the cross section for the $e^+e^- \rightarrow t\bar{t}H$ process is 2-3 fb even near its maximum reached at around $\sqrt{s} = 700$ GeV, most of the past studies assumed the measurement energy in this region[1]. It has been pointed out, however, that the QCD threshold correction enhances the cross section significantly[2] and might open up the possibility of measuring the top Yukawa coupling at $\sqrt{s} = 500$ GeV, which is within the scope of the first phase of the ILC project. In order to investigate this possibility we have implemented this threshold enhancement into our $t\bar{t}H$ event generator and carried out Monte Carlo simulations.

In the next section we begin with clarifying the signatures of the $t\bar{t}H$ production and list up possible background processes that might mimic the signal. We then describe our analysis framework used for event generations and detector simulations in section 3. The
event selection procedure for the generated events is elaborated in section 4, considering characteristic features of the background processes. The results of the event selection are given in section 5. Section 6 summarizes our results and concludes this report.

2 Signal and Possible Background

The Feynman diagrams for the $e^+e^- \rightarrow ttH$ process followed by $t(\bar{t}) \rightarrow b(\bar{b})W$ decays are shown in Figure 1. Notice that the first and second diagrams contain the top Yukawa coupling, which we want to measure. The signatures of $ttH$ events depend on how the $H$ and the Ws decay. In this study we concentrate on the dominant decay mode: $H \rightarrow bb$ (68%). The signal events hence have four $b$ jets and two Ws. The $ttH$ events can then be classified into 3 groups (8-jet, 1-lepton+6-jet, and 2-lepton+4-jet modes) corresponding to the combinations of leptonic and hadronic decays of the two Ws. For Ws that decayed leptonically we cannot reconstruct their invariant masses due to missing neutrinos. On the other hand, for the Ws that decayed hadronically we can reconstruct their masses and use them as a signature. For the $t$ or the $\bar{t}$ with a hadronically decayed W we can also use the invariant mass of the 3-jet system to test if it is consistent with the top mass.

Possible background processes that might mimic the signatures of the $ttH$ production include $e^+e^- \rightarrow t\bar{t}Z$, $t\bar{t}$, and $t\bar{t}g$ followed by $g \rightarrow b\bar{b}$. The cross sections for these background processes are plotted in Fig.2 together with that of the signal. Notice the smallness of the contribution from the third diagram in Fig.1, which does not contain the top Yukawa coupling. We can hence determine the top Yukawa coupling by just counting the number of signal events unless they are swamped by the background; the signal cross section is only $\sim 0.5\text{ fb}$ with no beam polarization.

The production cross section for the $t\bar{t}Z$ background is $1.3\text{ fb}^*$ with no beam polarization. It has four $b$-jets and two Ws in the final state just like the signal, if the $Z$ boson decays into $b\bar{b}$ (15%). In this case the only difference that one can tell on an event-by-event basis lies in the invariant mass of the $b\bar{b}$ system, which should be consistent with $M_H$ for the signal and $M_Z$ for the background. The $tt$ production, on the other hand, has only two $b$-jets in the final state. If reconstructed correctly, it could not be the background. Since the $t\bar{t}$ production cross section ($\sim 500\text{ fb}$) is much larger than that of the signal, however, a small fraction of mis-reconstruction or failure in $b$-tagging may lead to significant background.

\footnote{\textsuperscript{*}This value is with QCD threshold enhancement similar to that expected for the signal process. Without the correction the cross section is 0.7 fb.}
contamination. The $t\bar{t}g$ production followed by $g \rightarrow bb$ decay has the same signatures as the signal in terms of the number of $b$-jets and the number of $W$s. As with the $t\bar{t}Z$ background the only difference is the invariant mass of the $bb$ system. Its production cross section is also of the same order, 0.7 fb, as that of the $t\bar{t}Z$ background.

3 Analysis Framework

For Monte Carlo simulations, we generated signal and background events by using an event generator package (physsim[3]), which is based on full helicity amplitudes calculated with HELAS[4] including gauge boson decays, thereby correctly taking into account angular distributions of the decay products. The 4-momenta of the final-state quarks and leptons were passed to Pythia6.4[5] for parton showering and hadronization. The resultant particles were then swum through a detector model (see Table 1 for detector parameters) defined in our fast Monte Carlo detector simulator (QuickSim[6]). In the event generations we used $\alpha(M_Z) = 1/128$, $\sin^2\theta_W = 0.230$, $\alpha_s = 0.120$, $M_W = 80.0 \text{ GeV}$, $M_Z = 91.18 \text{ GeV}$, $M_t = 175 \text{ GeV}$, and $M_H = 120 \text{ GeV}$. We have included the initial state radiation and beam-strahlung in the event generations. The unique point of this study is the inclusion of the QCD threshold enhancement to the $tt$ system (see Fig.3) for the signal event generation, which plays an important role especially in a low energy experiment: about a factor of 2 enhancement at $\sqrt{s} = 500 \text{ GeV}$.
Figure 3: Invariant mass distribution for the $t\bar{t}$ sub-system.

Table 1: Detector Parameters, where $p, p_T$ and $E$ are measured in units of GeV

<table>
<thead>
<tr>
<th>Detector</th>
<th>Performance</th>
<th>Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex detector</td>
<td>$\sigma_b = 7.0 \oplus (20.0/p) \sin^{3/2} \theta_{\mu m}$</td>
<td>$</td>
</tr>
<tr>
<td>Central drift chamber</td>
<td>$\sigma_{p_T}/p_T = 1.1 \times 10^{-4} p_T \oplus 0.1%$</td>
<td>$</td>
</tr>
<tr>
<td>EM calorimeter</td>
<td>$\sigma_E/E = 15%/\sqrt{E} \oplus 1%$</td>
<td>$</td>
</tr>
<tr>
<td>Hadron calorimeter</td>
<td>$\sigma_E/E = 40%/\sqrt{E} \oplus 2%$</td>
<td>$</td>
</tr>
</tbody>
</table>

4 Event Selection

4.a Definition of our signal (1-lepton+6-jet mode on $t\bar{t}H$)

As explained in section 2 we can classify the $t\bar{t}H$ signal events into the following three decay modes according to how the two Ws from $t$ and $\bar{t}$ decay:

1. 8-jet mode (45%)
2. 1-lepton + 6-jet mode (35%)
3. 2-lepton + 4-jet mode (7%)

where the lepton is required to be either $e^\pm$ or $\mu^\pm$ and the final-state $H$ to decay into the dominant $b\bar{b}$ state. Notice that in all of these three modes we have four $b$-jets in the final states, which makes the separation of the $t\bar{t}$ background easier. In this study we concentrate on the 1-lepton + 6-jet mode as our first step because the branching ratio is not so low and the number of jets is not so high.

As shown in Figs.4 and 5 the signatures of our signal are

- an isolated energetic $e^\pm$ or $\mu^\pm$,
six jets including four $b$-jets, two of which form a $H$ boson,

- the remaining two jets being consistent with a $W$ boson, and

- one of the two unused $b$-jets together with this $W$ candidate comprising a $t$ quark.

Figure 4: Schematic diagram defining our signal signatures

Figure 5: Invariant mass distributions for the hadronically decayed $W$, $t$, and $H$, which are reconstructed using generator information.

In what follows we will elaborate selection cuts designed to single out these signatures.

4.4.1 Isolated lepton search

Our event selection starts with the search for a lepton coming from a $W \rightarrow l\nu$ decay. Such a lepton from $W$ tends to be energetic and isolated from the other tracks. In order to find such an isolated lepton, we consider a cone around each lepton track (see Fig.6) and define the cone energy to be the sum of the energies of the other tracks in the cone. Figure 7 plots the cone energy against the lepton energy. The energetic isolated leptons from $W$s have to have a high lepton energy and a low cone energy, hence populating the bottom edge region (black points), while leptons from heavy flavor jets are likely to be less energetic and
have a higher cone energy (gray points). The smooth curve in the figure is our cut to select energetic isolated leptons.

4.a.2 Forced 6-Jet clustering

After finding and eliminating an energetic isolated lepton, we perform jet clustering to make six jets. For the jet clustering we use a variable \( Y \) defined by

\[
Y = \frac{M_{\text{jet}}^2}{E_{\text{visible}}^2}.
\]

We keep putting tracks together to form a jet while \( Y < Y_{\text{cut}} \). By adjusting the \( Y_{\text{cut}} \) value, we can make arbitrary number of jets. We hence force the events to cluster into six jets by choosing an appropriate \( Y_{\text{cut}} \) value on the event-by-event basis (forced 6-jet clustering).

4.a.3 \( Y_{\text{cut}} \) cut

The \( Y_{\text{cut}} \) value for a \( t\bar{t} \) background event to form six jets should be lower than the one for a signal \( ttH \rightarrow t\bar{t}bb \) event because, after the energetic isolated lepton requirement, the \( t\bar{t} \) event can hardly have more than four jets. The difference in the \( Y_{\text{cut}} \) value distributions between \( ttH(H \rightarrow bb) \) and \( tt \) is shown in Fig.8. As seen in the figure, by cutting \( Y_{\text{cut}} \) values at 0.002 we can reduce the \( tt \) background effectively.

![Figure 8: \( Y_{\text{cut}} \) value distribution after isolated lepton finding](image)

4.a.4 mass cut

After performing the jet clustering, we try to identify which jet is coming from which parent parton. We want to separate the correct combination from the other combinatorial background. Mass cut comes in handy to reduce the combinatorial background. Looping over all the 2-jet combinations we look for a pair having an invariant mass within the window of \( \pm 15 \) GeV from the nominal \( W \) mass of 80.0 GeV. From the remaining four jets we pick up one and attach it to the just found pair making a \( W \) candidate to see if the resultant 3-jet system has an invariant mass within \( \pm 25 \) GeV from the nominal \( t \) mass of 175 GeV.
If it does we search for a pair from the three jets left over that is within the mass window of ±15 GeV from the nominal $H$ mass of 120 GeV. Since these mass cuts are rather loose there is a significant chance to have multiple combinations that pass them. For such a case we define a $\chi^2$ variable with

$$\chi^2 = \left( \frac{M_{2\text{-}jet(W)} - M_W}{\sigma_{M_W}} \right)^2 + \left( \frac{M_{3\text{-}jet(t/\bar{t})} - M_t}{\sigma_{M_t}} \right)^2 + \left( \frac{M_{2\text{-}jet(H)} - M_H}{\sigma_{M_H}} \right)^2,$$

and select the combination with the smallest $\chi^2$ value. Fig. 9 shows the mass distributions for the best combinations. Although $W$ and $t/\bar{t}$ peaks are present for both the signal and the $t\bar{t}$ background, a $H$ peak is seen only for the signal process. The $H$ peak is, however, swamped in the $t\bar{t}$ background.

4.a.5 $b$-tagging by the $n$-sig. method

For the $t\bar{t}$ background rejection, $b$-tagging is very powerful since the signal $t\bar{t}H(H \to b\bar{b})$ process has four $b$-jets, while the $t\bar{t}$ background process has only two $b$-jets. For $b$-tagging we use the so called $n$-sig. method described as follows.

Figure 10 sketches a jet from the interaction point (IP), which includes a $b$-hadron. The $b$-hadron decays at distance from the IP due to its long-life. It makes the $b$-jet to have some tracks which are away from the IP. When the distance ($\ell$) between the IP and a track is larger than a given value ($m\sigma_\ell$), the track is defined as an off-vertex track. A jet is recognized as a $b$-jet if the number of such significantly off-vertex tracks exceeds a certain cut value ($n$). In this analysis, we define tight $b$-tagging with a tagging condition: $(m,n) = (3.0,2)$ and loose $b$-tagging with $(m,n) = (2.0,2)$, and require all of the four $b$-jet candidates to satisfy the loose $b$-tagging condition and there has to be at least one tight $b$-tagged jet from each of the $H$ and $t/\bar{t}$ candidates.
Figure 10: n-sig. method

The mass distributions after the $b$-tagging are shown in Fig. 11. We can see that the $t\bar{t}$
background has been suppressed effectively. As mentioned above the $t\bar{t}Z$ and $t\bar{t}g \ (g \to b\bar{b})$
background events have similar signatures as a signal and can be separated only with the
invariant mass of the $H$ candidate. In the next section we summarize the results of our
event selection including these remaining background processes.

5 Results

In order to estimate the feasibility of measuring the top Yukawa coupling we need to specify
the beam polarization and the integrated luminosity. In this study we assume an integrated
luminosity of $1 \text{ ab}^{-1}$. As for the beam polarization, it is worth noting that only the left-right
or right-left combination contributes to the signal and background cross sections because of
the $\gamma\mu$ coupling of the beam particles to the vector bosons ($\gamma/Z$) in the intermediate states.
It is hence sufficient to know the cross sections for the beam polarization combinations:
$(e^-, e^+) = (-1, +1), (+1, -1)$. Table 2 shows these cross sections.

For both of the beam polarization combinations: $(-1, +1)$ and $(+1, -1)$, we have gen-
erated 50k events each for the $t\bar{t}H$, $t\bar{t}Z$, and $t\bar{t}g \ (g \to b\bar{b})$ processes, and 5M events for
the $t\bar{t}$ background. We performed the event selection described in the previous section and
tabulated the results normalized to an integrated luminosity $1 \text{ ab}^{-1}$ in Table 3 assuming the
cross section shown in Table 2.

The corresponding distributions for the reconstructed $W$, $t/\bar{t}$, and $H$ candidates are
shown in Fig. 12 for the beam polarization combination: $(-0.8, +0.3)$. We can see a clear
Table 2: Cross sections at $\sqrt{s} = 500$ GeV. $tH$ and $tZ$ are with QCD threshold enhancement. $(-1,+1)/(+1,-1)$ corresponds to $(e_L^-, e_R^+)/ (e_R^-, e_L^+)$, respectively.

<table>
<thead>
<tr>
<th>Beam Polarization</th>
<th>$tH$</th>
<th>$tZ$</th>
<th>$tt$</th>
<th>$ttg (g \rightarrow bb)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-1,+1)$</td>
<td>1.24 fb</td>
<td>2.18 fb</td>
<td>720 fb</td>
<td>1.93 fb</td>
</tr>
<tr>
<td>$(+1,-1)$</td>
<td>0.540 fb</td>
<td>0.712 fb</td>
<td>309 fb</td>
<td>0.859 fb</td>
</tr>
</tbody>
</table>

Table 3: Cut Statistics (normalized to 1 ab$^{-1}$)

<table>
<thead>
<tr>
<th>Beam Polarization</th>
<th>No Cut</th>
<th>$N_{\text{iso.lep}=1}$</th>
<th>$Y_{\text{cut}} (6 \text{ jets}) &gt; 0.002$</th>
<th>$\text{btag \&amp; mass cut}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processes</td>
<td>$tH$</td>
<td>$tZ$</td>
<td>$ttg (bb)$</td>
<td>$tH$</td>
</tr>
<tr>
<td></td>
<td>(0.0,0.0)</td>
<td>(0.8,+0.3)</td>
<td>(0.0,0.0)</td>
<td>(0.8,+0.3)</td>
</tr>
<tr>
<td>No Cut</td>
<td>449.0 fb</td>
<td>1340.0 fb</td>
<td>514030.5 fb</td>
<td>697.5 fb</td>
</tr>
<tr>
<td>$N_{\text{iso.lep}=1}$</td>
<td>159.4 fb</td>
<td>435.9 fb</td>
<td>209718.4 fb</td>
<td>242.2 fb</td>
</tr>
<tr>
<td>$Y_{\text{cut}} (6 \text{ jets}) &gt; 0.002$</td>
<td>139.2 fb</td>
<td>307.8 fb</td>
<td>22851.3 fb</td>
<td>152.5 fb</td>
</tr>
<tr>
<td>$\text{btag \&amp; mass cut}$</td>
<td>23.0 fb</td>
<td>12.2 fb</td>
<td>11.9 fb</td>
<td>6.9 fb</td>
</tr>
</tbody>
</table>

Note: $Y_{\text{cut}} (6 \text{ jets}) > 0.002$ corresponds to $W$ masses $60, 80, 100, 120$ GeV and $H$ masses $140, 160, 180, 200$ GeV.

Figure 12: Mass distributions (cumulative) for the final selected sample for the beam polarization combination: $(-0.8, +0.3)$.

In the case of no beam polarization 23.0 signal events are left with 31.0 background events total. On the other hand we have 38.9 signal events with 52.8 background events total at the end of the event selection. The signal significance is 4.1 $\sigma$ for the polarization combination: (0, 0) and $5.4 \sigma$ for the polarization combination: $(-0.8, +0.3)$. Since the number of the signal events is proportional to the square of the top Yukawa coupling $(g_Y)$, we can easily translate these numbers to its expected precisions: $\Delta g_Y / g_Y = \pm 0.12$ and $\pm 0.093$ for the beam polarization polarization combinations: (0, 0) and $(-0.8, +0.3)$, respectively.
6 Summary and Conclusion

We have performed a feasibility study of measuring the top Yukawa coupling at $\sqrt{s} = 500$ GeV, taking advantage of the QCD threshold enhancement to the $t\bar{t}$ sub-system. For this study we have implemented the threshold enhancement in the $t\bar{t}H$ and $t\bar{t}Z$ event generators in the physsim package. It is found that for an integrated luminosity of $1 \text{ ab}^{-1}$ we can observe the $t\bar{t}H$ process with a significance of $4.1 \sigma$ without beam polarization, and $5.4 \sigma$ with the beam polarization combination: $(e^-, e^+) = (-0.8, +0.3)$. These numbers show that we can measure the top Yukawa coupling to an accuracy of about 10% at $\sqrt{s} = 500$ GeV, which is the energy already available in the first stage of the ILC.

7 Acknowledgments

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References

Higgs boson pair production at the Photon Linear Collider in the two Higgs doublet model

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We calculate the cross section of the lightest Higgs boson pair production at the Photon Linear Collider in the two Higgs doublet model. We focus on the scenario in which the lightest Higgs boson has the standard model like couplings to gauge bosons. We take into account the one-loop correction to the $hhh$ coupling as well as additional one-loop diagrams due to charged bosons to the $\gamma\gamma \rightarrow hh$ helicity amplitudes. We discuss the impact of these corrections on the $hhh$ coupling measurement at the Photon Linear Collider.

1 Introduction

The Higgs sector is the last unknown part of the standard model (SM). In the SM, the tree level Higgs self-coupling $\lambda_{hhh} = 3m_h^2/v$ and $\lambda_{hhhh} = 3m_h^2/v^2$ are uniquely determined by the Higgs boson mass $m_h$, where $v$ is vacuum expectation value (VEV) of the Higgs boson. The effective Higgs potential is written as

$$V = \frac{1}{2} m_h^2 h^2 + \frac{1}{3!} \lambda_{hhh} h^3 + \frac{1}{4!} \lambda_{hhhh} h^4 + \cdots,$$

where the effective Higgs self-couplings $\lambda_{hhh}$ and $\lambda_{hhhh}$ are given by precision measurement of $hhh$ and $hhhh$ couplings. If the deviation from the SM tree level Higgs self-coupling ($\lambda_{hhh}$ and $\lambda_{hhhh}$) is found, it can be regarded as an evidence of new physics beyond the SM. The origin of the spontaneous electroweak symmetry breaking (EWSB) would be experimentally tested after the discovery of a new scalar particle by measuring its mass and self-couplings. The Higgs self-coupling measurement is one of main purposes at the International Linear Collider (ILC). The structure of the Higgs potential depends on the scenario of new physics beyond the SM, so that precision measurement of the $hhh$ coupling can be a probe of each new physics scenario\cite{1, 2}.

It is known that the measurement of the triple Higgs boson coupling is rather challenging at the CERN Large Hadron Collider (LHC). At the SLHC with luminosity of 3000 fb$^{-1}$,
the $hhh$ coupling can be determined with an accuracy of 20-30% for $160 \text{ GeV} \leq m_h \leq 180 \text{ GeV}[3, 4]$. At the ILC, the main processes for the $hhh$ measurement are the double Higgs boson production mechanisms via the Higgs-strahlung and the W-boson fusion[5, 6]. At the ILC with a center of mass energy of 500 GeV, the double Higgs strahlung process $e^+e^- \rightarrow Zhh$ is dominant. On the other hand, W-boson fusion process $e^+e^- \rightarrow hh\nu\bar{\nu}$ becomes dominant due to its $t$-channel nature at 1 TeV or higher energies[7]. Sensitivity to the $hhh$ coupling in these processes becomes rapidly worse for greater Higgs boson masses. In particular, for the intermediate mass range ($140 \text{ GeV} \leq m_h \leq 200 \text{ GeV}$), it has not yet been known how accurately the $hhh$ coupling can be measured by the electron-positron collision. The Photon Linear Collider (PLC) is an optional experiment of the ILC. The possibility of measuring the $hhh$ coupling via the process of $\gamma\gamma \rightarrow hh$ has been discussed in Ref. [8]. In Ref. [9] the statistical sensitivity to the $hhh$ coupling constant has been studied especially for a light Higgs boson mass in relatively low energy collisions.

In this paper, we study the double Higgs production process at the PLC. In Sect. 2, we discuss the statistical sensitivity to the $hhh$ coupling constant via the process of $e^−e^− \rightarrow \gamma\gamma \rightarrow hh$ at the PLC in the SM. In Sect. 3, we study the new particle effects on the $\gamma\gamma \rightarrow hh$ process in the two Higgs doublet model (THDM).

2 The statistical sensitivity to the $hhh$ coupling constant

We study the statistical sensitivity to the $hhh$ coupling constant for wide regions of the Higgs boson masses and the collider energies at the PLC. The $\gamma\gamma \rightarrow hh$ process is an one-loop induced process. The Feynman diagrams for this process in the SM are given in Ref. [8]. There are two types of diagrams, which are the pole diagrams and the box diagrams. The amplitude of the pole diagrams describes as $M_{\text{pole}} \propto \tilde{\lambda}_{hhh}/s$, where $s$ is the center of mass energy of the $\gamma\gamma$ system. It is suppressed by $1/s$ at the high energy region, so that the statistical sensitivity to the $hhh$ coupling becomes rapidly worse for this region. On the other hand, the box diagrams do not depend on the $hhh$ coupling.

In Fig. 1, we present the statistical sensitivity on the Higgs self-coupling constant at the PLC. We modify the triple Higgs coupling constant as $\tilde{\lambda}_{hhh} = \lambda_{hhh}(1 + \delta\kappa)$, where $\delta\kappa$ represents deviation from the SM prediction. We assume that the efficiency of the particle tagging is $100\%$ with an integrated luminosity of $1/3 \text{ ab}^{-1}$ and $E_{ee}$ is the center of mass energy of the $e^-e^-$ system. We plot $\delta\kappa$ based on statistical error of the event number in the $e^-e^- \rightarrow \gamma\gamma \rightarrow hh$ process in the SM. Namely, $\delta\kappa$ is determined by

$$|N(\delta\kappa) - N(\delta\kappa = 0)| = \sqrt{N(\delta\kappa = 0)}$$

for assumed luminosity. Notice that $\delta\kappa$ is not symmetric with respect to $\delta\kappa = 0$ because there is interference between pole and box diagrams. The cases for $\delta\kappa > 0$ and $\delta\kappa < 0$ are shown separately. The left [right] figure shows the sensitivity as a function of $m_h$ [$E_{ee}$]. It is found that when the collision energy is limited to be lower than $500-600 \text{ GeV}$ the statistical sensitivity to the $hhh$ coupling can be better for the process in the $\gamma\gamma$ collision than that in the electron-positron collision for the Higgs boson with the mass of $160 \text{ GeV}[10]$.

3 The $\gamma\gamma \rightarrow hh$ process in the THDM

We consider the new particle effects on the $\gamma\gamma \rightarrow hh$ process in the THDM, in which additional CP-even, CP-odd and charged Higgs boson appear. It is known that non-decoupling
Figure 1: The statistical sensitivity to the $hhh$ coupling constant at the PLC. In the left [right] figure, the statistical sensitivity is shown as a function of $m_h$ [$E_{ee}$] for each value of $E_{ee}$ [$m_h$]. Solid [Dotted] lines correspond to $\delta \kappa > 0$ [$\delta \kappa < 0$] case.

The statistical effect of extra Higgs bosons shift the $hhh$ coupling value from the SM by $O(100)$%[1]. In the $\gamma\gamma \rightarrow hh$ helicity amplitudes, there are additional one-loop diagrams by the charged Higgs boson loop to the ordinary SM diagrams (the W-boson loop and the top quark loop). It is found that both the charged Higgs boson loop contribution to the $\gamma\gamma \rightarrow hh$ amplitudes and the non-decoupling effect on the $hhh$ coupling can enhance the cross section from its SM value significantly[11].

In order to study the new physics effect on $\gamma\gamma \rightarrow hh$ process, we calculate the helicity amplitudes in the THDM. The THDM Higgs potential is given by

$$V_{\text{THDM}} = \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 - (\mu_3^2 |\Phi_1|^2 |\Phi_2|^2 + \text{h.c.})$$

$$+ \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1|^2 |\Phi_2|^2 + \frac{\lambda_5}{2} \left( |\Phi_1|^2 |\Phi_2|^2 + \text{h.c.} \right),$$

where $\Phi_1$ and $\Phi_2$ are two Higgs doublets with hypercharge $+1/2$. The Higgs doublets are parametrized as

$$\Phi_i = \left[ \frac{\omega_i}{\sqrt{2}}(v_i + h_i + iz_i) \right], \quad (i = 1, 2),$$

where VEVs $v_1$ and $v_2$ satisfy $v_1^2 + v_2^2 = v^2 \simeq (246 \text{ GeV})^2$. The mass matrices can be diagonalized by introducing the mixing angles $\alpha$ and $\beta$, where $\alpha$ diagonalizes the mass matrix of the CP-even neutral bosons, and $\tan \beta = v_2/v_1$. Consequently, we have two CP-even ($h$ and $H$), a CP-odd ($A$) and a pair of charged ($H^\pm$) bosons. We define $\alpha$ such that $h$ is the SM-like Higgs boson when $\sin(\beta - \alpha) = 1$.

We concentrate on the case with so called the SM-like limit [$\sin(\beta - \alpha) = 1$], where the lightest Higgs boson $h$ has the same tree-level couplings as the SM Higgs boson, and the
other bosons do not couple to gauge bosons and behave just as extra scalar bosons. In this limit, the masses of Higgs bosons are

\[
m_h^2 = \left\{ \lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + 2(\lambda_3 + \lambda_4 + \lambda_5) \cos^2 \beta \sin^2 \beta \right\} v^2, \hspace{1cm} (3.16)
\]

\[
m_H^2 = M^2 + \frac{1}{8} \left( \lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4 + \lambda_5) \right) (1 - \cos 4\beta) v^2, \hspace{1cm} (3.17)
\]

\[
m_A^2 = M^2 - \lambda_5 v^2, \hspace{1cm} (3.18)
\]

\[
m_{H^\pm}^2 = M^2 - \frac{\lambda_4 + \lambda_5}{2} v^2, \hspace{1cm} (3.19)
\]

where \( M = |\mu_3|/\sqrt{\sin \beta \cos \beta} \) represents the soft breaking scale for the discrete symmetry, and determines the decoupling property of the extra Higgs bosons. When \( M \to 0 \), the extra Higgs bosons \( H, A \) and \( H^\pm \) receive their masses from the VEV, so that the masses are proportional to \( \lambda_i \). Large masses cause significant non-decoupling effect in the radiative correction to the \( hhh \) coupling. On the other hand, when \( M \gg v \) the masses are determined by \( M \). In this case, the quantum effect decouples for \( M \to \infty \).

It is known that in the THDM \( \lambda_{hhh} \) can be changed from the SM prediction by the one-loop contribution of extra Higgs bosons due to the non-decoupling effect (when \( M \to 0 \)). In the following analysis, we include such an effect on the cross sections. The effective \( hhh \) coupling \( \Gamma_{\text{THDM}}^\text{hhh}(\hat{s}, m_h^2, m_H^2) \) is evaluated at the one-loop level as \[1\]

\[
\Gamma_{\text{THDM}}^\text{hhh}(\hat{s}, m_h^2, m_H^2) \simeq \frac{3m_h^2}{v} \left[ 1 + \sum_{\Phi = H, A, H^+, H^-} \frac{m_\Phi^4}{12\pi^2 v^2 m_h^2} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 - \frac{N_c m_t^4}{3\pi^2 v^2 m_h^2} \right]. \hspace{1cm} (3.20)
\]

The exact one-loop formula for \( \Gamma_{\text{THDM}}^\text{hhh} \) is given in Ref. [2], which has been used in our actual numerical analysis.

In Fig. 2, we plot the cross sections of \( \gamma \gamma \to hh \) for the helicity set \((+,+)\) as a function of the photon-photon collision energy \( E_{\gamma \gamma} \). The five curves correspond to the following cases,

(a) THDM 2-loop: the cross section in the THDM with additional one-loop corrections to the \( hhh \) vertex, \( \Gamma_{\text{THDM}}^\text{hhh} \).

(b) THDM 1-loop: the cross section in the THDM with the tree level \( hhh \) coupling constant \( \lambda_{hhh} \).

(c) SM 2-loop: the cross section in the SM with additional top loop correction to the \( hhh \) coupling \( \Gamma_{\text{SM}}^\text{hhh} \) given in Ref. [2].

(d) SM 1-loop: the cross section in the SM with the tree level \( hhh \) coupling constant \( \lambda_{hhh} \) (= \( \lambda_{hhh} \) for \( \sin(\beta - \alpha) = 1 \)).

(e) For comparison, we also show the result which corresponds to the SM 1-loop result with the effective \( hhh \) coupling \( \Gamma_{\text{THDM}}^\text{hhh} \).

In the left figure, there are three peaks in the case (a) (THDM 2-loop). The one at the lowest \( E_{\gamma \gamma} \) is the peak just above the threshold of \( hh \) production. There the cross section is by about factor three enhanced as compared to the SM prediction due to the effect of \( \Delta \Gamma_{\text{THDM}}^\text{hhh} / \Gamma_{\text{SM}}^\text{hhh} \approx 120\% \) because of the dominance of the pole diagrams in \( \gamma \gamma \to hh \). The
Figure 2: The cross section $\hat{\sigma}(++, +)$ for the sub process $\gamma\gamma \to hh$ with the photon helicity set $(+, +)$ as a function of the collision energy $E_{\gamma\gamma}$. In the left [right] figure the parameters are taken to be $m_h = 120$ [160] GeV for $m_\Phi(\equiv m_H = m_A = m_{H^\pm}) = 400$ GeV, $\sin(\beta - \alpha) = 1$, $\tan \beta = 1$ and $M = 0$.

The second peak at around $E_{\gamma\gamma} \sim 400$ GeV comes from the top quark loop contribution which is enhanced by the threshold of top pair production. Around this point, the case (a) can be described by the case (e) (SM+$\Gamma_{hhh}^{\mathrm{THDM}}$). For $E_{\gamma\gamma} \sim 400$-600 GeV, the cross section in the case (a) deviates from the case (c) (SM 2-loop) due to both the charged Higgs loop effect and the effect of $\Delta \Gamma_{hhh}^{\mathrm{THDM}}/\Gamma_{hhh}^{\mathrm{SM}}$. The third peak at around $E_{\gamma\gamma} \sim 850$ GeV is the threshold enhancement of the charged Higgs boson loop effect, where the real production of charged Higgs bosons occurs. The contribution from the non-pole one-loop diagrams are dominant.

In the right figure, we can see two peaks around $E_{\gamma\gamma} \sim 350$-400 GeV and 850 GeV. At the first peak, the contribution from the pole diagrams is dominant so that the cross section is largely enhanced by the effect of $\Delta \Gamma_{hhh}^{\mathrm{THDM}}/\Gamma_{hhh}^{\mathrm{SM}}$ by several times 100% for $E_{\gamma\gamma} \sim 350$ GeV. It also amounts to about 80% for $E_{\gamma\gamma} \sim 400$ GeV. For $E_{\gamma\gamma} < 600$-700 GeV, the result in the case (e) gives a good description of that in the case (a). The second peak is due to the threshold effect of the real $H^+H^-$ production as in the left figure.

In Fig. 3, the full cross section of $e^-e^- \to \gamma\gamma \to hh$ is given from the sub cross sections by convoluting the photon luminosity spectrum[8]. In our study, we set $x = 4E_\omega/\omega_0^2 = 4.8$ where $E_\omega$ is the energy of electron beam, $\omega_0$ is the laser photon energy and $m_e$ is the electron mass. In order to extract the contribution from $\hat{\sigma}(++, +)$ that is sensitive to the $hhh$ vertex, we take the polarizations of the initial laser beam to be both $-1$, and those for the initial electrons to be both $+0.45$. The full cross section for $m_\Phi = 400$ GeV has similar energy dependences to the sub cross section $\hat{\sigma}(++, +)$ in Fig. 2, where corresponding energies are rescaled approximately by around $\sqrt{s} \sim E_{\gamma\gamma}/0.8$ due to the photon luminosity spectrum. For smaller $m_\Phi$, the peak around $\sqrt{s} \sim 350$ GeV becomes lower because of smaller $\Delta \Gamma_{hhh}^{\mathrm{THDM}}/\Gamma_{hhh}^{\mathrm{SM}}$.

In Fig. 4, five curves correspond to the cases (a) to (e) in Fig. 2. In the left figure, one
can see that the cross section is enhanced due to the enlarged $\Gamma_{\text{THDM}}^{hhh}$ for larger values of $m_\Phi$ which is proportional to $m_\Phi^4$ (when $M \approx 0$). This implies that the cross section for these parameters is essentially determined by the pole diagram contributions. The effect of the charged Higgs boson loop is relatively small since the threshold of charged Higgs boson production is far. Therefore, the deviation in the cross section from the SM value is smaller for relatively small $m_\Phi$ (10-20% for $m_\Phi < 300$ GeV due to the charged Higgs loop effect) but it becomes rapidly enhanced for greater values of $m_\Phi$ ($\mathcal{O}(100)$% for $m_\Phi > 350$ GeV due to the large $\Delta \Gamma_{hhh}^{\text{THDM}}$). A similar enhancement for the large $m_\Phi$ values can be seen in the right figure. The enhancement in the cross section in the THDM can also be seen for $m_\Phi < 250$ GeV, where the threshold effect of the charged Higgs boson loop appears around $\sqrt{s} \approx 600$ GeV in addition to that of the top quark loop diagrams. For $m_\Phi = 250-400$ GeV, both contributions from the charged Higgs boson loop contribution and the effective $hhh$ coupling are important and enhance the cross section from its SM value by 40-50%.

## 4 Conclusions

In this paper, we have analysed the new physics loop effects on the cross section of $\gamma \gamma \rightarrow hh$ in the THDM with SM-like limit including the next to leading effect due to the extra Higgs boson loop diagram in the $hhh$ vertex. Our analysis shows that the cross section can be largely changed from the SM prediction by the two kinds of contributions; i.e., additional contribution by the charged Higgs boson loop effect, and the effective one-loop $hhh$ vertex $\Gamma_{hhh}^{\text{THDM}}$ enhanced by the non-decoupling effect of extra Higgs bosons. The cross section strongly depends on $m_h$ and $\sqrt{s}$ and also on $m_\Phi$. The approximation of the full cross section in the case (a) (THDM 2-loop) by using the result in the case (e) (SM+$\Gamma_{hhh}^{\text{THDM}}$) is a good description for $\sqrt{s} \ll 2m_\Phi/0.8$. On the other hand, in a wide region between
threshold of top pair production and that of charged Higgs boson pair production, both the contributions (those from charged Higgs boson loop effect and from $\Gamma_{\text{THDM}}^{hhh}$) are important. In the region below the threshold of the real production of extra Higgs bosons, cross section is largely enhanced from the SM value by the effects of the charged Higgs boson loop and the effective $\Gamma_{\text{THDM}}^{hhh}$ coupling. These New Physics effects would be detectable at the future Photon Linear Collider.

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References


Measuring Higgs boson associated Lepton Flavour Violation in electron-photon collisions at the ILC *

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We study the LFV Higgs production processes \(e^-\gamma \rightarrow \ell^-\varphi\) (\(\ell = \mu, \tau; \varphi = H, A\)) as a probe of Higgs mediated LFV couplings at an electron-photon collider, where \(H\) and \(A\) are extra CP even and odd Higgs bosons, respectively, in the two Higgs doublet model. Under the constraints from the current data of muon and tau rare decay, the cross section can be significantly large. It would improve the experimental upper bounds on the effective LFV coupling constants. In addition, the chirality nature of the LFV Higgs coupling constants can be measured by selecting electron beam polarizations.

1 Introduction

Lepton Flavour Violation (LFV) is clear evidence of new physics beyond the standard model (SM). It can be naturally induced in various new physics scenarios such as supersymmetric extensions of the SM. The origin of LFV would be related to the structure of the fundamental theory at high energies. Therefore, new physics models can be explored by measuring the LFV processes. In the minimal supersymmetric SM with heavy right-handed neutrinos (MSSMRN), the LFV Yukawa interactions can be radiatively generated via the slepton mixing \([2, 3]\). The slepton mixing can be induced by the running effect from the neutrino Yukawa interaction even when flavour blind structure is realized at the grand unification scale \([2]\).

The experimental bound on the effective LFV Yukawa couplings have been studied extensively \([4, 5, 6]\). These constraints will be improved at PSI MEG \([7]\) and J-PARC COMET \([8]\) experiments via muon rare decays, and at CERN LHCb \([9]\) and KEK super-B factory \([10]\) via tau rare decays. In addition, collider signatures of the LFV phenomena have also been investigated at the CERN Large Hadron Collider (LHC) \([11]\), the International Linear Collider (ILC) \([12]\), and the Neutrino Factory \([13]\). These collider experiments would be useful to test the Higgs-boson-associated LFV couplings \([14, 15, 6, 16]\).

In this report, we discuss the physics potential of the LFV Higgs boson production process \(e^-\gamma \rightarrow \ell^-\varphi\) (\(\ell = \mu, \tau; \varphi = h, H, A\)) where \(h, H\) and \(A\) are neutral Higgs bosons. It can be an useful tool for measuring Higgs-boson-mediated LFV parameters in two Higgs doublet models (THDMs) including Minimal Supersymmetric SMs (MSSMs). The total cross sections for these processes can be large for allowed values of the LFV couplings under the constraint from the current experimental data. Measuring these processes, the bounds for the Higgs boson associated LFV coupling constants can be improved significantly.

\*This proceeding paper is based on Ref \([1]\).
Furthermore, the chirality of these couplings can be measured by using the polarized initial electron beam.

2 Higgs boson associated LFV coupling constants

The effective Yukawa interaction for charged leptons is given in the general framework of the THDM by [5, 6]

\[ \mathcal{L}_{\text{lepton}} = - \delta R_i \{ Y_{\ell_i} \delta_{ij} \Phi_1 + (Y_{\ell_i} e_{ij}^L + e_{ij}^R Y_{\ell_i}) \Phi_2 \} \cdot \ell_j + \text{H.c.,} \]

(2.21)

where \( \ell_{R_i} (i = 1 - 3) \) represent isospin singlet fields of right-handed charged leptons, \( L_i \) are isospin doublets of left-handed leptons, \( Y_{\ell_i} \) are the Yukawa coupling constants of \( \ell_i \), and \( \Phi_1 \) and \( \Phi_2 \) are the scalar iso-doublets with hypercharge \( Y = 1/2 \). Parameters \( e_{ij}^X \) \((X = L, R)\) can induce LFV interactions in the charged lepton sector in the basis of the mass eigenstates. In Model II THDM [17], \( e_{ij}^X \) vanishes at the tree level, but it can be generated radiatively by new physics effects [3]. The effective Lagrangian can be rewritten in terms of physical Higgs boson fields. Assuming the CP invariant Higgs sector, there are two CP even Higgs bosons \( h \) and \( H \) \((m_h < m_H)\), one CP odd state \( A \) and a pair of charged Higgs bosons \( H^{\pm} \).

From Eq. (2.21), interaction terms can be deduced to [3, 6]

\[ \mathcal{L}_{\text{LFV}} = - \frac{m_{l_\ell}}{v \cos^2 \beta} \left( \kappa_{11}^L P_L e + \kappa_{11}^R \tau_P L_\ell \right) \{ \cos(\alpha - \beta)h + \sin(\alpha - \beta)H - iA \} + \text{H.c.,} \]

(2.22)

where \( P_L \) is the projection operator to the left-handed fermions, \( m_{l_\ell} \) are mass eigenvalues of charged leptons, \( v = \sqrt{2} \sqrt{(\Phi_1^L)^2 + (\Phi_1^R)^2} \) \((\approx 246 \text{ GeV})\), \( \alpha \) is the mixing angle between the CP even Higgs bosons, and \( \tan \beta \equiv \langle \Phi_2^0 \rangle / \langle \Phi_1^0 \rangle \).

Once a new physics model is assumed, \( \kappa_{ij}^X \) can be predicted as a function of the model parameters. In supersymmetric SMs, LFV Yukawa coupling constants can be radiatively generated by slepton mixing. Magnitudes of the LFV parameters \( \kappa_{ij}^X \) can be calculated as a function of the parameters of the slepton sector. For the scale of the dimensionful parameters in the slepton sector to be of TeV scales, we typically obtain \( |\kappa_{ij}^X|^2 \sim (1-10) \times 10^{-7} \) [2, 3]. In the MSSM only \( \kappa_{ij}^R \) are generated by the quantum effect via the neutrino Yukawa couplings assuming flavour conservation at the scale of right-handed neutrinos.

Current experimental bounds on the effective LFV parameters \( \kappa_{ij}^X \) are obtained from the data of non-observation for various LFV processes [18]. For \( e^\mu \tau \) mixing, we obtain the upper bound from the semi-leptonic decay \( \tau \to e\gamma \) [5]: \( |\kappa_{31}^L|^2 + |\kappa_{31}^R|^2 \leq 6.4 \times 10^{-6} \left( \frac{m_A}{100 \text{ GeV}} \right)^4 \left( \frac{m_\tau}{m_\mu} \right)^4 \), for \( \tan \beta \gtrsim 20 \) and \( m_A \approx m_H \gtrsim 160 \text{ GeV} \) (with \( \sin(\beta - \alpha) \approx 1 \)). The most stringent bound on \( e^\mu \mu \) mixing is derived from \( \mu \to e\gamma \) data [19] as \((4/9)|\kappa_{31}^L|^2 + |\kappa_{31}^R|^2 \lesssim 4.3 \times 10^{-4} \left( \frac{50}{\tan \beta} \right)^4 \left( \frac{m_\mu}{100 \text{ GeV}} \right)^4 \), for \( \tan \beta \gtrsim 20 \) and \( m_A \approx m_H \gtrsim 160 \text{ GeV} \) (with \( \sin(\beta - \alpha) \approx 1 \)). The upper bound on \((4/9)|\kappa_{31}^L|^2 + |\kappa_{13}^R|^2 \) is expected to be improved at future experiments such as MEG and COMET for rare muon decays by a factor of \( 10^{2-3} \), while that on \(|\kappa_{31}^L|^2 + |\kappa_{13}^R|^2 \) is by \( 10^{1-2} \) at LHCb and SuperKEKB via rare tau decays [7, 8, 9, 10].

3 LFV Higgs production processes

We now discuss the lepton flavour violating Higgs boson production processes \( e^-\gamma \to \ell^-\varphi \) \((\ell = \mu, \tau; \varphi = h, H, A)\) in \( e\gamma \) collisions. The differential cross section is calculated
Figure 1: The production cross section of $e^- \gamma \rightarrow \tau^- A$ as a function of the center-of-mass energy $\sqrt{s_{ee}}$ of the electron-electron system. Solid curve represents the result in the THDM with the maximal allowed value of $|\kappa_{31}|^2$ under the current experimental data in both figures.

by using the effective LFV parameters $\kappa_{ij}^X$ as

$$
\frac{d\hat{\sigma}_{e^-\gamma \rightarrow \ell^- \phi}(\sqrt{s_{ee}})}{d \cos \theta} = \frac{G_F \alpha_{EM} m_e^2 \beta_{\ell \phi}}{16\sqrt{2}s_{ee}} \left| \kappa_{11} \right|^2 \eta_2 \left( \eta_2^2 + 4z^2 \right) - 16z \frac{m_e^2}{s_{ee}}, \tag{3.23}
$$

where $z = (m_e^2 - m^2) / s_{ee}$ and $\beta_{\ell \phi} = \sqrt{\lambda(m_e^2 / s_{ee}, m_e^2 / s_{ee})}$ with $\lambda(a, b) = 1 + a^2 + b^2 - 2a - 2b - 2ab$. The functions are defined as $\eta_2 = 1 + z \pm \beta_{\ell \phi} \cos \theta$ where $\theta$ is the scattering angle of the outgoing lepton from the beam direction. The effective LFV parameters can be written by

$$
|\kappa_{11}|^2 = \left| \kappa_{11}^R \right|^2 (1 - P_e) + \left| \kappa_{11}^L \right|^2 (1 + P_e) \times \begin{cases} 
\cos^2(\alpha - \beta) & \text{for } h \\
\sin^2(\alpha - \beta) & \text{for } H \\
1 & \text{for } A
\end{cases}, \tag{3.24}
$$

where $P_e$ is the polarization of the incident electron beam: $P_e = -1 \ (1)$ represents that electrons in the beam are 100% left- (right-) handed.

At the ILC, a high energy photon beam can be obtained by Compton backward-scattering of laser and an electron beam [20]. The full cross section can be evaluated from that for the sub process by convoluting with the photon structure function as [20]

$$
\sigma (\sqrt{s_{ee}}) = \int_{x_{min}}^{x_{max}} dx \ F_{\gamma/e} (x) \hat{\sigma}_{e^-\gamma \rightarrow \ell^- \phi}(\sqrt{s_{ee}}), \tag{3.25}
$$

where $x_{max} = \xi / (1 + \xi)$. $x_{min} = (m_e^2 + m^2) / s_{ee}$, $\xi = 4E_e \omega_0 / m_e^2$ with $\omega_0$ to be the frequency of the laser and $E_e$ being the energy of incident electrons, and $x = \omega / E_e$ with $\omega$ to be the photon energy in the scattered photon beam. The photon distribution function is given in Ref. [20]. We note that when $\sin(\beta - \alpha) \simeq 1$ and $m_H \simeq m_A$ (In the MSSM, this automatically realizes for $m_A \geq 160 \text{ GeV}$) signal from both $e^- \gamma \rightarrow \ell^- H$ and $e^- \gamma \rightarrow \ell^- A$ can be used to measure the LFV parameters, while the cross section for $e^- \gamma \rightarrow \ell^- h$ is suppressed.

In FIG. 1, we show the full cross sections of $e^- \gamma \rightarrow \tau^- A$ as a function of the center-of-mass energy of the $e^-e^-$ system for $\tan \beta = 50$ and $m_A = 350 \text{ GeV}$. Scattered leptons

60
mainly go into the forward direction, however most of events can be detected by imposing the escape cut $\epsilon \leq \theta \leq \pi - \epsilon$ where $\epsilon = 20$ mrad [21]. The cross section can be around 10 fb with the maximal allowed values for $|\kappa_{21}|^2$ under the constraint from the $\tau \to e\gamma$ data. The results correspond that, assuming the integrated luminosity of the $e\gamma$ collision to be 500 fb$^{-1}$ and the tagging efficiencies of a $b$ quark and a tau lepton to be 60% and 30%, respectively, about $10^3$ of $\tau^\pm b\bar{b}$ events can be observed as the signal, where we multiply factor of two by adding both $e^-\gamma \to \ell^- A \to \ell^- b\bar{b}$ and $e^-\gamma \to \ell^- H \to \ell^- b\bar{b}$. Therefore, we can naively say that non-observation of the signal improves the upper bound for the $e\tau$ mixing by 2–3 orders of magnitude if the backgrounds are suppressed. In FIG. 1 (left), those with a set of the typical values of $|\kappa_{21}|^2$ and $|\kappa_{13}|^2$ in the MSSMRN are shown for $P_{c} = -0.9$ (dashed), $P_{c} = +0.9$ (long dashed), and $P_{e} = 0$ (dotted), where we take $(|\kappa_{21}|^2, |\kappa_{23}|^2) = (2 \times 10^{-7}, 0)$. The cross sections are sensitive to the polarization of the electron beam. They can be as large as 0.5 fb for $P_{c} = -0.9$, while it is around 0.03 fb for $P_{c} = +0.9$. In FIG. 1 (right), the results with $(|\kappa_{21}|^2, |\kappa_{13}|^2) = (2 \times 10^{-7}, 1 \times 10^{-7})$ in general supersymmetric models are shown for each polarization of the incident electrons. The cross sections are a few times 1 fb and not sensitive for polarizations. Therefore, by using the polarized beam of the electrons we can separately measure $|\kappa_{21}|^2$ and $|\kappa_{13}|^2$ and distinguish fundamental models with LFV.

In FIG. 2, the full cross sections of $e^-\gamma \to \mu^- A$ are shown for $\tan \beta = 50$ and $m_A = 350$ GeV. Those with the maximally allowed values for $|\kappa_{21}|^2 = |\kappa_{21}|^2 + |\kappa_{12}|^2$ from the $\mu \to e\gamma$ decay can be 7.3 fb where we have adopted the same escape cut as before discussed. This means that about a few times $10^3$ of the signal $\mu^- b\bar{b}$ can be produced for the integrated luminosity of the $e\gamma$ collision to be 500 fb$^{-1}$, assuming tagging efficiencies to be 60% for a $b$ quark and 100% for a muon, and using both $e^-\gamma \to \mu^- A$ and $e^-\gamma \to \mu^- H$. These results imply that $e\gamma$ collider can improve the bound on the $e\mu$ by a factor of $10^2$–$10^3$. Obtained sensitivity can be as large as those at undergoing MEG and projected COMET experiments. Because of the different dependencies on the parameters in the model, $\mu \to e\gamma$ can be sensitive than the LFV Higgs boson production for very high $\tan \beta (\gtrsim 50)$ with fixed Higgs boson mass. We also note that rare decay processes can measure the effect of
other LFV origin when Higgs bosons are heavy. Therefore, both the direct and the indirect measurements of LFV processes are complementary to each other. In FIG. 2 (left), those in the MSSM are shown for \( P_e = -0.9 \) (dashed), \( P_e = +0.9 \) (long dashed), and \( P_e = 0 \) (dotted), where we take \((|\kappa_{21}^L|^2,|\kappa_{12}^L|^2) = (2 \times 10^{-7}, 0)\). They can be as large as a few times \(10^{-3} \) fb for \( P_e = -0.9 \) and \( P_e = 0 \), while it is around \(10^{-4} \) fb for \( P_e = +0.9 \). In FIG. 2 (right), the results with \((|\kappa_{21}^L|^2,|\kappa_{12}^L|^2) = (2 \times 10^{-7}, 1 \times 10^{-7})\) are shown in general supersymmetric models in a similar manner.

It is understood that these processes are clear against backgrounds. For the processes of \(e^-\gamma\to\tau^-\nu\to\tau^-bb\). The tau lepton decays into various hadronic and leptonic modes. The main background comes from \(e^-\gamma\to W^-Z\nu\), whose cross section is of the order of \(10^2 \) fb. The backgrounds can strongly be suppressed by the invariant mass cut for \(bb\). The backgrounds for the process \(e^-\gamma\to\mu^-\nu\to\mu^-bb\) also comes from \(e^-\gamma\to W^-Z\nu\to \mu^-bb\bar{\nu}\bar{\nu}\) which is small enough. Signal to background ratios are better than \(O(1)\) before kinematic cuts. They are easily improved by the invariant mass cut, so that our signals can be almost background free.

4 Conclusion

We have studied the Higgs boson associated LFV at an electron photon collider. Lots of new physics model can predict the LFV Yukawa interactions. The cross section for \(e^-\gamma\to \ell^-\phi\ (\ell = \mu, \tau; \phi = H, A)\) can be significant for the allowed values of the effective LFV couplings under the current experimental data. By measuring these processes at the ILC, the current upper bounds on the effective LFV Yukawa coupling constants are expected to be improved in a considerable extent. Such an improvement can be better than those at MEG and COMET experiments for the \(e^-\mu^-\phi\) vertices, and those at LHCb and SuperKEKB for the \(e^-\tau^-\phi\) vertices. Moreover, the chirality of the LFV Higgs coupling can be separately measured via these processes by using the polarized electron beam. The electron photon collider can be an useful tool of measuring Higgs boson associated LFV couplings.

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Feasibility study of Higgs pair creation in $\gamma\gamma$ collider

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We studied a feasibility of measuring Higgs boson pair production in a Photon Linear Collider. The optimum energy of $\gamma\gamma$ collision was estimated with a realistic luminosity distribution. We also discussed simulation study for detecting the signal against W boson pair backgrounds.

1 Introduction

As a possible option of the International Linear Collider, feasibility of physics orotundities of high energy photon-photon interaction has been considered. In the high energy photon linear colliders (PLCs), high energy photon beams are generated by inverse Compton scattering between the electron and the laser beams as illustrated in figure 1. Feasibility of the PLC for both physics and technical aspect, has been studied and summarized in [1]. In these study, one assumed integrated luminosity of 3-4 years PLC operation which, for example, may happens after initial operation of $e^+e^-$ mode of the ILC at $\sqrt{s} = 500$GeV.

In this study, we investigated a feasibility of self-coupling of the Higgs boson as an example of a precise measurement with the PLC by assuming an ultimate integrated luminosity, i.e., 10 years operation with a high luminosity parameters.

The Higgs boson self-coupling constant is represented by $\lambda = \lambda^{SM}(1 + \delta\kappa)$ which contributes Higgs boson pair production via a diagram shown in figure 2. Here, $\lambda^{SM}$ is Higgs boson self-coupling constant which is included in the Standard Model. $\delta\kappa$ represent the deviation from the Standard Model.

The self-coupling of the Higgs boson can also be studied in $e^+e^-$ collision via the diagram shown in figure 3. Comparing with the
$e^+e^- \rightarrow ZHH$ channel, where Higgs boson pairs are associated with the $Z$ boson production, the Higgs bosons are produced by $s$ channel via loop diagrams in $\gamma\gamma$ collision. Therefore, contribution of the $\delta\kappa$ to the production cross section is difference for the $e^+e^-$ and for $\gamma\gamma$ and studies in these two modes will be complementary each other. Detail of the theoretical background in this analysis can be found in [2].

2 Sensitivity Study

For optimization photon-photon collision energy, we defined the sensitivity for the $\delta\kappa$ as;

$$sensitivity = \frac{|N(\delta\kappa) - N_{SM}|}{\sqrt{N_{obs}}} = \frac{L|\eta\sigma(\delta\kappa) - \eta\sigma_{SM}|}{\sqrt{L(\eta\sigma(\delta\kappa) + \eta\sigma_{SM})}}$$

where, $N(\delta\kappa)$ is a expected number of events as a function of $\delta\kappa$ and $N_{SM}$ is the number of events expected from the Standard Model. $L, \eta, \sigma(\delta\kappa), \sigma_{SM}, \eta_B$ and $\sigma_B$ are integrated luminosity, detection efficiency of signal, cross section with $\delta\kappa$, cross section with the Standard model, detection efficiency for background events and the cross section for background processes, respectively. For $\eta = 1, \eta_B = 0$, sensitivity is written;

$$sensitivity = \sqrt{L}\frac{[\sigma(\delta\kappa) - \sigma_{SM}]}{\sqrt{\sigma(\delta\kappa)}}$$

The Higgs boson mass of 120GeV and the integrated luminosity of 1000fb$^{-1}$ was assumed in the study. The cross section is calculated by the formula which is described in [3] with a theoretically calculated PLC luminosity spectrum. The sensitivity as a function of the center of mass energy of the $\gamma\gamma$ collision for $\delta\kappa = 1$ and -1 is plotted in figure 4.

From the figure, the optimum energy for the $\gamma\gamma$ collision for Higgs boson mass of 120GeV was found to be around 270GeV.

3 Background

Figure 5 shows cross section as a function of collision energy for photon-photon interactions. Figure 5 indicates that $\gamma\gamma \rightarrow WW$ is main background with the production cross section of about 90pb. On the other hand, $\gamma\gamma \rightarrow HH$.
signal cross section is 0.044fb at optimized energy. Therefore, observation of signal requires background suppression of $10^{-7}$. The other reaction that has large cross section such as $\gamma \gamma \rightarrow WWZ$ and $\gamma \gamma \rightarrow tt$. However, the optimum energy for $\gamma \gamma \rightarrow HH$ is below this threshold for these channel.

### Table 1: Input parameters to CAIN. This parameters set make luminosity peak at optimum energy.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_e[GeV]$</td>
<td>190</td>
</tr>
<tr>
<td>$N/10^{10}$</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma_z[mm]$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\gamma_{x/y}/10^{-6}[mrad]$</td>
<td>2.5/0.03</td>
</tr>
<tr>
<td>$\beta_{x/y}[mm/IP]$</td>
<td>1.5/0.3</td>
</tr>
<tr>
<td>$\sigma_{x/y}[mm]$</td>
<td>96/4.7</td>
</tr>
<tr>
<td>$\lambda_L[nm]$</td>
<td>1054</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Particle</th>
<th>Branching ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{b}b$</td>
<td>0.6774</td>
</tr>
<tr>
<td>$\mu\mu$</td>
<td>0.00024</td>
</tr>
<tr>
<td>$\bar{c}c$</td>
<td>0.02982</td>
</tr>
<tr>
<td>$\tau\tau$</td>
<td>0.06916</td>
</tr>
<tr>
<td>$s\bar{s}$</td>
<td>0.00051</td>
</tr>
<tr>
<td>$gg$</td>
<td>0.0713</td>
</tr>
<tr>
<td>$\gamma\gamma$</td>
<td>0.00223</td>
</tr>
<tr>
<td>$\gamma Z$</td>
<td>0.001084</td>
</tr>
<tr>
<td>$WW$</td>
<td>0.1331</td>
</tr>
<tr>
<td>$ZZ$</td>
<td>0.0152</td>
</tr>
</tbody>
</table>

### 4 Simulation Framework

JLC Study Framework (JSF) is used as simulation framework in this study [15].

The helicity amplitude for the signal is calculated by theoretical calculation program [5]. The helicity amplitude for background processes were calculated by a helicity amplitude calculation package; HELAS [6].

The luminosity distribution used in the analysis were generated using the CAIN [7] program with the input parameters shown in table 1 [8]. The luminosity spectrum simulated with the CAIN is shown in figure 6.

From these helicity amplitude and luminosity spectrum, BASES/SPRING integrated and generated events. Pythia made parton shower and hadronized. Quick detector simulator read particle list that from pythia. Finally, data from Quick Detector Simulator is analyzed.

With this spectrum, we expect signal of 16 event/year, while $10^7$ event/year for background.

### 5 Analysis

The decay branching ratio of the Higgs boson of 120GeV is shown in table 2. Since main decay mode of the Higgs boson of 120GeV is $b$-quark pairs with the branching ratio of about 0.67, we tried the case that both Higgs boson decayed into $b$-quark pairs.

For each event, we applied forced four jets analysis in which a clustering algorithm is applied to an event by changing the clustering parameter until the event is catego-
rized as a four jets event. After the forced four jets analysis, invariant masses for jet pairs were calculated. For a four jets event, we must choose a right jets pairs originating from parent Higgs (or W for the background) bosons out of three possible combinations. For this purpose, we defined $\chi^2$s as:

$$\chi^2_H = \frac{(M_1 - M_H)^2}{\sigma_{j_1}^2} + \frac{(M_2 - M_H)^2}{\sigma_{j_H}^2}$$

$$\chi^2_W = \frac{(M_1 - M_W)^2}{\sigma_{j_1}^2} + \frac{(M_2 - M_W)^2}{\sigma_{j_W}^2}$$

where, $M_1$ and $M_2$ are reconstructed particle mass, $M_H$ and $M_W$ are Higgs boson and W boson mass respectively, with $\sigma_{j_H}$ and $\sigma_{j_W}$ being their resolutions. The jet of the least $\chi^2$ was chosen to be the most probable combination for an event. Figure 7 shows correlation of $\chi^2_H$ and $\chi^2_W$ for the most probable combination. To enhance Higgs boson from the W boson events, we choosed an event satisfies $-140/20 \times \chi^2_H + 140 \geq \chi^2_W$. The mass distributions for the Higgs and W boson events after $\chi^2$ cut are shown in figure 8.

6 b-tagging

By the $\chi^2$ analysis described in previous section, the W boson background was suppressed by 0.0541 while keeping the 46% efficiency for the Higgs boson events. In order further improve signal to background ratio, we applied b-quark tagging method for remaining events.

Figure 9 illustrates a b-quark tagging method we applied. For each track in a reconstructed jet, $N_{\text{sig}} = L/\sigma_L$ was calculated, where L is the least approach to the interaction point of the track in the plane perpendicular to the beam and $\sigma_L$ being its resolution. Then, $N_{\text{off}}(a)$, number of track which has $N_{\text{sig}} > a$, is calculated for each jet as a function of $a$. In current analysis we requied all jets must satisfy $N_{\text{off}}(3.5) \geq 2$. Figure 10 is the $\chi^2$ plot after b-tagging but before $\chi^2$ cuts. We obtained background suppression of $1.35 \pm 0.18 \times 10^{-6}$ and efficiency of signal of $0.1454 \pm 0.0044$, where the errors...
are from statistic of the Monte Carlo simulation. For remaining events, $\chi^2$ cut were applied. As a results, no WW events survived out of $3.85 \times 10^7$ simulated events while keeping signal efficiency of $0.1096 \pm 0.0014$.

7 Summary and prospect

We studied feasibility of measurement of Higgs self-coupling constant at the PLC. For Higgs mass of 120GeV, optimum photon-photon collision energy for observe $\gamma\gamma \rightarrow HH$ was found to be about 270GeV. With a parameters of PLC(TESLA-optimistic), 16events/year is expected for Higgs boson events while main background of $\gamma\gamma \rightarrow WW$ is about $10^7$events/year.

We tried an event selection with kinematical parameters and b-quark tagging by the simulation and found that background suppression of $10^{-7}$ with keeping signal efficiency of about 10% seemed to be possible.

For further analysis, we plan to improve signal efficiency by:
- optimization of selection criteria for $HH \rightarrow b\bar{b}b\bar{b}$ mode.
- study for $HH \rightarrow b\bar{b}WW^*$ decay.

For the background, it is necessary to estimate contribution from $\gamma\gamma \rightarrow ZZ$ events.

8 Acknowledgement

The authors would like to thank the ILC physics working group for valuable discussion and suggestion.

References


Figure 10: A result of b-tag selection. Number of b-tagged jets = 4 is required. Black indicates signal event. Red indicates background event. Number of remained background event is 52.
Analysis of Tau-pair process in the ILD reference detector model

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Tau-pair process has been analyzed in the ILD detector model as a benchmark process for LoI. Results of background rejection, forward-backward asymmetry and polarization measurements are obtained with full detector simulation.

1 Goals for LoI

Tau-pair process \((e^+e^- \rightarrow Z^*, \gamma \rightarrow \tau^+\tau^-)\) at \(\sqrt{s} = 500\) GeV is one of the benchmark processes\(^{[1]}\) proposed by Research Director. According to the report, this process is a good sample to examine detector performances of

- tau reconstruction, aspects of particle flow,
- \(\pi_0\) reconstruction,
- tracking of very close-by tracks.

In this process, tau leptons are highly boosted \((\gamma \sim 140)\), thus decay daughters (mainly charged and neutral pions, muons and electrons) are concentrated in a very narrow angle. Reconstruction of \(\pi_0\) from two photons is especially challenging for the ILC detectors.

Observables for the LoI are cross section, forward-backward asymmetry and polarization of tau leptons. The polarization measurement requires identification of tau decays, including reconstruction of \(\pi_0\). Efficiency and purity of event selection cuts should be also a good measure of detector performance.

For physics motivation, tau-pair process is important as a precision measurement of the electroweak theory. For example, measuring cross section and forward-backward asymmetry of tau-pair process very precisely can probe existence of heavy \(Z'\) boson.

2 Analysis framework and events

2.a Event samples

Events of ILD 0.0 LoI mass production\(^{[2]}\) are used for this study. Events reconstructed and listed at DESY by approximately end of February are used in this analysis. 10.3 M SM events generated in SLAC are processed for background estimation with appropriate event weight.

Since the SLAC events have a polarization issues for tau-pair events, tau-pair events generated in DESY are used instead of SLAC events in this analysis. For other modes including tau, SLAC events are used. Whizard 1.51 and TAUOLA\(^{[14]}\) are used to generate
the DESY events. Statistics of the signal channel is $500 \text{ fb}^{-1}$ both for $e_L^- e_R^+$ and $e_R^- e_L^+$ (total 2.3 M events).

Bhabha process ($e^+e^-$ elastic scattering) is an important background for tau-pair analysis. Since the cross section of Bhabha process is too large ($\sim 17 \text{ nb}$ for each polarization in SLAC events), following preselection is applied to the SLAC events before simulation.

- $|\cos \theta|$ of electron or positron must be smaller than 0.96.
- Opening angle between electron and positron must be larger than 165 deg.

After the preselection, the cross section is reduced to 50-90 pb. $\sim 1.0 \text{ fb}^{-1}$ of preselected Bhabha events are simulated.

Preselection is also applied to $\gamma \gamma \rightarrow \tau \tau$ events with following cuts:

- Opening angle between two taus must be larger than 170 deg.
- Energy sum of two taus is greater than 30 GeV.

The total cross section after the cuts is around 18 pb. Around 100 k events passing preselection are processed.

Integrated luminosity is assumed to be $500 \text{ fb}^{-1}$ each for two polarization setups, $e_L^- e_R^+$ and $e_R^- e_L^+$. Assumed polarization ratio is 80% for electron and 30% for positron (i.e. for $e_L^- e_R^+$ setup 90% of electrons are lefty polarized and 65% of positrons are righty polarized).

2.2 Tau clustering

For tau clustering, an original clustering processor (TaJet) is applied to the output of PandoraPFA. Following is a procedure of the processor.

1. Sort particles in energy order.
2. Select the most energetic charged particle (a tau candidate).
3. Search particles to be associated to the tau candidate. Criteria is:
   (a) Opening angle to the tau candidate is smaller than 50 mrad., or
   (b) Opening angle to the tau candidate is not larger than 1 rad. and invariant mass with the tau candidate is less than 2 GeV ($m_\tau = 1.777 \text{ GeV}$).
4. Combine energy and momentum of the tau candidate and associated particle and treat the combined particle as the new tau candidate.
5. Repeat from 3.
6. After all remaining particles do not meet the criteria, remaining most energetic charged particle is the next tau candidate. (Repeat from 2.)
7. After all charged particles are associated to tau candidates, remaining neutral particles are independently included in the cluster list as neutral fragments.

In the clustering stage, events with $> 6$ tracks are pre-cut to accelerate clustering since $> 99\%$ of tau decays have $\leq 3$ charged particles. Event with only one positive and one negative tau clusters are processed with latter analysis.

70
Table 1: Cut statistics for background suppression. Preselection (See Section 2.a for details) is applied for Bhabha events before these cuts. Number of events are normalized to 500 fb⁻¹. The same statistics is used for (a) and (b): only event weighting is different.

3 Background suppression

Main background of tau-pair analysis is Bhabha (e⁺e⁻ → e⁺e⁻), WW → ℓ⁺ℓ⁻ and γγ → τ⁺τ⁻. Since cross sections of Bhabha and two-photon events are huge (about 10⁴ and 10³ larger than signal, respectively), we need tight selection cuts for those background events. Following cuts are applied to signals and all SM background events after the tau clustering.

1. Number of tracks ≤ 6. Included as a pre-cut in tau clustering processor.

2. Only one positive and one negative tau clusters must exist in the event. (Neutral clusters are allowed.)

3. Opening angle of two tau candidates must be > 178 deg. This cut efficiently suppresses WW → ℓ⁺ℓ⁻ background.

4. ee and µµ events are rejected.

Charged particles depositing > 90% of their energy in ECAL are identified as electrons, and charged particles depositing < 70% of their energy (estimated by curvature of their tracks) in ECAL+HCAL are identified as muons. Events with two electrons or two muons are rejected in this cut. This cut is especially for suppressing Bhabha and e⁺e⁻ → µ⁺µ⁻ events. Signal loss is about 6%.

5. |cosθ| < 0.95 for both tau clusters.

   t-channel Bhabha events are almost completely suppressed by this cut. 20% of signal events are lost.

6. 70 < E_{vis} < 450 GeV. E_{vis} does not include energy of neutral clusters.

   Lower bound suppresses γγ → τ⁺τ⁻ events, and upper bound suppresses Bhabha events. Signal lost is negligibly small.
Figure 1: Distribution of cut values. Left column shows $e^-_L e^+_R$ distribution and Right column shows $e^-_R e^+_L$ distribution. Cuts are applied from top, and ee and $\mu\mu$ veto cuts are applied between second and third rows, whose distributions are omitted.
Figure 2: Angular distribution of $\tau^+$ momentum direction. Number of events are normalized to 500 fb$^{-1}$. Error bars stand for statistical errors in current MC statistics. The same statistics is used for (a) and (b); only event weighting is different.

Table 1 shows the result of these cuts and Figure 1 shows distribution of cut values. Most of the background is effectively cut off by the cuts, $\sim$10% level of the signal. Remaining background is mainly Bhabha, $\gamma\gamma \rightarrow \tau\tau$ and $WW \rightarrow l\nu l\nu$.

Purity of tau selection is 92.4% in $e^-_L e^+_R$ sample and 93.6% in $e^-_R e^+_L$. The difference is mainly from difference of the cross section between each polarization.

Selection efficiency of tau-pair events is literally low (15.8% in $e^-_L e^+_R$ and 16.3% in $e^-_R e^+_L$). However, the ‘nocut’ number contains radiative events, which have effectively lower $\sqrt{s}$ and should not be used in the analysis. These radiative events are cut by the opening angle selection. The real efficiency varies by the definition of the events. The acceptance of ‘softly-radiated’ tau-pair events is determined by the opening angle cuts. Loosing the cut accepts more events, although $\gamma\gamma \rightarrow \tau\tau$ and $WW \rightarrow l\nu l\nu$ background significantly increase.

4 Cross section

Cross section can be easily obtained by count-based method since background amount is low. Assuming background subtraction can be performed in the error of statistics, we obtain number of signal event as $125400 \pm 368$ ($e^-_L e^+_R$) and $103197 \pm 332$ ($e^-_R e^+_L$), ie. 0.29% and 0.32% statistical error, respectively. The statistical error is dominated by signal statistics, so poor statistics of background events in the current MC sample can only have small effect on these numbers (0.30% and 0.33% statistical error, even if background is doubled). Systematic error can be introduced by polarization error, MC incorrespondance to real detector etc., but it cannot be accurately estimated in this stage of detector development and thus not considered now.

5 Forward-backward asymmetry

Figure 2 shows a result on angular distribution of $\tau^+$ leptons ($\tau^-$ events are essentially the same event-by-event since we require opening angle $> 178$ deg.).
Assuming that background can be subtracted effectively, forward-backward asymmetry is calculated by following formulae.

\[
A_{FB} = \frac{N_F - N_B}{N_F + N_B},
\]

\[
\sigma A_{FB} = \sqrt{\left(\frac{\partial A_{FB}}{\partial N_F}\sigma_{N_F}\right)^2 + \left(\frac{\partial A_{FB}}{\partial N_B}\sigma_{N_B}\right)^2},
\]

\[
\sigma N_F = \sqrt{N_F + N_{FBG}}, \quad \sigma N_B = \sqrt{N_B + N_{BBG}},
\]

where \(N_B\) is number of signal events in backward region (\(\cos \theta < 0\)), \(N_F\) is number of signal events in forward region (\(\cos \theta > 0\)), \(N_{BBG}\) is number of background events in the backward region and \(N_{FBG}\) is number of background events in the forward region. The formulae can be reduced to

\[
\sigma A_{FB} = 2\sqrt{\frac{N_B^2(N_F + N_{FBG}) + N_F^2(N_B + N_{BBG})}{(N_F + N_B)^2}}.
\]

Result of the calculation is:

\[
\begin{align*}
\epsilon_L^+ e_R^- : & \quad N_F = 95529, N_B = 29872, N_{FBG} = 9201, N_{BBG} = 1130, A_{FB} = 52.36 \pm 0.25 \text{(30)} \\
\epsilon_R^+ e_L^- : & \quad N_F = 75556, N_B = 27640, N_{FBG} = 5477, N_{BBG} = 1605, A_{FB} = 44.19 \pm 0.28 \text{(31)}
\end{align*}
\]

Statistical accuracy of \(A_{FB}\) is 0.48% and 0.63%, respectively.

6 Decay mode separation

Separating decay modes of tau is essential for the polarization measurement. There are five dominant decay modes of tau, \(\tau^+ \rightarrow e^+ \nu_e \nu_\tau (17.9\%), \tau^+ \rightarrow \mu^+ \nu_\mu \nu_\tau (17.4\%), \tau^+ \rightarrow \pi^+ \nu_\tau (10.9\%), \tau^+ \rightarrow \rho^+ \nu_\tau \rightarrow \pi^+ \nu_\tau (25.2\%),\) and \(\tau^+ \rightarrow a_1^+ \nu_\tau \rightarrow \pi \pi \pi \nu_\tau (9.3\% \text{ (1-prong)}\) and \(9.0\% \text{ (3-prong)})\). Other decay modes (10.3%) include Kaons and multi-pions in other resonant modes or continuum.

We utilize a neural network for the decay mode selection. Two separate networks are trained for 1-prong and 3-prong events. 1-prong decay includes \(e^+ \nu_e \nu_\tau, \mu^+ \nu_\mu \nu_\tau, \pi^+ \nu_\tau, \rho^+ \nu_\tau\) and \(a_1^+ \nu_\tau\) modes. Input variables of the 1-prong neural net are as follows.

- Two lepton-ID values. Likelihood-based lepton ID software was developed, but due to the known issues of the event production the lepton ID is not properly worked on the mass production sample. As a simpler lepton ID, we use ratio between the energy deposit of the electromagnetic calorimeter (ECAL) and the total deposit energy for the electron ID, and ratio between the calorimeter energy deposit and the track momentum for the muon ID. These two variables are included in the neural network. (Variable (a) and (b).)

- Energy of the charged particle and two kinds of energy sums of the neutral particles. The neutral energy sums contain particles whose ECAL energy deposit is > 80%, and < 80% of the total energy deposit, respectively. Particles with ECAL energy deposit < 80% are considered to be hadrons, which contain more spurious particles from a mis-fragmentation of energetic charged particles mainly at HCAL. The energy sums are especially expected to discriminate \(\pi\) mode. (Variable (c), (d) and (e).)
Number of neutral particles except neutral hadrons. Number of photons is a powerful information to separate $\rho$ (expected number of photons is 2) and $a_1$ (expected number of photons is 4). (Variable (f).)

Invariant masses of all reconstructed particles except neutral hadrons and invariant masses of photons. Invariant masses of all reconstructed particles should equal to the masses of intermediate particles, $\rho$ and $a_1$. If photons are reconstructed properly, invariant masses of photons are close to that of $\pi_0$. For the photon / hadron separation, above criteria is used again. (Variable (g) and (h).)

Energy of the third-energetic neutral particle. This variable is also to separate $\rho$ and $a_1$. Since $\rho$ can have at most two photons, energy of the third photon should be small even if it exists in the $\rho$ mode. (Variable (i).)

We use two hidden layers, first layer has 18 neurons and second has 9 neurons. The output neurons are likelihood value of $e^+\pi^-\nu_\tau$, $\mu^+\pi^-\nu_\tau$, $\pi^+\nu_\tau$, $\rho^+\nu_\tau$ and $a_1^-\nu_\tau$ modes (5 neurons), which is set to 1(true)/0(false) by the MC information in the training samples.

For the 3-prong events, only $a_1$ is the discriminating decay mode. Input variables (a), (b), (c), (d), (f), (g), (h) (noted with (a')-(h') in Fig. 4) in the 1-prong selection are also included in the 3-prong selection. There is one additional variable, which is:

Invariant mass of all charged particles. This should equals to the mass of $a_1$ if the decay is $a_1$ mode. (Variable (j').)

We use two hidden layers with 16 and 5 neurons. The only output neuron stands for likelihood value of $a_1$ mode, set to 1/0 in the training samples as well.

For the training, half of the tau-pair events in the mass production are used. Number of epochs is 1000 for both 1-prong and 3-prong network.

Fig. 3 and 4 shows distributions of the input variables, and Fig. 5 shows distribution of the output neurons. The mode selection is applied based on the values of the output neurons, as follows.

- If one or more of the values of the output neurons exceed 0.5, The neurons which gives the highest output value is used as the selection.
- If no output values exceed 0.5, the event is classified as ‘others’.

Table 2 shows the obtained efficiency and purity for the mode selection. These values are obtained with the half of the tau-pair events which are not used for the training. >90% efficiency and purity is obtained for all decay modes except 1-prong $a_1$ decay.

7 Polarization Measurement

7.a Optimal Observable

To identify tau polarization, optimal observables[4] are used for $e^+\pi^-\nu_\tau$, $\mu^+\pi^-\nu_\tau$, $\pi^+\nu_\tau$ and $\rho^+\nu_\tau$ decay modes. Decay distribution of all tau decay can be described as the same form,

$$W = \frac{1}{2}(1 + p\cos \theta_h)$$

(7.32)
Figure 3: Distributions of the input variables for the 1-prong neural network. $e_L^-$ (80%) $e_R^+$ polarization is used for the plots.
Figure 4: Distributions of the input variables for the 3-prong neural network. $e_L^- (80\%) e_R^+$ polarization is used for the plots.
Figure 5: Output variables for the neural net selection. (a)-(e) are the output of the 1-prong neural net for $e^+\nu_e\nu_\tau$, $\mu^+\nu_\tau$, $\pi^+\nu_\tau$, $\rho^+\nu_\tau$ and $a_1^+\nu_\tau$ modes, respectively. (f) is the output of the 3-prong neural net for $a_1^+\nu_\tau$ identification. $e_L^+$ (80%) $e_R^-$ polarization is used for the plots.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Purity</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e\nu\nu$</td>
<td>98.9%</td>
<td>98.9%</td>
</tr>
<tr>
<td>$\mu\nu\nu$</td>
<td>98.8%</td>
<td>99.3%</td>
</tr>
<tr>
<td>$\pi\nu$</td>
<td>96.0%</td>
<td>89.5%</td>
</tr>
<tr>
<td>$\rho\nu$</td>
<td>91.6%</td>
<td>88.6%</td>
</tr>
<tr>
<td>$a_1\nu$ (1-prong)</td>
<td>67.2%</td>
<td>73.4%</td>
</tr>
<tr>
<td>$a_1\nu$ (3-prong)</td>
<td>91.1%</td>
<td>88.9%</td>
</tr>
</tbody>
</table>

Table 2: Purity and efficiency of the tau decay mode selection with neural networks. Process background is not included in the purity & efficiency numbers.
where \( p \) is polarization of \( \tau \) (-1 to 1) and \( \theta_h \) is the opening angle of polarimator vector \( \vec{h} \) with respect to the \( \tau \) momentum vector. Explicit notation of \( \vec{h} \) varies by the decay modes: for pure-leptonic decay, flight direction of antineutrino is \( \vec{h} \) and for \( \pi^+\nu_\tau \) decay, flight direction of pion is \( \vec{h} \). For the multipion decay, \( \vec{h} \) is constructed from the hadronic current.

To reconstruct \( p \) from a set of observables, we split \( W \) to \( p \)-dependent and \( p \)-independent components such as

\[
W(\xi) = f(\xi) + pg(\xi),
\]

and the optimal observable \( \omega \) is defined as

\[
\omega = \frac{g(\xi)}{f(\xi)}.
\]

By definition, probability density \( P \) at \( \omega \) for the polarization \( p \) gives

\[
P(\omega; p) - P(\omega; p = 0) = \omega
\]

and \( p \) can be easily obtained from the \( \omega \) distribution.

The explicit formula of \( \omega \) for each decay mode is as follows[5].

1. Pure-leptonic decay:

\[
\omega_\tau = \frac{1 + x - 8x^2}{5 + 5x - 4x^2}
\]

where \( x \) is the lepton energy divided by \( \tau \) energy (250 GeV in this case). Since the pure-leptonic decay mode has two missing neutrinos, polarization discrimination power is weaker than semi-leptonic decay modes.

2. \( \pi^+\nu_\tau \) decay:

\[
\omega_{\pi} = 2x - 1.
\]

This mode has maximum polarization discrimination power since \( \vec{h} \) can be fully reconstructed.

3. \( p^+\nu_\tau \) decay: This decay mode has multiple observable particles and thus more complicated formula to describe \( \omega \). Tau momentum direction is unobservable in this decay, so it is integrated out in the \( \omega \) formulation. The explicit formula is:

\[
\omega_p = \frac{(1 + m_\pi^2/Q^2 + 2)(1 + m_\pi^2/Q^2)2\cos^2\theta - 1 + 3\cos^2\beta - 1) - 3\sin^2\theta - 1 + 3\cos^2\beta - 1) - 3\sin^2\theta - 1 + 3\cos^2\beta - 1)} {2\cdot m_\pi^2 - 2\cdot (1 + m_\pi^2/Q^2)(3\cos^2\theta - 1 + 3\cos^2\beta - 1)}
\]

\[
\cos \psi = \frac{x(m_\pi^2 + Q^2) - 2Q^2}{(m_\pi^2 - Q^2)\sqrt{x^2 - 4Q^2/s}}
\]

\[
x = \frac{2E_h}{\sqrt{s}}
\]

where \( E_h \) is the energy sum of \( \rho \) (which equals to the cluster energy), \( Q^2 \) is the invariant mass of the visible particles (should equals to \( m_\rho = 0.77 \) GeV but obtained from the event), \( \sqrt{s} \) is the center-of-mass energy (500 GeV), \( \theta \) is the angle of the \( \rho \) flight direction with respect to \( \tau \) direction in \( \tau \)-rest frame, and \( \beta \) is the angle of the charged pion flight direction with respect to \( \rho \) direction in \( \rho \)-rest frame.
7. b Polarization measurement

Figure 6 shows the $\omega$ distribution for each decay mode passing the neural net selection. For the leptonic mode, most of the events are concentrated on the $\omega \approx 0$ region, reflecting the weak discrimination power. For the $\pi^+ \nu_\tau$ and $\rho^+ \nu_\tau$ modes, $\omega$ distribution is broadly distributed and large difference between left and right polarization can be seen.

Polarization $p$ can be obtained by following procedure.

1. Mode and process background is eliminated from each bin of the $\omega$ histograms and statistical error of background remains included in the error of each bin.

2. Histograms from all decay modes are summed into one histogram.

3. The histogram with polarizing sample (polarization $p$) is divided by non-polarizing sample after normalizing both histograms.

4. Perform linear fit passing $(0, 1)$ to the divided histogram (one parameter fit). Obtained slope stands for $p$.

Figure 7 shows the combined $\omega$ distribution and Figure 8 shows the linear fits to obtain $p$ value. Obtained $p$ is $-63.82 \pm 0.66\%$ ($e_L^+ e_R^-$, 80% and 30%) and $50.83 \pm 0.79\%$ ($e_R^- e_L^+$, 80% and 30%).

8 Summary

Tau-pair process has been analysed in the ILD_00 detector model. After the tau selection cuts, statistical error of cross section measurement is 0.29% ($e^- e^+_L e^-_R$, with 80% and 30% polarization, respectively) and 0.32% ($e^- e^+_R e^-_L$). Process background can be suppressed to around 10% of signal events. Forward-backward asymmetry can be determined with 0.48% and 0.63% statistical error.

Polarization measurement needs separation of decay modes. The neural net selection gives > 91% efficiency and > 88% purity of mode selection for all major decay modes except $a_1 \nu$ 1 prong mode. Polarization analysis of $e^+ e^- \nu_\tau$, $\mu^+ \mu^- \nu_\tau$, $\pi^+ \nu_\tau$ and $\rho^+ \nu_\tau$ decay mode is performed using the optimal observable method, and it results in $P(\tau) = -63.82 \pm 0.66\%$ ($e_L^+ e_R^-$) and $P(\tau) = 50.83 \pm 0.79\%$ ($e_R^- e_L^+$).

The $a_1 \nu_\tau$ mode is not included in the current polarization measurement. For the 3-prong $a_1$ decay, $\tau$ direction can be reconstructed from the vertex information and it can improve the analysis power to the same level as $\pi^+ \nu_\tau$ mode. However, since the branching ratio of 3-prong $a_1$ decay is only about 9%, the expected improvement with 3-prong $a_1$ decay is about 20%.

References

Figure 6: Distribution of the optimal observable for each decay mode. The left column shows distribution of $e^-_L$ (80%) $e^+_R$ (30%) events, and the right column shows distribution of $e^-_R$ (80%) $e^+_L$ (30%) events.
Figure 7: Distribution of the optimal observable after summing up all decay modes.

Figure 8: Ratio of the polarizing sample to the non-polarizing sample. Linear fit is applied to obtain $p$ value.
Chargino and Neutralino Separation with the ILD Experiment

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One of the benchmark processes for the optimisation of the detector concepts proposed for the International Linear Collider is Chargino and Neutralino pair production in an mSugra scenario where $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^0$ are mass degenerate and decay into $W^\pm \tilde{\chi}_1^0$ and $Z^0 \tilde{\chi}_1^0$, respectively. In this case the separation of both processes in the fully hadronic decay mode is very sensitive to the jet energy resolution and thus to the particle flow performance. The mass resolutions and cross-section uncertainties achievable with the ILD detector concept are studied in full simulation at a center of mass energy of 500 GeV, an integrated luminosity of 500 \( fb^{-1} \) and beam polarisations of \( P(e^+, e^-) = (30\%, -80\%) \). For the $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^0$ pair production cross-sections, statistical precisions of 0.84\% and 2.75\% are achieved, respectively. The masses of $\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ can be determined with a statistical precision of 2.9 GeV, 1.7 GeV and 1.0 GeV, respectively.

1 Introduction

In anticipation of the International Linear Collider (ILC), a proposed $e^+ e^-$ collider with center-of-mass energies between 90 and 500 GeV, upgradable to 1 TeV, and polarised beams, several detector concepts are being discussed. In order to evaluate the performance of these concepts, benchmark processes have been chosen which are challenging for key aspects of the detector designs [1].

In order to test the jet energy resolution, a supersymmetric scenario which assumes non-universal soft SUSY-breaking contributions to the Higgs masses has been defined. In this scenario, the mass differences between the lightest SUSY particle (LSP) and the heavier gauginos become large, while at the same time the sleptons are so heavy that gaugino decays into sleptons are kinematically forbidden. The corresponding benchmark point has been defined in [1] as “Point 5” with the following SUSY parameters:

\[
m_0 = 206 \text{ GeV}, \quad m_{1/2} = 293 \text{ GeV}, \quad \tan \beta = 10, \quad A = 0, \quad \mu = 375 \text{ GeV}
\]

With a top quark mass of $M_t = 178 \text{ GeV}$, the following gaugino masses are obtained by Spheno [2]:
\[ M_{\tilde{\chi}_1^0} = 115.7 \text{ GeV}, \quad M_{\tilde{\chi}_1^\pm} = 216.5 \text{ GeV}, \quad M_{\tilde{\chi}_2^0} = 216.7 \text{ GeV}, \quad M_{\tilde{\chi}_3^0} = 380 \text{ GeV}. \quad (1.42) \]

The lightest sleptons are even heavier than the gauginos, thus leading to branching fractions of 99.4% for the decay \( \tilde{\chi}_1^\pm \rightarrow W^\pm \tilde{\chi}_1^0 \) and 96.4% for \( \tilde{\chi}_2^0 \rightarrow Z^0 \tilde{\chi}_1^0 \):

\[ M_{\tilde{\tau}_1} = 230.8 \text{ GeV} \quad M_{\tilde{e}_R} = 237.4 \text{ GeV} \quad (1.43) \]

In order to benchmark the jet energy reconstruction, the fully hadronic decay mode of the gauge bosons is considered here. In this mode, Chargino and Neutralino events can only be separated via the mass of the vector bosons they decay into. The motivation of this study is not to evaluate the final precision which could be achieved at the ILC by combining several final states, or even by performing threshold scans, but to test the detector performance in the most challenging decay mode.

The analysis is performed at a center of mass energy of 500 GeV for an integrated luminosity of 500 fb\(^{-1}\) with beam polarisations of \( P(e^+, e^-) = (30\%, -80\%) \). It is based on a detailed simulation of the ILD detector based on GEANT4 [3], which is described briefly in the next section. Section 3 discusses the event reconstruction and selection procedure, including a pure Standard Model control selection. The results for the cross-section and mass measurement are presented in sections 4 and 5, respectively.

2 The ILD Detector Concept and its Simulation

The proposed ILD detector has been described in detail in the ILD Letter of Intent [4]. Its main characteristics comprise a time projection chamber as a main tracking device, which is complemented by silicon tracking and vertexing detectors, and highly granular electromagnetic and hadronic calorimeters as required for the particle flow approach [5]. Both, tracking system and calorimeters, are included in a solenoidal magnetic field with a strength of 3.5 T provided by a superconducting coil. The magnetic flux is returned in an iron yoke, which is instrumented for muon detection. Special calorimeters at low polar angles complement the hermeticity of the detector and provide luminosity measurement.

While previous studies were based on fast simulation programs which smear four-vectors with expected resolutions, we have used a full GEANT4 based simulation of all ILD components. Many details are included, in particular gaps in the sensitive regions and realistic estimates of dead material due to cables, mechanical support, cooling and so on.

With this detector simulation, the following performance has been achieved [4]: For tracks with a transverse momentum \( p_t \) larger than 1 GeV, the tracking efficiency is 99.5% across almost the entire polar angle range of \(|\cos \theta| < 0.995\) covered by the tracking detectors, with a \( p_t \) resolution of better than \( \sigma_{p_t}/p_t = 2 \times 10^{-5} \oplus 1 \times 10^{-3}/(p_t \sin \theta) \). The calorimetric system has been designed to deliver a jet energy resolution of 3.0% to 3.7% over a large range of energies from 250 GeV down to 45 GeV for polar angles \( \theta \) in the range \(|\cos \theta| < 0.9\). The luminosity is expected to be known to \( 10^{-3} \) from measurements of the Bhabha scattering cross-sections at small angles. The beam polarisations and the beam energies will be measured to \( \delta P/P = 0.25\% \) and \( 2 \times 10^{-4} \), respectively by dedicated instrumentation in the beam delivery system.

The event sample used in this analysis has been generated using the matrix element generator Whizard [6]. It comprises all Standard Model processes plus all kinematically
accessible SUSY processes in the chosen scenario. In total, about $12 \times 10^6$ events have been generated and processed through the full simulation and reconstruction chain for this analysis.

3 Event Reconstruction and Selection

The reconstruction and also the first event selection steps are implemented in the Marlin-Reco framework [7]. The central part of the reconstruction for this analysis is the particle flow algorithm Pandora [5], which forms charged and neutral particle candidates - so-called “particle flow objects” or PFOs - from tracks and calorimeter clusters. The resulting list of PFOs for each event is forced into a 4-jet configuration using the Durham algorithm. The jet energy scale is raised by 1%, determined from dijet samples. No special treatment of b-quark jets is considered here.

As a final step of the reconstruction, a constrained kinematic fit [8], which requires the two dijet masses of the event to be equal, is performed on each event. All three possible jet pairings are tested. The resulting improvement in mass resolution is evaluated on Standard Model events, as described in section 3.b.

3.a SUSY Selection

The major part of the Standard Model events is rejected by applying the following selection to all events in the SUSY and SM samples:

- In order to eliminate pure leptonic events, the total number of tracks in the event should be larger than 20 and each jet has to contain at least two tracks.
- Since the two LSPs escape undetected, the visible energy of the event $E_{\text{vis}}$ should be less than 300 GeV. In order to remove a substantial fraction of 2-photon events with very low visible energy, $E_{\text{vis}} > 100$ GeV is required as well.
- To ensure a proper jet reconstruction, each jet should have a reconstructed energy of at least 5 GeV and a polar angle $\theta$ fulfilling $\cos(\theta_{\text{jet}}) > 0.99$.
- 2-jet events are rejected by requiring the distance parameter of the Durham jet algorithm for which the event flips from 4-jet to 3-jet configuration, $y_{34}$ to be larger than 0.001.
- Coplanar events (e.g. $W^+W^-$ with ISR/beamstrahlung photons) are removed by requiring $|\cos(\theta)|$ of the missing momentum to be smaller than 0.99.
- No lepton candidate with an energy larger than 25 GeV is allowed in order to suppress semi-leptonic events.

The upper part of table 1 shows the reduction for these cuts. The selection efficiency of hadronic Chargino and Neutralino pair events is very high, 88.1% and 90.8%, respectively. Therefore, we will refer to this stage in the selection process as “high efficiency” selection. Although the SM background is significantly reduced already by these cuts, the contribution from 4-fermion events is still large, about 6 times the Chargino signal.

Figure 1a) shows the reconstructed boson mass distribution as obtained by the constrained kinematic fit after these selection cuts. A large fraction of the remaining Standard
Model background features low invariant dijet masses, but nevertheless a sizable amount of background remains also in the signal region.

For the cross-section measurement, the sample is therefore cleaned further by four additional cuts:

- The number of particle flow objects (PFOs) in each jet should be \( N_{\text{PFO}} > 3 \) in order to reject \( \tau \) jets more effectively.
- The direction of the missing momentum should fulfill \( |\cos \theta_{\text{miss}}| < 0.8 \): This cut is quite powerful to reject all kinds of SM backgrounds, which tend to peak in the forward region, while the signal follows a flat \( \cos \theta_{\text{miss}} \) distribution. Nevertheless, it reduces the signal efficiency substantially, which could be avoided for example by placing a more stringent cut on the missing mass instead (see next item). However, the missing mass distribution of the signal directly depends on the LSP mass, thus it should not be too finely tuned to specific mass values, since we want to measure the gaugino masses. The prediction of a flat \( \cos \theta_{\text{miss}} \) distribution depends only on the spin, and can thus be considered model-independent (within SUSY).
- The missing mass should be larger than 220 GeV to further reject 6-fermion events (semi-leptonic \( t\bar{t} \)). The value of this cut is chosen such that it is in a region with no SUSY contribution, i.e. where the data should agree with the SM expectation. Thus in a real experiment an adequate cut position could be found from the data. For this reason, no upper cut is placed on \( M_{\text{miss}} \), since other SUSY processes contribute there, and it would not be trivial to determine a suitable cut value from real data.
- The kinematic fit constraining the two dijet masses to be equal should converge for at least one jet pairing: This is necessary in order to use the fit result for further analysis. The efficiency and resolution of the fit can be cross-checked easily on real data, for instance with the control selection described in the previous section.

The obtained reduction due to these cuts is shown in the last four lines of table 1. The final distribution of the reconstructed boson mass, again obtained by the constrained kinematic fit, is displayed in figure 1b. It illustrates the achieved boson mass resolution and thus \( W \) and \( Z \) pair separation, however at significantly reduced efficiency. Fitting the total spectrum by a fourth order polynomial for the background plus the sum of two Breit-Wigner functions folded with a Gaussian for the \( W \) and \( Z \) contributions, the mass resolutions can be determined to 3.4 %.

Table 2 shows the final purity and efficiency of signal and major background processes. According to this table, \( e^+e^- \rightarrow qqqq \) is the dominant process in the remaining background.

3.6 Standard Model Control Selection

Since the Chargino and Neutralino separation relies on reconstructing the masses of the \( W \) and \( Z \) bosons from their decay products, the dijet mass resolution is a crucial parameter in this analysis and has to be determined from Standard Model \( W \) and \( Z \) pair events. For this purpose, the “high efficiency” selection from above is applied to all simulated data, inverting only the cut on the visible energy to \( E_{\text{vis}} > 300 \) GeV. This yields an event sample which is vastly dominated by 4-fermion events, with a small contribution from 6-fermion events, but no SUSY events. The corresponding dijet mass spectrum is shown in figure 2.
The mass resolution has been determined for two cases:

a) The jet pairing is chosen such that the difference between the two dijet masses in each event is minimized.

b) A kinematic fit, which constrains the two dijet masses in each event to be equal, is performed for all three possible jet-boson associations. The jet pairing which yields the highest fit probability is chosen.

The resulting mass distributions are fitted with the sum of two Breit-Wigner functions convoluted with a Gaussian, fixing the $W$ and $Z$ widths as well as the $Z$ pole mass to their PDG values and having the same $\sigma$ for both Gaussians, plus a forth order polynomial for all non-resonant contributions.

Figure 3 shows the fitted spectra and the resulting fit parameters. In case a), without the kinematic fit, the dijet mass resolution is determined as $\sigma_{m}^{a} = 3.5$ GeV, while it is reduced to $\sigma_{m}^{b} = 3.0$ GeV when the kinematic fit is applied.

These mass resolutions are even better than in the SUSY case, since the kinematics of the events is more favourable here. While the SM gauge boson pairs are highly boosted and thus finding the correct jet pairing is relatively easy, the bosons in our SUSY scenario are produced nearly at rest, resulting in a higher combinatorical background and a slightly worse boson mass resolution. Nevertheless, a SM control selection will be crucial to demonstrate the level of detector understanding, since the actual SUSY measurement will rely on template distributions and selection efficiencies determined from simulations.

4 Cross-Section Measurement

The cross-sections of $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-$ and $e^+e^- \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0$ can be measured by determining the amount of $W$ and $Z$ pair like events. For the hadronic events we are concerned with here, a 2-dimensional fit in the plane of the two dijet masses per event is performed to obtain the amount of $W$ and $Z$ pair candidates.

Figure 4 shows the dijet mass distributions without the kinematic fit. All three possible jet-boson associations are taken into account in the histograms. 4a shows the dijet mass distribution of all Standard Model and SUSY point5 events passing the selection cuts; 4b is the SM part of 4a; 4c and 4d are statistically independent template samples for $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^0$, made by 500 fb$^{-1}$. Before the fitting, the SM contribution (4b) is subtracted from the distribution of all events (4a). SUSY contributions other than $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^0$ pair are not corrected for, but the contribution is negligibly small.

Figure 4e shows the result of a fit using a linear combination of the Chargino and Neutralino template distributions depicted 4c and d in. The residuals of the fit are displayed in figure 4f. They are sufficiently small and don’t show any specific structures, indicating a well working fit.

While it can be assumed that the SM distribution is well known and can be controlled for instance with the SM selection above, the assumption that the shape of the Chargino and Neutralino spectra is known is not evident. However, the shape of the dijet mass distribution on generator level is quite independent of the details of the SUSY scenario, as long as the decay into real $W$ and $Z$ bosons is open. As discussed already in section 3.b, the shape of the reconstructed dijet mass distribution is influenced by the mass differences between $\tilde{\chi}_1^\pm / \tilde{\chi}_2^0$ and the LSP, which determines the boost of the vector bosons and thus has an
effect on the amount of combinatorical background and the mass resolution. As shown in
the next section, the masses of the gauginos can be measured purely from edge positions in
the energy spectra of the gauge bosons, without any assumption on the cross-section. Thus,
with the gaugino masses measured, we are confident that enough is known about the SUSY
scenario at hand to apply the template method.

The background subtraction and the fit have been performed 10000 times, varying the
bin contents of the SUSY and the SM distribution according to their statistical errors. The
fitted fractions of Chargino and Neutralino contribution have been averaged over all fit
outcomes, while the expected uncertainty is estimated from the variance of the fit results.
Expressed in percent of the expected cross-section, this procedure yields 99.97 ± 0.84% for
the Chargino and 97.50 ± 2.75% for the Neutralino case. In terms of absolute cross-
sections this is equivalent to \( \sigma(e^+e^- \rightarrow \tilde{\chi}^\pm_1\tilde{\chi}^\mp_1) = 124.80 \pm 1.05 \text{ fb}^{-1} \) (MC: 124.84 fb\(^{-1}\)), and
\( \sigma(e^+e^- \rightarrow \tilde{\chi}^0_2\tilde{\chi}^0_2) = 21.90 \pm 0.62 \text{ fb}^{-1} \) (MC: 22.46 fb\(^{-1}\)).

If we use a best jet pairing rather than all combinations for the dijet mass, the statistical
error grows by about 10%. This illustrates the fact that the true jet-boson association
cannot always be found and that the jet pairings not classified as “best” still contain valuable
information.

5 Mass Measurement

The masses of gauginos can be obtained via the energy spectrum of the W and Z boson
candidates, since the distribution of gauginos is box-like with edges determined by the masses
and the center-of-mass energy. Deviations from the pure box shape are due to the finite
width of the W and Z bosons, the beam energy spectrum and the detector resolution. For
the mass measurement, we have to separate the sample on an event-by-events basis into
\( \tilde{\chi}^\pm_1 \) and \( \tilde{\chi}^0_2 \) pair candidates. This is done via the dijet masses, as described in the next
subsection. Afterwards, the edge positions are fitted for both the Chargino and Neutralino
selected sample. Finally, the actual masses are calculated from the edge positions.

5.a Dijet Selection

For each event, the jet pairing with the highest probability in the kinematic fit is chosen.
An event is selected as a Chargino or Neutralino candidate using the following \( \chi^2 \) variables,
which are constructed from the invariant masses calculated from the four-vectors before the
kinematic fit:

\[
\begin{align*}
\chi^2_W(m_1, m_2) &= \frac{(m_1 - m_W)^2 + (m_2 - m_W)^2}{\sigma^2} \\
\chi^2_Z(m_1, m_2) &= \frac{(m_1 - m_Z)^2 + (m_2 - m_Z)^2}{\sigma^2},
\end{align*}
\]

where \( m_1 \) and \( m_2 \) are dijet masses of selected jet-pairs, \( m_W \) and \( m_Z \) are the nominal W
and Z pole masses and \( \sigma = 5 \text{ GeV} \). Events with \( \chi^2_W < 4 \) are classified as \( \tilde{\chi}^\pm \), while events
with \( \chi^2_Z > 4 \) are classified as \( \tilde{\chi}^0_2 \).

Figure 5a) shows the energy spectrum of the selected W candidates, while figure 5b)
presents the same spectrum for the Z candidates. The edge positions can be seen in the
spectra, although the four-fermion background is still large, especially in the Z energy
distribution. The SM background can be fitted separately, as described below.
5. b Fitting the Edges

In the next step, the energy spectra of the \( W \) and \( Z \) candidates are fitted according to the following procedure.

1. First, the Standard Model contribution is fitted with the following function:

\[
f_{SM}(x; t_0, a_{0-2}, \sigma, \Gamma) = \int_{t_0}^{\infty} (a_2 t^2 + a_1 t + a_0) V(t - x, \sigma, \Gamma) dt \tag{5.46}
\]

Here, \( x \) denotes the boson energy, and \( V(x, \sigma, \Gamma) \) is the Voigt function, i.e., a Breit-Wigner function of width \( \Gamma \) convoluted with a Gaussian of resolution \( \sigma \). The \( t_0 \) parameter adjusts the threshold position, while the parameters \( a_0, a_1 \) and \( a_2 \) are used to describe the shape of the plateau with a second order polynomial. The result of this fit is shown in figure 5.

2. Since the available statistics of the Standard Model sample is limited, the actual background used in the SUSY fit is generated from the fitted functions, including fluctuations according to the statistical errors expected from 500 fb\(^{-1} \) of integrated luminosity.

3. Finally, the sum of the SUSY spectra and the SM spectra generated in the previous step are fitted. The SUSY part of the fitting function is similar to the one used on the Standard Model, but this time also an upper edge position \( t_1 \) is introduced. Furthermore, the Gaussian resolution \( \sigma \) is allowed to have two different values at the edge positions, namely \( \sigma_0 \) and \( \sigma_1 \), with intermediate values obtained by linear interpolation.

\[
f(x; t_{0-1}, b_{0-2}, \sigma_{0-1}, \Gamma) = f_{SM} + \int_{t_0}^{t_1} (b_2 t^2 + b_1 t + b_0) V(t - x, \sigma(t), \Gamma) dt \tag{5.47}
\]

\[
\sigma(t; \sigma_0, \sigma_1) = \sigma_0 + \frac{(\sigma_1 - \sigma_0)(t - 80)}{40}. \tag{5.48}
\]

All parameters of \( f_{SM} \) are fixed to the values obtained in the first step. For the \( \tilde{\chi}_1^0 \) fit, \( b_2 \) is also fixed to 0.

Figure 5 shows the results of the SM fit as well as the results of SUSY mass fit for both the Chargino and the Neutralino selection.

To obtain edge positions, the fit is performed 100 times with different Standard Model spectra generated from the SM fit function. As final result, the averaged edge position and error are given:

- \( \tilde{\chi}_1^+ \) lower edge: 79.88 ± 0.19 (MC: 79.80) GeV,
- \( \tilde{\chi}_1^+ \) upper edge: 131.49 ± 0.74 (MC: 132.77) GeV,
- \( \tilde{\chi}_0^0 \) lower edge: 92.34 ± 0.44 (MC: 93.09) GeV, and
- \( \tilde{\chi}_0^0 \) upper edge: 127.67 ± 0.76 (MC: 129.92) GeV.

There is a tendency that the fitted numbers are slightly smaller than MC numbers. Better jet energy correction or modification of the fitting function can reduce the shift, but principally the shift could be corrected with a dedicated MC study.
5.c Mass Determination from Edge Positions

The relation between the gaugino masses and the energy endpoints of the gauge bosons is determined by pure kinematics. Neglecting radiation losses, the energy of the gauginos is equal to the beam energy: \( E_\chi = E_{\text{beam}} \). In the gaugino restsystem, denoted with \(^*\), the energy of the vector boson (i.e. \( W \) or \( Z \)) is given by the usual formula for two-body decays:

\[
E^*_V = \frac{M^2_\chi + M^2_V - M^2_{\text{LSP}}}{2 \cdot M_\chi},
\]

(5.49)

where subscript \( \chi \) denotes the decaying gaugino (i.e. \( \tilde{\chi}^\pm_1 \) or \( \tilde{\chi}^0_2 \)), \( V \) the vector boson (i.e. \( W \) or \( Z \)) and the LSP \( \tilde{\chi}^0_1 \). Boosting this into the laboratory system yields:

\[
E_V = \gamma E^*_V \pm \gamma \beta \sqrt{E^*_V^2 - M^2_V}
\]

(5.50)

The Lorentz boost \( \gamma \) is given by \( \gamma = E_\chi / M_\chi \), and \( \beta = \sqrt{1 - 1/\gamma^2} \). The plus sign will give the upper edge of the allowed energy range, \( E_+ \), and the minus sign the lower one, \( E_- \). For further calculations it is useful to introduce the center point of the allowed energy range, \( E_M \), and its width \( E_D \):

\[
E_M = \frac{E_+ + E_-}{2}, \quad E_D = \frac{E_+ - E_-}{2}
\]

(5.51)

In solving equation 5.50 for the gaugino masses, it is useful to note that \( \gamma \cdot E^*_V = E_M \). With this relation, \( E^*_V \) can be eliminated and thus the LSP mass is obtained from \( E_D \):

\[
E_D = \gamma \sqrt{1 - 1/\gamma^2} \sqrt{E^*_V^2 - M^2_V}
\]

(5.52)

\[
= \sqrt{1 - 1/\gamma^2} \sqrt{\gamma^2 \cdot E^*_V^2 - \gamma^2 \cdot M^2_V}
\]

(5.53)

\[
= \sqrt{1 - 1/\gamma^2} \sqrt{E^2_M - \gamma^2 \cdot M^2_V}
\]

(5.54)

This is a quadratic equation in \( \gamma^2 \), which has two solutions:

\[
\gamma^2 = \frac{1}{2 \cdot M^2_V} \left[ (E_+ \cdot E_- + M^2_V) \pm \sqrt{(E^2_+ + M^2_V)(E^2_+ + M^2_V)} \right]
\]

(5.55)

Inserting this into \( \gamma \cdot E^*_V = E_M \), the LSP mass can be solved for:

\[
M^2_{\text{LSP}} = M^2_V + \frac{E^2_{\text{beam}}}{\gamma^2} \left( 1 - \frac{E_+ + E_-}{E_{\text{beam}}} \right)
\]

(5.56)

For a single energy spectrum, we thus have two solutions in the general case. However with the constraint that the LSP mass has to be the same for both the Chargino and the Neutralino decay, a unique solution can be determined - in this case the one with the upper sign.

For the point5 SUSY parameters, the lower edge of the \( W \) energy spectrum is just equal to the \( W \) rest mass, meaning that the \( W \) bosons from the decay can be produced at rest, with the LSP carrying away all the momentum. This case has to be distinguished from
a configuration where the boost is so large that the $W$ could actually fly into the same
direction as the LSP in the laboratory frame. In this case, since the energy cannot become
lower than the $W$ rest mass, the lower part of the spectrum would be “folded over” and
create a second falling edge above the $W$ mass, precisely at $E_V = \sqrt{M_W^2 + p_{V,min}^2}$, where
\[ p_{V,min} = -\gamma \beta E_V^* + \gamma \sqrt{E_V^* - M_W^2}. \]
Moreover, this case of $E_- = M_W$ corresponds to the
case where the equation for $\gamma^2$ has only one solution, with the $\pm$ term of equation 5.55
vanishing. At this point, the partial derivative $\partial E_-/\partial M_{\tilde{\chi}_\pm^0}$ becomes zero. So the inverse
derivative which appears in the error propagation becomes undefined - or more realistically,
with $E_- = M_W$ not exactly fulfilled, at least very large.

Since the discrimination between models is beyond the scope of this paper, but will be
subject of future studies, we ignore here possible information from the lower edge of the $W$
energy spectrum. Instead, the lower and upper edge of the $Z$ energy spectrum are used to
calculate the masses of $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$. In a second step, the Chargino mass is calculated from
the LSP mass and the upper edge of the $W$ spectrum.

The error propagation is done by using a toy Monte Carlo, taking into account the
correlations between the two masses determined from one energy spectrum. It calculates
the gaugino masses by above equations with edge positions varying randomly according to
their errors obtained from the edge fit. For the center edge positions two patterns were
tried, the fitted edge positions and the MC truth positions.

Table 3 shows the obtained mass values and errors. Without correction of the edge
position, the average value of obtained masses deviates by 3-4 GeV from the MC truth.
This might be due to the fact that phase space was not considered, and could be reduced
by an improved fitting function. With better fitting functions. Without the kinematic fit,
the mass resolution is worse by typically 400 to 500 MeV, which corresponds to 15 to 40% of
the errors, depending on the gaugino considered.

6 Summary

The physics performance of the ILD detector concept has been evaluated using a SUSY
benchmark scenario referred to as “Point 5”, where $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^0$ are nearly mass degenerate
and decay into real $W^\pm$ and $Z^0$ bosons, respectively, plus a $\tilde{\chi}_1^0$. The cross-sections for
Chargino and Neutralino pair production have been obtained by a fit to the two-dimensional
dijet mass spectrum relying on Monte-Carlo templates. The resulting statistical errors are
0.84% in the Chargino case and 2.75% in the Neutralino case.

The gaugino masses have been determined from a fit to the edges of the energy spectra
of the $W^\pm$ and $Z^0$ bosons obtained by a kinematic fit. The resulting mass resolutions are
2.9 GeV, 1.7 GeV and 1.0 GeV for $\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$, respectively. Without the kinematic fit,
the mass resolution is worse by 400 to 500 MeV.

Acknowledgements

We thank Frank Gaede, Steve Aplin, Jan Engels and Ivan Marchesini for simulating the
event samples for this study, and we thank Timothy Barklow and Mikael Berggren for
generating the corresponding four-vector files for SM backgrounds and SUSY, respectively.
We further thank François Richard and Tohru Takeshita for the fruitful discussions about the
analysis. This work has been supported by the Emmy-Noether programme of the Deutsche Forschungsgemeinschaft (grant LI-1560/1-1).

References


\[ \tilde{\chi}_\pm^1 \rightarrow \text{hadrons} \]
\[ \tilde{\chi}_0^2 \rightarrow \text{hadrons} \]
\[ \text{other SUSY} \]
\[ \text{SM } \gamma \gamma \]
\[ \text{SM 6f} \]
\[ \text{SM 4f} \]
\[ \text{SM 2f} \]

### Table 1: Event numbers after each of the selection cuts, normalized to 500 fb\(^{-1}\) and \(P(e^+, e^-) = (30\%, -80\%).\)

<table>
<thead>
<tr>
<th>Processes</th>
<th>No cut</th>
<th>all cuts</th>
<th>Purity</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tilde{\chi}_1^\pm \tilde{\chi}_1^\pm \rightarrow \text{hadrons} )</td>
<td>28529</td>
<td>16552</td>
<td>58%</td>
<td>58%</td>
</tr>
<tr>
<td>(\tilde{\chi}_2^0 \tilde{\chi}_2^0 \rightarrow \text{hadrons} )</td>
<td>5488</td>
<td>3607</td>
<td>13%</td>
<td>65%</td>
</tr>
<tr>
<td>Other SUSY point5</td>
<td>74650</td>
<td>77</td>
<td>0.27%</td>
<td>1.0 \times 10^{-3}</td>
</tr>
<tr>
<td>qqqq (WW, ZZ)</td>
<td>4.29e+06</td>
<td>5885</td>
<td>21%</td>
<td>1.4 \times 10^{-4}</td>
</tr>
<tr>
<td>qq\ell\nu (WW)</td>
<td>5.19e+06</td>
<td>561</td>
<td>2.0%</td>
<td>1.1 \times 10^{-4}</td>
</tr>
<tr>
<td>qqqq\ell\nu (tt)</td>
<td>216996</td>
<td>489</td>
<td>1.7%</td>
<td>2.3 \times 10^{-4}</td>
</tr>
<tr>
<td>(\gamma \gamma \rightarrow qqqq )</td>
<td>26356</td>
<td>397</td>
<td>1.4%</td>
<td>1.5%</td>
</tr>
<tr>
<td>qqqq\nu (WWZ)</td>
<td>9262</td>
<td>268</td>
<td>0.94%</td>
<td>2.9%</td>
</tr>
<tr>
<td>qqq\nu (ZZ)</td>
<td>307779</td>
<td>76</td>
<td>0.27%</td>
<td>2.1 \times 10^{-4}</td>
</tr>
<tr>
<td>qq</td>
<td>9.77e+06</td>
<td>76</td>
<td>0.27%</td>
<td>7.8 \times 10^{-6}</td>
</tr>
<tr>
<td>Other background</td>
<td>3.68e+09</td>
<td>438</td>
<td>1.5%</td>
<td>1.2 \times 10^{-4}</td>
</tr>
</tbody>
</table>

### Table 2: Purity and efficiency of signal and major background sources after the selection cuts and with an invariant dijet mass larger than 65 GeV. The processes in pathentheses indicate the dominant intermediate states.

<table>
<thead>
<tr>
<th>Observables</th>
<th>Obtained value</th>
<th>Error</th>
<th>Error at the true mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m(\tilde{\chi}_1^\pm))</td>
<td>220.90 GeV</td>
<td>2.90 GeV</td>
<td>3.34 GeV</td>
</tr>
<tr>
<td>(m(\tilde{\chi}_2^0))</td>
<td>220.56 GeV</td>
<td>1.72 GeV</td>
<td>1.39 GeV</td>
</tr>
<tr>
<td>(m(\tilde{\chi}_1^0))</td>
<td>118.97 GeV</td>
<td>1.02 GeV</td>
<td>0.95 GeV</td>
</tr>
</tbody>
</table>

### Table 3: Performance on gaugino masses and associated errors. The last column shows errors on masses when the true edge positions are used in the error propagation. MC truth masses are 216.7, 216.5 and 115.7 GeV for \(\tilde{\chi}_1^\pm, \tilde{\chi}_2^0\) and \(\tilde{\chi}_1^0\), respectively.
Figure 1: a) Reconstructed mass of the vector boson candidates after all selection cuts and kinematic fit for the jet pairing with the highest fit probability. b) Same distribution after some additional cuts to enhance the purity.

Figure 2: Dijet mass spectrum for Standard Model selection. The event sample is dominated by 4-fermion events, with a small contribution from 6-fermion events, but doesn’t contain any SUSY events.
Figure 3: Dijet mass distributions a) without and b) with kinematic fit. Fitting the distributions with the sum of two Breit-Wigner functions folded with Gaussian plus a forth order polynomial for the non-resonant background yields dijet mass resolutions of 3.5 GeV (case a) and 3.0 GeV (case b).
Figure 4: Dijet mass distribution for cross-section fit. For (a) and (b) the same events are used, while (c) and (d) are statistically independent of (a).
Figure 5: Mass determination: a) Energy spectrum of the $W^\pm$ candidates reconstructed from events selected as $\tilde{\chi}_1^\pm$ pairs and b) Energy spectrum of the $Z^0$ candidates reconstructed from events selected as $\tilde{\chi}_2^0$ pairs. In both cases, the Standard Model contribution has been fitted separately before fitting the total spectrum.
Hidden particle production at the ILC

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In a class of new physics models, new physics sector is completely or partly hidden, namely, singlet under the Standard Model (SM) gauge group. Such a new physics sector, which we call “hidden sector” throughout this paper, includes some singlet fields. These hidden sector fields, in general, couple with the SM fields through higher dimensional operators. If the cutoff scale of the higher dimensional operators lies around the TeV scale, effects of the hidden fields are accessible at future colliders such as the Large Hadron Collider (LHC) and the International Linear Collider (ILC).

There have been several new physics models proposed that include hidden fields. The most familiar example would be the Kaluza-Klein (KK) modes of graviton in extra dimension scenarios [1] [2]. A singlet chiral superfield in the next to Minimal Supersymmetric Standard Model (MSSM) [3] is also a well-known example, which has interesting implications, in particular, on Higgs phenomenology in collider physics [4]. Another example is the supersymmetry breaking sector of the model proposed in Ref. [5], where a singlet scalar field couples with the SM fields through higher dimensional operators with a cutoff around $\Lambda = 1 - 10$ TeV and its collider phenomenology at the LHC and ILC has been discussed. A very recently proposed scenario [6], “unparticle physics”, belongs to this class of models, whose phenomenological aspects have been intensively studied by many authors.

In this paper, we introduce a hidden scalar field and investigate the hidden scalar production at the ILC. We assume that the hidden scalar couples with only the SM gauge fields through higher dimensional operators suppressed by a TeV-scale cutoff. In this case, at the ILC, this hidden scalar can be produced through the similar process to the SM Higgs boson.
production and with the production cross sections comparable to the Higgs boson one. Thus, the hidden scalar production has interesting implications on the Higgs phenomenology. The crucial difference of the hidden scalar from the Higgs boson lies in that the hidden scalar has nothing to do with the electroweak symmetry breaking. This feature reflects the fact that the hidden scalar couples with mostly the transverse mode of the weak gauge bosons while the Higgs boson couples with mostly their longitudinal modes. Also, the branching ratio of the scalar is quite different from the Higgs boson one. Supposing the hidden scalar is found at the ILC, it is an interesting issue how to distinguish it from the Higgs boson and in this paper, we tackle this issue. Based on realistic Monte Carlo simulations, we study the feasibility of measurements for the hidden scalar productions and its couplings to the SM particles, and show how well the hidden scalar can be distinguished from the Higgs boson at the ILC.

This paper is organized as follows. In the next section, we introduce our theoretical framework and present formulas relevant to our studies. In Sec. 3, we show the results from our Monte Carlo simulations and demonstrate how accurately the ILC can measure the typical features of the scalar and distinguish it from the Higgs boson. The last section is devoted to summary and discussions.

2 Theoretical framework

In this paper, we introduce a real scalar field $\chi$ as a hidden field, which communicates with the SM sector through interactions of the form,

$$\mathcal{L}_{\text{int}} = \frac{c_i}{\Lambda^{d_{\text{SM}}-3}} \chi \mathcal{O}_i^{\text{SM}},$$

where $c_i$ is a dimensionless coefficient, $\Lambda$ is a cutoff scale, and $\mathcal{O}_i^{\text{SM}}$ is an operator of the SM fields with mass dimension $d_{\text{SM}}$. We consider the case that the cutoff, which is naturally characterized by a new physics scale, is around the TeV scale. As an example, it would be easy to imagine a model like the large extra-dimension models [1] whose fundamental scale is in the TeV range or a model with warped extra dimensions [2] where the effective cutoff scale is warped down to the TeV range from the 4-dimensional Planck scale. For a more concrete example, see Ref. [5]. In these models, the above effective interaction can be introduced at tree level.

The theoretical requirements for the SM operator $\mathcal{O}_i^{\text{SM}}$ are that it should be a Lorentz scalar operator and be singlet under the SM gauge group. Although there are many possibilities for such operators, we assume that the hidden scalar couples with only the SM gauge bosons through the operators described as follows*:

$$\mathcal{L}_{\text{int}} = -\frac{1}{2} \sum_A c_A \frac{\chi}{\Lambda} \text{tr} [F_A^{\mu\nu} F_A_{\mu\nu}],$$

where $c_A$ is a dimensionless parameter, and $F_A$'s ($A = 1, 2, 3$) are the field strengths of the corresponding SM gauge groups, U(1)$_Y$, SU(2)$_L$, and SU(3)$_C$. After the electroweak symmetry breaking, Eq. (2.58) is rewritten as interactions between $\chi$ and gluons, photons,

*In fact, it is easy to construct a simple model which can realize this situation. We give comments on this respect in the last section.
$Z$- and $W$-bosons.

\[ \mathcal{L}_{\text{int}} = - \frac{c_{gg}}{4} G_{\mu \nu}^a G_{\mu \nu}^a - \frac{c_{WW}}{2} W_\mu^+ W_\mu^- - \frac{c_{ZZ}}{4} Z_\mu^+ Z_\mu^- \]

where $G_{\mu \nu}^a$, $W_\mu^+$, $Z_\mu^+$ and $F_{\mu \nu}$ are the field strengths of gluon, $W$-boson, $Z$-boson and photon, respectively. The couplings $c_{gg}$ etc. can be described in terms of the original three couplings, $c_1$, $c_2$ and $c_3$, and the weak mixing angle $\theta_w$ as follows:

\[
\begin{align*}
  c_{gg} &= c_3, \\
  c_{WW} &= c_2, \\
  c_{ZZ} &= c_1 \sin^2 \theta_w + c_2 \cos^2 \theta_w, \\
  c_{\gamma \gamma} &= c_1 \cos^2 \theta_w + c_2 \sin^2 \theta_w, \\
  c_{ZZ} &= (-c_1 + c_2) \sin \theta_w \cos \theta_w.
\end{align*}
\]

The hidden scalar can be produced at the ILC through these interactions. The dominant $\chi$ production process is the associated production, $e^+ e^- \rightarrow \gamma^*, Z^* \rightarrow \chi Z$ and $e^+ e^- \rightarrow \gamma^*, Z^* \rightarrow \chi Z$. First, let us consider the process $e^+ e^- \rightarrow \chi Z$. The cross section is calculated as

\[
\frac{d\sigma}{d\cos \theta}(e^+ e^- \rightarrow \chi Z) = \frac{1}{68 \pi s} \sqrt{E_Z^2 - m_Z^2} \frac{s}{s}
\times \left[ \left( c_{ZZ}^2 (g_L^2 + g_R^2) \left( \frac{s}{s - m_Z^2} \right)^2 - c_{ZZ} c_{\gamma \gamma} (g_L + g_R) e \left( \frac{s}{s - m_Z^2} \right) + c_{\gamma \gamma}^2 \right) \frac{E_Z^2}{\Lambda^2} (1 + \cos^2 \theta) \right]
\]

where $\cos \theta$ is the scattering angle of the final state $Z$-boson, $g_L = 2(m_Z/s)(1/2 + \sin^2 \theta_w)$, $g_R = 2(m_Z/s) \sin^2 \theta_w$, and $E_Z = \sqrt{s} \left( 1 + \frac{m_Z^2 - m_\chi^2}{s} \right)$. It is interesting to compare this $\chi$ production process to the similar process of the associated Higgs production (Higgsstrahlung), $e^+ e^- \rightarrow Zh$, through the Standard Model interaction $\mathcal{L}_{\text{int}} = \frac{m_Z^2}{2} h Z^\mu Z_\mu$. In Figure 1, we show the ratio of the total cross sections between $\chi$ and Higgs boson productions as a function of $\Lambda$ at the ILC with the collider energy $\sqrt{s} = 500$ GeV. Here we have taken $c_1 = c_2$ and $m_\chi = m_h = 120$ GeV. The ratio, $\sigma(e^+ e^- \rightarrow \chi Z)/\sigma(e^+ e^- \rightarrow Zh)$, becomes one for $\Lambda_R \simeq 872$ GeV, and it decreases proportionally to $1/\Lambda^2$. Note that in the high energy limit, the $\chi$ production cross section becomes energy-independent.

The coupling manner among $\chi$ and the $Z$-boson pair is different from that of the Higgs boson. As can be understood from Eq. (2.59), $\chi$ couples with the transverse modes of the $Z$-bosons, while the Higgs boson mainly couples with the longitudinal modes. This fact reflects into the difference of the angular distribution of the final state $Z$-boson. In the high energy limit, we find $\frac{d\sigma}{d\cos \theta}(e^+ e^- \rightarrow \chi Z) \propto 1 + \cos^2 \theta$, while $\frac{d\sigma}{d\cos \theta}(e^+ e^- \rightarrow Zh) \propto 1 - \cos^2 \theta$. Figure 2 shows the angular distributions of the associated $\chi$ and Higgs boson productions, respectively. Even if $m_\chi = m_h$ and the cross sections of $\chi$ and Higgs boson productions are
comparable, the angular dependence of the cross section can distinguish the $\chi$ production from the Higgs boson one.

The formula for the process $e^+e^-\to\gamma^*, Z^*\to\gamma\chi$ can be easily obtained from Eq. (2.61) for $Z$-boson by the replacements: $c_{ZZ}\to c_{Z\gamma}$, $c_{Z\gamma\gamma}\to c_{\gamma\gamma}$ and $m_Z\to 0$. As a result, the cross section of the process $e^+e^-\to\gamma\chi$ is found to be

$$\frac{d\sigma}{d\cos\theta} (e^+e^- \to \gamma\chi) = \frac{1}{128\pi s} \sqrt{\frac{E_\chi^2 - m_\chi^2}{s}} \times$$

$$\left(\frac{c_{Z\gamma}^2 (g_L^2 + g_R^2) \left(\frac{s}{s - m_Z^2}\right)^2 - c_{Z\gamma} c_{\gamma\gamma} (g_L + g_R) e \left(\frac{s}{s - m_Z^2}\right) + c_{\gamma\gamma}^2 e^2}{1 + c_{\gamma\gamma}^2 e^2}\right) \frac{E_\gamma^2}{\Lambda_{IR}^2} \left(1 + \cos^2\theta\right)$$

where $E_\gamma = \frac{\sqrt{s}}{2} \left(1 - \frac{m_\chi^2}{s}\right)$. For example, $\sigma(e^+e^-\to\gamma\chi) = 105$ fb at $\sqrt{s} = 500$ GeV with the parameter set: $m_\chi = 120$ GeV, $c_1 = c_2 = 1$ and $\Lambda = 1$ TeV. For the Higgs production, the process $e^+e^-\to\gamma^*\to\gamma h$ is negligible.

Next, we consider $\chi$ decay processes into a pair of gauge bosons. Partial decay widths are given by

$$\Gamma(\chi \to gg) = \frac{c_{gg}^2 m_\chi^3}{8\pi \Lambda^2},$$

$$\Gamma(\chi \to \gamma\gamma) = \frac{c_{\gamma\gamma}^2 m_\chi^3}{64\pi \Lambda^2},$$

$$\Gamma(\chi \to ZZ) = \frac{c_{ZZ}^2 m_\chi^3}{512\pi \Lambda^2} \beta_Z \left(3 + 2\beta_Z^2 + 3\beta_Z^4\right),$$

$$\Gamma(\chi \to WW) = \frac{c_{WW}^2 m_\chi^3}{256\pi \Lambda^2} \beta_W \left(3 + 2\beta_W^2 + 3\beta_W^4\right),$$

$$\Gamma(\chi \to Z\gamma) = \frac{c_{Z\gamma}^2 \tan^2(2\theta_w) m_\chi^3}{128\pi \Lambda^2} \left(1 - \frac{m_\gamma^2}{m_\chi^2}\right)^3,$$  \hspace{1cm} (2.63)

where $\beta_Z = \sqrt{1 - 4(m_Z/m_\chi)^2}$, and $\beta_W = \sqrt{1 - 4(m_W/m_\chi)^2}$. In Figure 3. the branching ratio of the $\chi$ decay is depicted. We see that the branching ratio of the $\chi$ decay is quite different from that of the Higgs boson. In particular, the branching ratio of $\chi \to \gamma\gamma$ can be large, $\text{Br}(\chi \to \gamma\gamma) \approx 0.1$ for the parameter set in Figure 3. On the other hand, the branching ratio of the Higgs boson into two photons in the SM is at most $10^{-3}$, since the coupling between the Higgs boson and two photons are induced through one-loop radiative corrections.

There are several models where the branching ratio of the Higgs boson into two photons is enhanced due to new physics effects. For example, in the MSSM with a large $\tan\beta$ [7], the lightest Higgs boson almost coincides with the up-type Higgs boson of the weak eigenstate. As a result, the Yukawa coupling to bottom quark is suppressed and two-photon branching ratio is relatively enhanced. Another example is the Next to MSSM (NMSSM), where a pseudo scalar ($A^0$) couples to the lightest (SM-like) Higgs boson. In this model, the Higgs boson can decay into two pseudo scalars ($h \to A^0 A^0$) with a sizable branching ratio. If the pseudo scalar is extremely light (lighter than twice the pion mass), it dominantly decays into two photons ($A^0 \to \gamma\gamma$), so that Higgs boson decays into four photons. Since the pseudo-scalar is very light, two photons produced in its decay are almost collinear and will
be detected as a single photon \[4\]. As a result, the Higgs decay into two pseudo-scalars, followed by \(A^0 \rightarrow \gamma\gamma\), effectively enhances the Higgs branching ratio into two photons \[4\]. Therefore, the anomalous branching ratio alone is not enough to distinguish such a Higgs boson from \(\chi\) (in the associated production with a \(Z\)-boson) and the measurements of angular distribution and polarization of the final state \(Z\)-boson are crucial.

Here, let us consider current experimental constraints on the parameters in our framework. Since the hidden scalar \(\chi\) has the properties similar to the Higgs boson, we can use the current experimental limits of the Higgs boson search to constrain model parameters. The most severe constraint is provided by the Higgs boson search in the two-photon decay mode at Tevatron with the integrated luminosity 1 fb\(^{-1}\), which is found to be \(\sigma_h \mathrm{Br}(h \rightarrow \gamma\gamma) \lesssim 0.1\) pb for a light Higgs boson with a mass around 120 GeV \[8\]. Here, \(\sigma_h\) is the Higgs boson production cross section at Tevatron, which is dominated by the gluon fusion process. The Standard Model predicts \(\sigma_h \mathrm{Br}(h \rightarrow \gamma\gamma) \sim 10^{-3}\) pb, far below the bound. However, when this bound is applied to the \(\chi\) production with \(\Lambda \approx 1\) TeV, we obtain a severe constraint on the model parameters. Comparing the couplings between gluons and \(\chi\) to the one between gluons and the Higgs boson in the SM, we find the ratio of the production cross sections at Tevatron as \(\sigma_\chi / \sigma_h \sim 100c_{gg}^2\). For \(m_\chi = 120\) GeV, for example, the main decay mode of the \(\chi\) will be into two gluons and photons, and the branching ratio into two photons is estimated as \(\mathrm{Br}(\chi \rightarrow \gamma\gamma) \simeq (c_{\gamma\gamma}/c_{gg})^2/9\). When we assume the universal coupling \(c_1 = c_2 = c_3\) (equivalent to \(c_{gg} = c_{\gamma\gamma} = c_{WW} = c_{ZZ}\) and \(c_{Z\gamma} = 0\)), the Tevatron bound leads us to \(c_1 = c_2 = c_3 \lesssim 0.1\). However, in this case, the \(\chi\) production cross section becomes two orders of magnitude smaller than the Higgs boson production cross section at the ILC.

There are many possible choices of the parameter set \((c_1, c_2, c_3)\) so as to satisfy the Tevatron bound, while keeping the \(\chi\) production cross section to be comparable to the Higgs boson one. To simplify our discussion, in this paper, we choose a special parameter set: \(c_1 = c_2 = 1\) and \(c_3 = 0\), namely the gluophobic but universal for \(c_1\) and \(c_2\). Therefore, the \(\chi\) production channel through the gluon fusion is closed. For \(m_\chi < 2m_W\), the hidden scalar has a 100\% branching ratio into two photons.

### 3 Monte Carlo Simulation

As estimated in the previous section, if the cutoff is around 1 TeV, the production cross section of the hidden scalar can be comparable to the Higgs boson production cross section at the ILC. There are two main production processes associated with a \(Z\)-boson or a photon. In the following, we investigate each process. In our analysis, we take the same mass for the
hidden scalar and the Higgs boson: $m_\chi = m_h = 120$ GeV, as a reference.

3.a Observables to be measured

The associated hidden scalar production with a Z-boson is very similar to the Higgs production process and their production cross sections are comparable for $\Lambda \simeq 1$ TeV. One crucial difference is that the hidden scalar couples to Z-bosons through Eq. (2.59) so that the Z-boson in the final state is mostly transversely polarized. On the other hand, in the Higgs boson production the interaction between the Higgs boson and the longitudinal mode of the Z-boson dominates. In order to distinguish the hidden scalar from the Higgs boson, we will measure

(1) the angular distribution of the Z-boson in the final state,

(2) the polarization of the Z-boson in the final state.

As shown in the previous section, the branching ratio of the hidden scalar decay is quite different from the Higgs boson one. In our reference parameter set, the hidden scalar decays 100% into two photons. The Higgs boson with $m_h = 120$ GeV dominantly decays into a bottom and anti-bottom quark pair. In order to distinguish the hidden scalar from the Higgs boson, we will measure

(3) the branching ratios into two photons and into the bottom and anti-bottom quark pair through b-tagging.

The associated hidden scalar production with a photon is unique and such a process for the Higgs boson is negligible. We will investigate similar things as in the Z-boson case.

3.b Analysis Framework

For Monte Carlo simulation studies of the hidden scalar productions and decays, we have developed event generators of the processes: $e^+e^- \rightarrow \gamma\chi$ and $e^+e^- \rightarrow Z\chi$ followed by the $\chi \rightarrow \gamma\gamma$ decay, which are now included in physsim-2007a [9]. In the helicity amplitude calculations, we retain the Z-boson wave function if any and replace it with the wave function composed with the daughter fermion-antifermion pair according to the HELAS algorithm [10]. This allows us to properly take into account the gauge boson polarization effects. The phase space integration and generation of parton 4-momenta are performed with BASES/SPRING [11]. Parton showering and hadronization are carried out using PYTHIA6.3 [12] with final-state tau leptons treated by TAUOLA [13] in order to handle their polarizations properly. The background $e^+e^- \rightarrow Zh$ events are generated using the $e^+e^- \rightarrow Z\chi$ generator with the $e^+e^- \rightarrow Z\chi$ helicity amplitudes replaced by corresponding $e^+e^- \rightarrow Zh$ amplitudes and the Higgs decay handled by PYTHIA6.3.

In the Monte Carlo simulations, we set the nominal center-of-mass energy at 500 GeV and assume no beam polarization. Effects of natural beam-energy spread and beamstrahlung
are taken into account according to the beam parameters given in [14]. We have assumed no crossing angle between the electron and the positron beams and ignored the transverse component of the initial state radiation. Consequently, the $Z\chi$ or $\gamma\chi$ system in our Monte-Carlo sample has no transverse momentum.

The generated Monte-Carlo events were passed to a detector simulator (JSF Quick Simulator [15]) which incorporates the ACFA-LC study parameters (see Table 1). The quick simulator created vertex-detector hits, smeared charged-track parameters in the central tracker with parameter correlation properly taken into account, and simulated calorimeter signals as from individual segments, thereby allowing realistic simulation of cluster overlapping. It should also be noted that track-cluster matching was performed to achieve the best energy-flow measurements.

<table>
<thead>
<tr>
<th>Detector</th>
<th>Performance</th>
<th>Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex detector</td>
<td>$\sigma_b = 7.0 \oplus (20.0/p) / \sin^{1/2} \theta$ $\mu$m $</td>
<td>\cos \theta</td>
</tr>
<tr>
<td>Central drift chamber</td>
<td>$\sigma_{pp}/p_T = 1.1 \times 10^{-4} p_T \oplus 0.1$ $%$ $</td>
<td>\cos \theta</td>
</tr>
<tr>
<td>EM calorimeter</td>
<td>$\sigma_E/E = 15 % / \sqrt{E} \oplus 1$ $%$ $</td>
<td>\cos \theta</td>
</tr>
<tr>
<td>Hadron calorimeter</td>
<td>$\sigma_E/E = 40 % / \sqrt{E} \oplus 2$ $%$ $</td>
<td>\cos \theta</td>
</tr>
</tbody>
</table>

Table 1: ACFA study parameters for an LC detector, where $p$, $p_T$, and $E$ are measured in units of GeV.

3.c Event Selection and Results

3.c.1 $e^+e^- \rightarrow Z\chi; \chi \rightarrow \gamma\gamma$ process

Data equivalent to 50 fb$^{-1}$ have been generated for both $e^+e^- \rightarrow Z\chi$ followed by $\chi \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow Zh$ followed by $h \rightarrow \gamma\gamma$. A typical event is displayed in Figure 4. For the $Z\chi \rightarrow q\bar{q}\gamma\gamma$ process, there are two jets and two photons in the final state. In the event selection, it is firstly required that the number of reconstructed particles ($N_{\text{particles}}$) is greater than 4. In the next, the number of photons reconstructed in the calorimeters ($N_{\text{gammas}}$) is greater than 2, and the two photons whose invariant mass is the closest to $m_\chi$ are selected. Finally, the number of jets ($N_{\text{jets}}$) is required to be equal to 2. These selection criteria are summarized in Table 2 together with efficiency of each cut. The distribution of the invariant mass of the two photons which are considered to come from a $\chi$ decay is shown in Figure 5 after imposing all the above selection criteria. In the figure, the grey histogram is for the $e^+e^- \rightarrow Zh$ process where the number of remaining events is much less than that of the

Figure 3: The branching ratio of the hidden scalar ($\chi$) as a function of its mass $m_\chi$ for $c_1 = c_2 = c_3 = 1$. Different lines correspond to the modes, $\chi \rightarrow gg$, $WW$, $\gamma\gamma$ and $ZZ$. 

Figure 4: A typical event for the $e^+e^- \rightarrow Z\chi; \chi \rightarrow \gamma\gamma$ process.
\( e^+e^- \rightarrow Z\chi \) process. Figures 6 and 7 show the \( \chi \) and Higgs production angles (left) and the angular distribution of the reconstructed jets from associated Z-boson decays (right) for the both processes, respectively. As can be seen from these plots, \( \chi \) couples with the transverse modes of the Z-bosons, while the Higgs boson couples with the longitudinal modes. The \( e^+e^- \rightarrow Zh \) followed by \( h \rightarrow A^0A^0 \) process is also analyzed with the same cut conditions and its cut statistics is summarized in Table 2. Here, we have assumed \( \text{Br}(h \rightarrow A^0A^0) = 0.1 \) and \( \text{Br}(A^0 \rightarrow \gamma\gamma) = 1.000 \). The distribution of the invariant mass of the two photons will be similar to Figure 5 in this model, but again we can discriminate the \( \chi \) from the Higgs by looking at the angular distributions. Figure 8 shows the Higgs production angle and the angular distribution of the reconstructed jets from associated Z-boson decays (right) for the \( h \rightarrow A^0A^0 \) process.

<table>
<thead>
<tr>
<th>Cut</th>
<th>( Z\chi; \chi \rightarrow \gamma\gamma )</th>
<th>( Zh; h \rightarrow \gamma\gamma )</th>
<th>( Zh; h \rightarrow A^0A^0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Cut</td>
<td>2187 (1.0000)</td>
<td>142 (1.000)</td>
<td>7087 (1.0000)</td>
</tr>
<tr>
<td>( N_{\text{particles}} \geq 4 )</td>
<td>1738 (0.7947)</td>
<td>106 (0.747)</td>
<td>5692 (0.8032)</td>
</tr>
<tr>
<td>( N_{\text{gammas}} \geq 2 )</td>
<td>1521 (0.8751)</td>
<td>96 (0.906)</td>
<td>4865 (0.8547)</td>
</tr>
<tr>
<td>Cut on ( M_{\gamma\gamma} )</td>
<td>1499 (0.9855)</td>
<td>95 (0.990)</td>
<td>4828 (0.9924)</td>
</tr>
<tr>
<td>( N_{\text{jets}} = 2 ) for ( Y_{\text{cut}} = 0.004 )</td>
<td>1498 (0.9993)</td>
<td>95 (1.000)</td>
<td>4825 (0.9994)</td>
</tr>
<tr>
<td>Total Efficiency</td>
<td>0.6850 ± 0.0099</td>
<td>0.669 ± 0.040</td>
<td>0.6808 ± 0.0055</td>
</tr>
</tbody>
</table>

Table 2: Cut statistics and breakdown of selection efficiency. The numbers inside and outside of parenthesis are the efficiency and the remaining number of events after each cut, respectively.

3.c.2 \( e^+e^- \rightarrow \gamma\chi; \chi \rightarrow \gamma\gamma \) process

Data equivalent to 5.7 \( \text{fb}^{-1} \) have been generated for both signal \( (e^+e^- \rightarrow \gamma\chi \) followed by \( \chi \rightarrow \gamma\gamma \)) and background \( (e^+e^- \rightarrow \gamma\gamma \) with an ISR photon) processes. A typical signal event is displayed in Figure 9. For the \( \gamma\chi \rightarrow \gamma\gamma \) process, there are three photons in the final state. The number of photons reconstructed in the calorimeters \( (N_{\text{gammas}}) \) is required to be equal to 3. It is also required that the energy and the cosine of the polar angle of each photon are greater than 1 GeV and less than 0.999, respectively. Among the photons, two photons whose invariant mass is within \( m_\chi \pm 25 \text{ GeV} \) are considered to be from a \( \chi \) decay. Finally, the cosines of the production angles of both \( \chi \) and the remaining photon are required to be less than 0.99. These selection criteria are summarized in Table 3 together with their efficiencies. The distribution of the invariant mass of two photons which are considered to come from a \( \chi \) decay (left) and the angular distribution of the \( \chi \) (right) are shown in Figure 10 after imposing all the above selection criteria. A peak at \( m_\chi \) can be clearly seen over the grey background histogram with the angular distribution consistent with \( 1 + \cos^2 \theta \).

4 Summary and discussions

If a hidden scalar field appears in a certain class of new physics models around the TeV scale, there are interesting implications for collider phenomenology. In particular, since the
scalar behaves like the Higgs boson in its production process, it is an interesting issue how to distinguish the scalar from the Higgs boson in future collider experiments. We investigated the hidden scalar production at the ILC and addressed this issue based on realistic Monte Carlo simulations.

With the $\chi$ production cross section comparable to the Higgs boson one, the invariant mass distribution reconstructed from two-photon final states due to the decay mode $\chi \rightarrow \gamma \gamma$ shows a clear peak at $m_\chi$. In the $\chi$ production associated with a $Z$-boson, the $\chi$ production angle and the angular distribution of the reconstructed jets from the associated $Z$-boson decay reveal that the hidden scalar couples to transversally polarized $Z$-bosons. On the other hand, the Higgs boson production associated with a $Z$-boson shows clearly different results in angular distributions and distinguishable from the hidden scalar production.

Some comments are in order here. In this paper, we have assumed that the hidden scalar couples with only the SM gauge bosons. In general, one can introduce couplings between the hidden scalar and the SM fermions and the Higgs doublets. Once these couplings are introduced, the phenomenology of $\chi$ productions can be drastically altered from those in this paper. In particular, the coupling of the hidden scalar to the Higgs doublets induces the mixing between $\chi$ and the Higgs boson. This mixing spoils the crucial difference between the hidden scalar and the Higgs boson that the former has nothing to do with the electroweak symmetry breaking while the latter is crucial for it. It is not natural but in practice, we can assume the above unwanted couplings to be small.

In fact, it is easy to introduce a setup where the couplings are naturally suppressed. As a simple example, let us consider a model in the context of the brane world scenario, where there are two different branes with the three spatial dimensions separated in extra-dimensional directions. Suppose that the SM gauge bosons live in the bulk and the hidden scalar resides on one brane while the SM fermions and the Higgs doublets on the other brane. In this setup, the couplings between the hidden scalar and the SM fermions and Higgs doublets are geometrically suppressed, while the hidden scalar couples with the bulk SM gauge bosons.

In our analysis, we have taken a special parameter set for simplicity, namely, the gluophobic ($c_3 = 0$) and universal ($c_2 = c_1$) couplings considering the Tevatron bound. In general, it is not necessary to take $c_3 = 0$ in order to avoid the Tevatron bound. For example, a parameter set, $c_{gg} = c_{\gamma \gamma} = 0.1$ and $c_{ZZ} = 1$, can be consistent with the Tevatron bound while keeping the $\chi$ production cross section comparable to that of Higgs boson. In this case, $c_{Z\gamma} \simeq 0.7$ and the hidden scalar decays into $Z\gamma$ with a sizable branching ratio. It is

<table>
<thead>
<tr>
<th>Cut</th>
<th>$\gamma \chi; \chi \rightarrow \gamma \gamma$</th>
<th>$\gamma \gamma$ with an ISR</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Cut</td>
<td>$600 \ (1.0000)$</td>
<td>$100000 \ (1.0000)$</td>
</tr>
<tr>
<td>$N_{\text{gammas}} = 3$</td>
<td>$575 \ (0.9583)$</td>
<td>$3746 \ (0.0375)$</td>
</tr>
<tr>
<td>$E_{\gamma\gamma} &gt; 1 \ \text{GeV}$</td>
<td>$575 \ (1.0000)$</td>
<td>$3730 \ (0.9959)$</td>
</tr>
<tr>
<td>$</td>
<td>\cos(\theta_j)</td>
<td>\leq 0.999$</td>
</tr>
<tr>
<td>$</td>
<td>M_{\gamma\gamma} - m_\chi</td>
<td>\leq 25 \ \text{GeV}$</td>
</tr>
<tr>
<td>$</td>
<td>\cos(\theta_\chi)</td>
<td>$ and $</td>
</tr>
<tr>
<td>Total Efficiency</td>
<td>$0.9533 \pm 0.0086$</td>
<td>$0.0127 \pm 0.0001$</td>
</tr>
</tbody>
</table>

Table 3: Similar to Table 2 for $e^+e^- \rightarrow \gamma \chi$ and $e^+e^- \rightarrow \gamma \gamma$ with an ISR photon.

106
interesting to study the $\chi$ production through its decay into $Z\gamma$. Also, in this parameter set, the decay mode into two gluons is sizable. It is hence an interesting issue how to distinguish $\chi$ from the Higgs boson through their hadronic decay modes. As can be seen from Eq. (2.61), the coupling between the hidden scalar and the longitudinal mode of the $Z$-boson is proportional to $m_Z/E_Z$ and is sizable at low energy. To measure this coupling through the energy-dependence of the production angle distribution may provide an additional handle to pin down the $\chi$ production. For this purpose, the ILC with low energy could be useful.

In this paper, we have concentrated on the hidden scalar production associated with a $Z$-boson or a photon. It is also interesting to investigate the weak boson fusion process. For example, in the $Z$-boson fusion process, measuring the correlations between the cross section and the azimuthal angle between the final state electron and positron can be used to distinguish the couplings between a scalar and the $Z$-boson with different polarizations.

5 Acknowledgments

The authors would like to thank all the members of the ILC physics subgroup [16] for useful discussions. This study is supported in part by the Creative Scientific Research Grant No. 18GS0202 of the Japan Society for Promotion of Science.
Figure 4: Event displays of $e^+e^- \rightarrow Z\chi$ followed by $\chi \rightarrow \gamma\gamma$. Two jets from the $Z$-boson decay and two photons from the $\chi$ decay can be clearly seen.
Figure 5: The distribution of the invariant mass of two photons which are considered to come from a $\chi$ decay.

Figure 6: The $\chi$ production angle (left) and the angular distribution of the reconstructed jets from associated $Z$-boson decays (right).
Figure 7: The Higgs production angle (left) and the angular distribution of the reconstructed jets from associated $Z$-boson decays (right) for $e^+e^- \rightarrow Zh$ followed by $H \rightarrow \gamma\gamma$.

Figure 8: The Higgs production angle (left) and the angular distribution of the reconstructed jets from associated $Z$-boson decays (right) for $e^+e^- \rightarrow Zh$ followed by $h \rightarrow A^0A^0$. 

110
Figure 9: Event displays of $e^+e^- \rightarrow \gamma \chi$ followed by $\chi \rightarrow \gamma \gamma$. 
Figure 10: The distribution of the invariant mass of two photons which are considered to come from a $\chi$ decay (left) and the angular distribution of the $\chi$ (right) for the $e^+e^- \rightarrow \gamma\chi$ process with background.

References


ICL phenomenology in a TeV scale radiative seesaw model for neutrino mass, dark matter and baryon asymmetry

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We discuss phenomenology in a new TeV scale model which would explain neutrino oscillation, dark matter, and baryon asymmetry of the Universe simultaneously by the dynamics of the extended Higgs sector and TeV-scale right-handed neutrinos. Tiny neutrino masses are generated at the three-loop level due to the exact \(Z_2\) symmetry, by which the stability of the dark matter candidate is guaranteed. The model provides various discriminative predictions in Higgs phenomenology, which can be tested at the Large Hadron Collider and the International Linear Collider.

1 Introduction

In spite of the success of the Standard Model (SM) for elementary particles, it is widely understood that a new model beyond the SM must be considered to explain the phenomena such as tiny neutrino masses and their mixing [1], the nature of dark matter (DM) [2] and baryon asymmetry of the Universe [3].

We here discuss the model in which these problems would be simultaneously explained by the TeV-scale physics [4]. Tiny neutrino masses are generated at the three-loop level due to an exact discrete symmetry, by which tree-level Yukawa couplings of neutrinos are prohibited. The lightest neutral odd state under the discrete symmetry is a candidate of DM. Baryon asymmetry can also be generated at the electroweak phase transition (EWPT) by additional CP violating phases in the Higgs sector [5]. In this framework, a successful model can be made without contradiction of the current data.

The original idea of generating tiny neutrino masses via the radiative effect has been proposed by Zee [6]. The extension with a TeV-scale right-handed (RH) neutrino has been discussed in Ref. [7], where neutrino masses are generated at the three-loop level due to the exact \(Z_2\) parity, and the \(Z_2\)-odd RH neutrino is a candidate of DM. This has been extended with two RH neutrinos to describe the neutrino data [8]. Several models with adding baryogenesis have been considered in Ref. [9]. The following advantages would be in the present model [4]: (a) all mass scales are at most at the TeV scale without large hierarchy, (b) physics for generating neutrino masses is connected with that for DM and baryogenesis, (c) the model parameters are strongly constrained by the current data, so that the model provides discriminative predictions which can be tested at future experiments.

In the following, we first explain the basic properties of the model, and discuss its phenomenology, in particular that at the International Linear Collider (ILC).
2 Model

Two scalar isospin doublets with hypercharge 1/2 (Φ₁ and Φ₂), charged singlet fields (S±), a real scalar singlet (η) and two generation isospin-singlet RH neutrinos (NαR with α = 1, 2) are introduced in our model [4]. We impose an exact Z₂ symmetry to generate tiny neutrino masses at the three-loop level, which we refer as Z₂. We assign Z₂-odd charge to S±, η and NαR, while ordinary gauge fields, quarks and leptons and Higgs doublets are Z₂ even. In order to avoid the flavor changing neutral current in a natural way, we impose another (softly-broken) discrete symmetry (˜Z₂) [10].

We employ so called Type-X Yukawa interaction [11], where ˜Z₂ charges are assigned such that only Φ₁ couples to leptons whereas Φ₂ does to quarks [12, 13, 14];

\[
\mathcal{L}_Y = -y_{eL} L^\dagger Q^i e^j_R - y_{uL} Q^i d^j_R - y_{dL} L^\dagger Q^i \Phi^i_R + \text{h.c.},
\]

(2.64)

where Q^i (L^i) is the ordinary i-th generation left-handed (LH) quark (lepton) doublet, and u^i_R and d^i_R (e^i_R) are RH-singlet up- and down-type quarks (charged leptons), respectively.

We summarize the particle properties under Z₂ and ˜Z₂ in Table 1.

The Yukawa coupling in Eq. (2.64) is different from that in the minimal supersymmetric SM (MSSM) [15]. In addition to the usual potential of the two Higgs doublet model (THDM) with the ˜Z₂ parity and that of the Z₂-odd scalars, we have the interaction terms between Z₂-even and -odd scalars:

\[
\mathcal{L}_{int} = -\sum_{a=1}^{2} \left( \rho_a |\Phi_a|^2 |S|^2 + \sigma_a |\Phi_a|^2 \frac{y^2}{2} \right) - \sum_{a,b=1}^{2} \kappa \epsilon_{ab} (\Phi^c_a \Phi^c_b S^- \eta + \text{h.c.})
\]

(2.65)

where \( \epsilon_{ab} \) is the anti-symmetric tensor with \( \epsilon_{12} = 1 \). The mass term and the interaction for NαR are given by

\[
\mathcal{L}_{Y_N} = \sum_{a=1}^{2} \left\{ \frac{1}{2} m_{N^a_R} N^a_R N^a_R - h^a_i (e^i_R)^c N^a_R S^- + \text{h.c.} \right\}
\]

(2.66)

Although the CP violating phase in the Lagrangian is crucial for successful baryogenesis at the EWPT [5], it does not much affect the following discussions. Thus, we neglect it for simplicity. We later give a comment on the case with the non-zero CP-violating phase.

As Z₂ is exact, the even and odd fields cannot mix. Mass matrices for the Z₂-even scalars are diagonalized as in the usual THDM by the mixing angles α and β, where α diagonalizes the CP-even states, and tan β = (\( \Phi^0_2 \))/\( \Phi^0_1 \) [15]. The Z₂ even physical states are two CP-even (h and H), a CP-odd (A) and charged (H±) states. We here define h and H such that h is always the SM-like Higgs boson when sin(β – α) = 1.

<table>
<thead>
<tr>
<th>Z₂ (exact)</th>
<th>Q^i</th>
<th>u^i_R</th>
<th>d^i_R</th>
<th>L^i</th>
<th>e^i_R</th>
<th>Φ₁</th>
<th>Φ₂</th>
<th>S±</th>
<th>η</th>
<th>N^α_R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z₂ (softly broken)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 1: Particle properties under the discrete symmetries.
Table 2: Values of $h_i^e$ for $m_{H^\pm}(m_{S^\pm}) = 100 \ (400)$ GeV, $m_\eta = 50$ GeV, $m_{N_N} = m_{N_2} = 3.0$ TeV for the normal hierarchy. For Set A (B), $\kappa \tan \beta = 29 \ (34)$ and $U_{e3} = 0 \ (0.14)$. Predictions on the branching ratio of $\mu \rightarrow e\gamma$ are also shown.

### 3 Neutrino Mass, Dark Matter, and Strongly 1st-Order Phase Transition

The LH neutrino mass matrix $M_{ij}$ is generated by the three-loop diagrams in Fig. 1. The absence of lower order loop contributions is guaranteed by $Z_2$, $H^\pm$ and $e^\pm_R$ play a crucial role to connect LH neutrinos with the one-loop sub-diagram by the $Z_2$-odd states. We obtain

$$M_{ij} = \sum_{\alpha = 1}^{2} C_{ij}^\alpha F(m_{H^\pm}, m_{S^\pm}, m_{N_N}, m_\eta),$$

(3.67)

where $C_{ij}^\alpha = 4\kappa/\tan^2 \beta (y^S_{i,j} h_i^e)(y^S_{i,j} h_i^\mu)$ with $y^S_{i,j} = \sqrt{2} m_{e_i}/v$ and $v \approx 246$ GeV. The factor of the three-loop integral function $F(m_{H^\pm}, m_{S^\pm}, m_{N_N}, m_\eta)$ includes the suppression factor of $1/(16\pi^2)^3$, whose typical size is $O(10^3)$ eV. Magnitudes of $\kappa/\tan \beta$ as well as $F$ determine the universal scale of $M_{ij}$, whereas variation of $h_i^\alpha$ ($i = e, \mu, \tau$) reproduces the mixing pattern indicated by the neutrino data [1].

Under the *natural* requirement $h_i^e \sim O(1)$, and taking the $\mu \rightarrow e\gamma$ search results into account [16], we find that $m_{N_N} \sim O(1)$ TeV, $m_{H^\pm} \lesssim O(100)$ GeV, $\kappa \tan \beta \gtrsim O(10)$, and $m_{S^\pm}$ being several times 100 GeV. On the other hand, the LEP direct search results indicate $m_{H^\pm}$ (and $m_{S^\pm}$) $\gtrsim 100$ GeV [1]. In addition, with the LEP precision measurement for the $\rho$ parameter, possible values uniquely turn out to be $m_{H^\pm} \simeq m_H$ (or $m_A$) $\approx 100$ GeV for $\sin(\beta - \alpha) \approx 1$. Thanks to the Yukawa coupling in Eq. (2.64), such a light $H^\pm$ is not excluded by the $b \rightarrow s\gamma$ data [17]. Since we cannot avoid to include the hierarchy among $y_i^S$, we only require $h_i^e y_i \sim O(y_i) \sim 10^{-5}$ for values of $h_i^e$. Our model turns out to prefer the normal hierarchy scenario. Several sets for $h_i^e$ are shown in Table 2 with the predictions.
on the branching ratio of $\mu \to e\gamma$ assuming the normal hierarchy $^*$.

The lightest $Z_2$-odd particle is stable and can be a candidate of DM if it is neutral. In our model, $N_\alpha^R$ must be heavy, so that the DM candidate is identified as $\eta$. When $\eta$ is lighter than the W boson, $\eta$ dominantly annihilates into $\bar{b}b$ and $\tau^+\tau^-$ via tree-level $s$-channel Higgs ($h$ and $H$) exchange diagrams, and into $\gamma\gamma$ via one-loop diagrams. From their summed thermal averaged annihilation rate ($\sigma v$), the relic density $\Omega_\eta h^2$ is evaluated. Fig. 2(Left) shows $\Omega_\eta h^2$ as a function of $m_\eta$. Strong annihilation can be seen near $50 \text{ GeV}$'s $m_H$ (60 GeV $m_h$) due to the resonance of $H (h)$ mediation. The data ($\Omega_{\text{DM}} h^2 \approx 0.11$ [2]) indicate that $m_\eta$ is around 40-65 GeV.

The model satisfies the necessary conditions for baryogenesis [3]. Especially, departure from thermal equilibrium can be realized by the strong first order EWPT. The free energy is given at a high temperature $T$ by

$$V_{\text{eff}}[\varphi, T] = D(T^2 - T_0^2)\varphi^2 - E T \varphi^3 + \frac{\lambda T}{4} \varphi^4 + ..., \quad (3.68)$$

where $\varphi$ is the order parameter. A large value of the coefficient $E$ is crucial for the strong first order EWPT with keeping $m_h \lesssim 120$ GeV. For sufficient sphaleron decoupling in the broken phase, it is required that [18]

$$\frac{\varphi_c}{T_c} \left( \sim \frac{2E}{\lambda T_c} \right) \gtrsim 1, \quad (3.69)$$

where $\varphi_c (\neq 0)$ and $T_c$ are the critical values of $\varphi$ and $T$ at the EWPT. In Fig. 2(Right), the allowed region under the condition of Eq. (3.69) is shown. The condition is satisfied when $m_{\tilde{S}^\pm} \gtrsim 350$ GeV for $m_A \gtrsim 100$ GeV, $m_h \simeq 120$ GeV, $m_H \approx m_{H^\pm} (\approx M) \approx 100$ GeV and $\sin(\beta - \alpha) \approx 1$, where $M$ represents the soft-breaking mass of extra Higgs bosons for $\tilde{Z}_2$ [4].

$^*$The predictions for $\mu \to e\gamma$ shown here are corrected ones from those in Ref. [4].
4 Phenomenology

A successful scenario which can simultaneously solve the above three issues under the data \([1, 16, 17]\) would be

\[
\sin(\beta - \alpha) \approx 1, \quad \kappa \tan \beta \approx 30, \quad m_h = 120 \text{ GeV}, \quad m_H \approx m_{H^\pm} \approx O(100) \text{ GeV}, \\
m_A \gtrsim O(100) \text{ GeV}, \quad m_{S^\pm} \sim 400 \text{ GeV}, \quad m_\eta \lesssim m_W, \quad m_{N^1} \approx m_{N^2} \approx 3 \text{ TeV}.
\]

This is realized without assuming unnatural hierarchy among the couplings. All the masses are between \(O(100) \text{ GeV}\) and \(O(1) \text{ TeV}\). The discriminative properties of this scenario are in order:

(I) \(h\) is the SM-like Higgs boson, but decays into \(\eta \eta\) when \(m_\eta < m_h/2\). The branching ratio is about 30\% for \(m_\eta \approx 43 \text{ GeV}\) and \(\tan \beta = 10\); see Fig. 3. This is related to the DM abundance, so that our DM scenario is testable at the CERN Large Hadron Collider (LHC) and the ILC by searching the missing decay of \(h\). Furthermore, \(\eta\) is potentially detectable by direct DM searches \([19]\), because \(\eta\) can scatter with nuclei via the scalar exchange \([20]\).

(II) For successful baryogenesis, the \(hhh\) coupling has to deviate from the SM value by more than 10-20\% \([21]\) (see Fig. 2), which can be tested at the ILC \([22]\).

(III) \(H\) (or \(A\)) can predominantly decay into \(\tau^+ \tau^-\) instead of \(b \bar{b}\) for \(\tan \beta \gtrsim 2\) because of the Type-X Yukawa interaction. For example, we have \(B(H(A) \rightarrow \tau^+ \tau^-) \approx 100\%\) and \(B(H(A) \rightarrow \mu^+ \mu^-) \approx 0.3\%\) for \(m_A = m_H = 130 \text{ GeV}, \sin(\beta - \alpha) = 1\) and \(\tan \beta = 10\). The scenario with light \(H^\pm\) and \(H\) (or \(A\)) can be directly tested at the LHC via \(pp \rightarrow W^* \rightarrow HH^\pm\) and \(AH^\pm\) \([23]\), and also \(pp \rightarrow HA\). Their signals are four lepton states \(\ell^- \ell^+ \tau^\pm \nu\) and \(\ell^- \ell^+ \tau^+ \tau^-\), where \(\ell\) represents \(\mu\) and \(\tau\) \([11]\). At the ILC, the process \(e^+ e^- \rightarrow HA\) would be useful to discriminate the model from the other new physics candidates. In Fig. 4, the production rate of \(e^+ e^- \rightarrow HA\) is shown for \(m_A = m_H\). For \(\sqrt{s} = 500 \text{ GeV}\), about 17,000 (110) of the \(\tau^+ \tau^- \tau^+ \tau^-\) (\(\mu^+ \mu^- \tau^+ \tau^-\)) events are then produced from the signal for \(m_A = m_H = 130 \text{ GeV}\) \([11]\), while about 60 (0) events are in the MSSM for the similar parameter set. The main background comes from \(ZZ\) production.
(about 400 fb), which is expected to be easily reduced by appropriate kinematic cuts.

(IV) The physics of $Z_2$-odd charged singlet $S^\pm$ is important to distinguish this model from the other models. At the LHC, they are produced in pair via the Drell-Yuan process [24]. The cross section amounts to 0.5 fb for $m_{S^\pm} = 400$ GeV at $\sqrt{s} = 14$ TeV, so that more than a hundred of the $S^+S^-$ events are produced for the integrated luminosity 300 fb$^{-1}$. The produced $S^\pm$ bosons decay as $S^\pm \rightarrow H^\pm \nu$, and $H^\pm$ mainly decay into $\tau^\pm \nu$. The signal would be a high-energy hadron pair [25] with a large missing transverse momentum.

The charged singlet scalar bosons $S^\pm$ in our model can also be better studied at the ILC via $e^+e^- \rightarrow S^+S^-$ shown in Fig. 5. The total cross sections are shown as a function of $m_{S^\pm}$ for $\sqrt{s} = 1$ TeV in Fig. 6. The other relevant parameters are taken as $m_{N_R^1} = m_{N_R^2} = 3$ TeV and $h_1^\alpha = h_2^\alpha = 2.0$. Both the contributions from the s-channel gauge boson ($\gamma$ and $Z$) mediation and the t-channel RH neutrino mediation are included in the calculation. The total cross section can amount to about 200 fb for $m_{S^\pm} = 400$ GeV at $\sqrt{s} = 1$ TeV due to the contributions of the t-channel RH neutrino-mediation diagrams with $O(1)$ coupling constants $h_\alpha^e$. The signal would be a number of energetic tau lepton pairs with large missing energies. Although several processes such as $e^+e^- \rightarrow W^+W^-$ and $e^+e^- \rightarrow H^+H^-$ can give backgrounds for this final state, we expect that the signal events can be separated by kinematic cuts.

Finally, there is a further advantage in testing our model at the $e^-e^-$ collision option of the ILC, where the dimension five operators $\bar{e'}\bar{e'} S^+S^+$, which appear in the sub-diagram of the three-loop induced masses of neutrinos in our model, can be directly measured. The production cross section for $e^-e^- \rightarrow S^-S^-$ [t-channel $N_R^2$ mediation: see Fig. 7] is given by

$$\sigma(e^-e^- \rightarrow S^-S^-) = \int_{t_{\text{min}}}^{t_{\text{max}}} dt \frac{1}{128\pi s} \left| \sum_{\alpha=1}^{2} (|h_\alpha^e|^2 m_{N_R^\alpha}) \left( \frac{1}{t - m_{N_R^\alpha}^2} + \frac{1}{u - m_{N_R^\alpha}^2} \right) \right|^2. \quad (4.71)$$

Due to the structure of our model that the tiny neutrino masses are generated at the three-loop level, the magnitudes of $h_\alpha^e$ ($\alpha = 1, 2$) are of $O(1)$, by which the cross section becomes very large. Furthermore, thanks to the Majorana nature of the t-channel diagram we obtain much larger cross section in the $e^-e^-$ collision than at the $e^+e^-$ collision when $m_{N_R^2}^2 \gg s$. Fig. 8 shows the production cross sections for $e^-e^- \rightarrow S^-S^-$ via the t-channel RH-neutrino.
The cross section can be as large as 30 pb for $m_{S^\pm} = 400$ GeV for $\sqrt{s}_{e^-e^-} = 1$ TeV, $m_{N_R^1} = m_{N_R^2} = 3$ TeV and $h^e_1 = h^e_2 = 2.0$. The backgrounds are expected to be much less than the $e^+e^-$ collision.

We emphasize that a combined study for these processes would be an important test for our model, in which neutrino masses are generated at the three-loop level by the $Z_2$ symmetry and the TeV-scale RH neutrinos $^\dagger$.

In the other radiative seesaw models in which the neutrino masses are induced at the one-loop level with RH neutrinos, the corresponding coupling constants to our $h^e_i$ couplings are necessarily one or two orders of magnitude smaller to satisfy the neutrino data, so that the cross section of the t-channel RH neutrino mediation processes are small due to the suppression factor ($h^e_i$)$^4$.

(V) The couplings $h^e_i$ cause lepton flavor violation such as $\mu \to e\gamma$ which would provide information on $m_{N_R^2}$ at future experiments.

Finally, we comment on the case with the CP violating phases. Our model includes the THDM, so that the same discussion can be applied in evaluation of baryon number at the EWPT [5]. The mass spectrum would be changed to some extent, but most of the features discussed above should be conserved with a little modification.

5 Summary

We have discussed the model with the extended Higgs sector and TeV-scale RH neutrinos, which would explain neutrino mass and mixing, DM and baryon asymmetry by the TeV scale physics. It gives specific predictions on the collider phenomenology. In particular, the predictions on the Higgs physics are completely different from those in the MSSM, so that the model can be distinguished at the LHC and also at the ILC.

References


$^\dagger$Unlike our model, in the model in Ref. [7], the coupling constants corresponding to our $h^e_i$ are small and instead those to $h^\mu_i$ are $O(1)$, so that its Majorana structure is not easy to test at $e^+e^-$ collisions.
Precision Measurements of the model parameters in the Littlest Higgs model with T-parity

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We investigate a possibility of precision measurements for parameters of the Littlest Higgs model with T-parity at the International Linear Collider (ILC). The model predicts new gauge bosons which masses strongly depend on the vacuum expectation value that breaks a global symmetry of the model. Through Monte Carlo simulations of production processes of new gauge bosons, we show that these masses can be determined very accurately at the ILC for a representative parameter point of the model. From the simulation result, we also discuss the determination of other model parameters at the ILC.

1 Introduction

The Little Higgs model \([1, 2]\) has been proposed for solving the little hierarchy problem. In this scenario, the Higgs boson is regarded as a pseudo Nambu-Goldstone (NG) boson associated with a global symmetry at some higher scale. Though the symmetry is not exact, its breaking is specially arranged to cancel quadratically divergent corrections to the Higgs mass term at 1-loop level. This is called the Little Higgs mechanism. As a result, the scale of new physics can be as high as 10 TeV without a fine-tuning on the Higgs mass term. Due to the symmetry, the scenario necessitates the introduction of new particles. In addition, the implementation of the \(Z_2\) symmetry called T-parity to the model has been proposed in order to avoid electroweak precision measurements \([3]\). In this study, we focus on the Littlest Higgs model with T-parity as a simple and typical example of models implementing both the Little Higgs mechanism and T-parity.

In order to test the Little Higgs model, precise determinations of properties of Little Higgs partners are mandatory, because these particles are directly related to the cancellation of quadratically divergent corrections to the Higgs mass term. In particular, measurements of heavy gauge boson masses are quite important. Since heavy gauge bosons acquire mass terms through the breaking of the global symmetry, precise measurements of their masses allow us to determine the most important parameter of the model, namely the vacuum expectation value (VEV) of the breaking. Furthermore, because the heavy photon is a candidate for dark matter \([8, 9]\), the determination of its property gives a great impact not only on particle physics but also on astrophysics and cosmology. However, it is difficult to
determine the properties of heavy gauge bosons at the Large Hadron Collider, because they have no color charge [6].

On the other hand, the ILC will provide an ideal environment to measure the properties of heavy gauge bosons. We study the sensitivity of the measurements to the Little Higgs parameters at the ILC based on a realistic Monte Carlo simulation [7]. We have used MadGraph [10] and Physsim [11] to generate signal and Standard Model (SM) events, respectively. In this study, we have also used PYTHIA6.4 [13], TAUOLA [14] and JSFQuickSimulator which implements the GLD geometry and other detector-performance related parameters [15].

2 Model

The Littlest Higgs model with T-parity is based on a non-linear sigma model describing an SU(5)/SO(5) symmetry breaking with a VEV, \( f \sim \mathcal{O}(1) \) TeV. An [SU(2) \times U(1)]^2 subgroup in the SU(5) is gauged, which is broken down to the SM gauge group SU(2)_L \times U(1)_{Y}. Due to the presence of the gauge and Yukawa interactions, the SU(5) global symmetry is not exact. The SM doublet and triplet Higgs bosons (\( H \) and \( \Phi \)) arise as pseudo NG bosons in the model. The triplet Higgs boson is T-odd, while the SM Higgs is T-even.

This model contains gauge fields of the gauged [SU(2) \times U(1)]^2 symmetry; The linear combinations \( W^a = (W^a_L + W^a_R)/\sqrt{2} \) and \( B = (B_1 + B_2)/\sqrt{2} \) correspond to the SM gauge bosons for the SU(2)_L and U(1)_Y symmetries. The other linear combinations \( W^a_H = (W^a_L - W^a_R)/\sqrt{2} \) and \( B_H = (B_1 - B_2)/\sqrt{2} \) are additional gauge bosons called heavy gauge bosons, which acquire masses of \( \mathcal{O}(f) \) through the SU(5)/SO(5) symmetry breaking. After the electroweak symmetry breaking, the neutral components of \( W^a_H \) and \( B_H \) are mixed with each other and form mass eigenstates \( A_H \) and \( Z_H \). The heavy gauge bosons (\( A_H, Z_H, \) and \( W_H \)) behave as T-odd particles, while SM gauge bosons are T-even.

To implement T-parity, two SU(2) doublets \( l^{(1)} \) and \( l^{(2)} \) are introduced for each SM lepton. The quantum numbers of \( l^{(1)} \) and \( l^{(2)} \) under the gauged [SU(2) \times U(1)]^2 symmetry are \( (2, -3/10; 1, -1/5; 2, -3/10) \) and \( (1, -1/5; 2, -3/10) \), respectively. The linear combination \( l_{SM} = (l^{(1)} - l^{(2)})/\sqrt{2} \) gives the left-handed SM lepton. On the other hand, another linear combination \( l_1 = (l^{(1)} + l^{(2)})/\sqrt{2} \) is vector-like T-odd partner which acquires the mass of \( \mathcal{O}(f) \). The masses depend on the \( \kappa_i; m_{\kappa_1} = \sqrt{2}k_1f, m_{\kappa_2} = (1/2)(\sqrt{2} + \sqrt{1 + c_f})\kappa_1f \approx \sqrt{2}k_1f \). In addition, new particles are also introduced in quark sector. (For details, see Ref. [13].)

3 Simulation study

The representative point used in our simulation study is \( (f, m_h, \lambda_2, \kappa_{1}) = (580 \text{ GeV}, 134 \text{ GeV}, 1.5, 0.5) \) where \((m_{A_H}, m_{W_H}, m_{Z_H}, m_{\Phi}) = (81.9 \text{ GeV}, 368 \text{ GeV}, 369 \text{ GeV}, 440 \text{ GeV})\) and \( \lambda_2 \) is an additional Yukawa coupling in the top sector. The model parameter satisfies not only the current electroweak precision data but also the WMAP observation [7]. Furthermore, no fine-tuning is needed at the sample point to keep the Higgs mass on the electroweak scale [15, 16].

In the model, there are four processes whose final states consist of two heavy gauge bosons: \( e^+ e^- \rightarrow A_H A_H, A_H Z_H, Z_H Z_H, \) and \( W_H^+ W_H^- \). The first process is undetectable. At the representative point, the largest cross section is expected for the fourth process, which is open at \( \sqrt{s} > 1 \) TeV. On the other hand, because \( m_{A_H} + m_{Z_H} \) is less than 500 GeV, the second process is important already at the \( \sqrt{s} = 500 \) GeV. We, hence, concentrate on
\[ e^+ e^- \rightarrow A_H Z_H \] and \[ e^+ e^- \rightarrow W_H^+ W_H^- \].

For the \( A_H Z_H \) production at \( \sqrt{s} = 500 \text{ GeV} \) with an integrated luminosity of 500 fb\(^{-1}\), we define \( A_H Z_H \rightarrow A_H A_H h \rightarrow A_H A_H b \bar{b} \) as our signal event. The \( A_H \) and \( Z_H \) boson masses can be estimated from the edges of the distribution of the reconstructed Higgs boson energies. The endpoints have been estimated by fitting the distribution with a line shape determined by a high statistics signal sample. The fit resulted in \( m_{A_H} = 83.2 \pm 13.3 \text{ GeV} \) and \( m_{Z_H} = 366.0 \pm 16.0 \text{ GeV} \), respectively.

For the \( W_H W_H \) production at \( \sqrt{s} = 1 \text{ TeV} \) with an integrated luminosity of 500 fb\(^{-1}\), we have used 4-jet final states, \( W_H^+ W_H^- \rightarrow A_H A_H W^+ W^- \rightarrow A_H A_H q q q q \). The masses of \( A_H \) and \( W_H \) bosons can be determined from the edges of the \( W \) energy distribution. The fitted masses of \( A_H \) and \( W_H \) bosons are \( 81.58 \pm 0.67 \text{ GeV} \) and \( 368.3 \pm 0.63 \text{ GeV} \), respectively. Using the process, it is also possible to confirm that the spin of \( W_H^\pm \) is consistent with one and the polarization of \( W_H^\pm \) from the \( W_H^\pm \) decay is dominantly longitudinal. Furthermore, the gauge charges of the \( W_H \) boson could be also measured using a polarized electron beam.

Figure 2 shows the probability contours for the masses of \( A_H \) and \( W_H \) at 1 TeV together with that of \( A_H \) and \( Z_H \) at 500 GeV. The mass resolution improves dramatically at \( \sqrt{s} = 1 \text{ TeV} \), compared to that at \( \sqrt{s} = 500 \text{ GeV} \).
4 Conclusion

The Littlest Higgs Model with T-parity is one of the attractive candidates for physics beyond the SM. We have shown that the masses of the heavy gauge bosons can be determined very accurately at the ILC. It is important to notice that these masses are obtained in a model-independent way, so that it is possible to test the Little Higgs model by comparing them with the theoretical predictions. Furthermore, since the masses of the heavy gauge bosons are determined by the VEV $f$, it is possible to accurately determine $f$. From the results obtained in our simulation study, it turns out that the VEV $f$ can be determined to accuracies of 4.3% at $\sqrt{s} = 500$ GeV and 0.1% at $\sqrt{s} = 1$ TeV. Another Little Higgs parameter $\kappa_l$ could also be estimated from production cross sections for the heavy gauge bosons, because the cross sections depend on the masses of heavy leptons. At the ILC with $\sqrt{s} = 500$ GeV and 1 TeV, $\kappa_l$ could be obtained within 9.5% and 0.8% accuracies, respectively.

Finally, we have also found that the thermal abundance of dark matter relics can be determined to 10% and 1% levels at $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1$ TeV, respectively. These accuracies are comparable to those of current and future cosmological observations such as the PLANCK satellite [17], implying that the ILC experiment will play an essential role to understand the thermal history of our universe.

5 Acknowledgments

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References

Measurement of Heavy Gauge Bosons in the Little Higgs Model with T-parity at ILC

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The Littlest Higgs Model with T-parity is one of the attractive candidates of physics beyond the Standard Model. One of the important predictions of the model is the existence of new heavy gauge bosons, where they acquire mass terms through the breaking of global symmetry necessarily imposed on the model. The determination of the masses are, hence, quite important to test the model. In this paper, the measurement accuracy of the heavy gauge bosons at ILC is reported.

1 Introduction

There are a number of scenarios for new physics beyond the Standard Model. The most famous one is the supersymmetric scenario. Recently, alternative one called the Little Higgs scenario has been proposed [1, 2]. In this scenario, the Higgs boson is regarded as a pseudo Nambu-Goldstone boson associated with a global symmetry at some higher scale. A $Z_2$ symmetry called T-parity is imposed on the models to satisfy constraints from electroweak precision measurements [3, 4, 5]. Under the parity, new particles are assigned to be T-odd (i.e. with a T-parity of $-1$), while the SM particles are T-even. The lightest T-odd particle is stable and provides a good candidate for dark matter. In this article, we focus on the Littlest Higgs model with T-parity as a simple and typical example of models implementing both the Little Higgs mechanism and T-parity.

In order to test the Little Higgs model, precise determinations of properties of Little Higgs partners are mandatory, because these particles are directly related to the cancellation of quadratically divergent corrections to the Higgs mass term. In particular, measurements of heavy gauge boson masses, Little Higgs partners for gauge bosons, are quite important. Since heavy gauge bosons acquire mass terms through the breaking of the global symmetry, precise measurements of their masses allow us to determine the most important parameter of the model, namely the vacuum expectation value of the breaking.

We studied the measurement accuracy of masses of the heavy gauge bosons at the international linear collider (ILC). In addition, the sensitivity to the vacuum expectation value (f) was estimated. In this paper, the status of the study is shown, and the detail of this study is described in [6].
Table 1: Cross sections for the production of heavy gauge bosons.

<table>
<thead>
<tr>
<th>( \sqrt{s} ) (GeV)</th>
<th>( e^+e^- \rightarrow A_HZ_H ) (fb)</th>
<th>( e^+e^- \rightarrow Z_HZ_H ) (fb)</th>
<th>( e^+e^- \rightarrow W_H^+W_H^- ) (fb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>1.91</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1 TeV</td>
<td>7.42</td>
<td>110</td>
<td>277</td>
</tr>
</tbody>
</table>

2 Representative point and target mode

In order to perform a numerical simulation at ILC, we need to choose a representative point in the parameter space of the Littlest Higgs model with T-parity. Firstly, the model parameters should satisfy the current electroweak precision data. In addition, the cosmological observation of dark matter relics also gives important information. Thus, we consider not only the electroweak precision measurements but also the WMAP observation \[7\] to choose a point in the parameter space. We have selected a representative point where Higgs mass and \( f \) are 134 GeV and 580 GeV, respectively. At the representative point, we have obtained \( \Omega_{DM}h^2 = 0.105 \). The masses of the heavy gauge bosons are \( (M_{A_H}, M_{W_H}, M_{Z_H}) = (81.9 \text{ GeV}, 368 \text{ GeV}, 369 \text{ GeV}) \), where \( A_H \), \( Z_H \), and \( W_H \) are the Little Higgs partners of a photon, Z boson, and W boson, respectively. Here, \( A_H \) plays the role of dark matter in this model \[8, 9\]. Since all the heavy gauge bosons are lighter than 500 GeV, it is possible to generate them at ILC.

There are four processes whose final states consist of two heavy gauge bosons: \( e^+e^- \rightarrow A_HA_H \), \( A_HZ_H \), \( Z_HZ_H \), and \( W_H^+W_H^- \). The first process is undetectable, thus not considered in this article. The cross sections of the other processes are shown in Table 1. Since \( m_{A_H} + m_{Z_H} \) is less than 500 GeV, \( A_HZ_H \) can be produced at the \( \sqrt{s} = 500 \) GeV. At \( \sqrt{s} = 1 \) TeV, we can observe \( W_H^+W_H^- \) with large cross section. We, hence, concentrate on \( e^+e^- \rightarrow A_HZ_H \) at \( \sqrt{s} = 500 \) GeV and \( e^+e^- \rightarrow W_H^+W_H^- \) at \( \sqrt{s} = 1 \) TeV. Feynman diagrams for the signal processes are shown in Fig. 1. Note that \( Z_H \) decays into \( A_Hh \), and \( W_H^\pm \) decays into \( A_HW^\pm \) with almost 100% branching fractions.

3 Simulation tools

We have used MadGraph \[10\] to generate \( e^+e^- \rightarrow A_HZ_H \) at \( \sqrt{s} = 500 \) GeV, while \( e^+e^- \rightarrow W_H^+W_H^- \) at \( \sqrt{s} = 1 \) TeV and all the standard model events have been generated by Physisim \[11\]. We ignored the initial- and final-state radiation, beamstrahlung, and the beam energy spread for study of \( e^+e^- \rightarrow A_HZ_H \) at \( \sqrt{s} = 500 \) GeV, whereas their effects were considered for study of \( e^+e^- \rightarrow W_H^+W_H^- \) at \( \sqrt{s} = 1 \) TeV where the beam energy spread is set to 0.14% for the electron beam and 0.07% for the positron beam. The finite cross-section angle between the electron and positron beams was assumed to be zero. In both event generators, the helicity amplitudes were calculated using the HELAS library \[12\], which allows us to deal with the effect of gauge boson polarizations properly.
Table 2: Signal and backgrounds processes considered in the $A_H Z_H$ analysis.

<table>
<thead>
<tr>
<th>Process</th>
<th>Cross sec. [fb]</th>
<th># of events</th>
<th># of events after all cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_H Z_H \rightarrow A_H A_H^{\pm} b\bar{b}$</td>
<td>1.05</td>
<td>525</td>
<td>272</td>
</tr>
<tr>
<td>$\nu \nu h \rightarrow \nu \nu bb$</td>
<td>34.0</td>
<td>17,000</td>
<td>3,359</td>
</tr>
<tr>
<td>$Z h \rightarrow \nu \nu bb$</td>
<td>5.57</td>
<td>2,785</td>
<td>1,406</td>
</tr>
<tr>
<td>$t\bar{t} \rightarrow WW^{\pm} b\bar{b}$</td>
<td>496</td>
<td>248,000</td>
<td>264</td>
</tr>
<tr>
<td>$Z Z \rightarrow \nu \nu bb$</td>
<td>25.5</td>
<td>12,750</td>
<td>178</td>
</tr>
<tr>
<td>$\nu \nu Z \rightarrow \nu \nu bb$</td>
<td>44.3</td>
<td>22,150</td>
<td>167</td>
</tr>
<tr>
<td>$\gamma Z \rightarrow \gamma bb$</td>
<td>1,200</td>
<td>600,000</td>
<td>45</td>
</tr>
</tbody>
</table>

Parton showering and hadronization have been carried out by using PYTHIA6.4 [13], where final-state tau leptons are decayed by TAUOLA [14] in order to handle their polarizations correctly. The generated Monte Carlo events have been passed to a detector simulator called JSFQuickSimulator, which implements the GLD geometry and other detector-performance related parameters [15].

4 Analysis

In this section, we present simulation and analysis results for heavy gauge boson productions. The simulation has been performed at $\sqrt{s} = 500$ GeV for the $A_H Z_H$ production and at $\sqrt{s} = 1$ TeV for the $W_H^+ W_H^-$ production with an integrated luminosity of 500 fb$^{-1}$.

4.a $e^+ e^- \rightarrow A_H Z_H$ at 500 GeV

$A_H$ and $Z_H$ are produced with the cross section of 1.9 fb at the center of mass energy of 500 GeV. Since $Z_H$ decays into $A_H$ and the Higgs boson, the signature is a single Higgs boson in the final state, mainly 2 jets from $h \rightarrow b\bar{b}$ (with a 55% branching ratio). We, therefore, define $A_H Z_H \rightarrow A_H A_H^{\pm} b\bar{b}$ as our signal event. For background events, contribution from light quarks was not taken into account because such events can be rejected to negligible level after requiring the existence of two $b$-jets, assuming a $b$-tagging efficiency of 80% for $b$-jets with 15% probability to misidentify a $c$-jet as a $b$-jet. This $b$-tagging performance was estimated by the full simulation, assuming a typical ILC detector. Signal and background processes considered in this analysis are summarized in Table 2. Figure 2 shows a typical $A_H Z_H$ event seen in the detector simulator.

The clusters in the calorimeters are combined to form a jet if the two clusters satisfy $y_{ij} < y_{\text{cut}}$. $y_{ij}$ is defined as

$$y_{ij} = \frac{2E_i E_j (1 - \cos \theta_{ij})}{E_{\text{vis}}^2},$$

where $\theta_{ij}$ is the angle between momenta of two clusters, $E_{i(j)}$ are their energies, and $E_{\text{vis}}$ is

![Figure 2: A typical event of $A_H Z_H$ in the simulator.](image)
the total visible energy. All events are forced to have two jets by adjusting $y_{\text{cut}}$. We have selected events with the reconstructed Higgs mass in a window of 100 – 140 GeV. Since Higgs bosons coming from the WW fusion process have the transverse momentum ($p_T$) mostly below W mass, $p_T$ is required to be above 80 GeV in order to suppress the $\nu\nu h \rightarrow \nu\nu bb$ background. Finally, multiplying the efficiency of double $b$-tagging (0.8 < 0.8 = 0.64), we are left with 272 signal and 5,419 background events as shown in Table 2, which corresponds to a signal significance of 3.7 (= 272/5419) standard deviations. The indication of the new physics signal can hence be obtained at $\sqrt{s} = 500$ GeV.

The masses of $A_H$ and $Z_H$ bosons can be estimated from the edges of the distribution of the reconstructed Higgs boson energies. This is because the maximum and minimum Higgs boson energies ($E_{\text{max}}$ and $E_{\text{min}}$) are written in terms of these masses,

$$E_{\text{max}} = \gamma_{Z_H} E_h^* + \beta_{Z_H} \gamma_{Z_H} p_h^*,$$

$$E_{\text{min}} = \gamma_{Z_H} E_h^* - \beta_{Z_H} \gamma_{Z_H} p_h^*,$$

where $\beta_{Z_H} (\gamma_{Z_H})$ is the $\beta (\gamma)$ factor of the $Z_H$ boson in the laboratory frame, while $E_h^* (p_h^*)$ is the energy (momentum) of the Higgs boson in the rest frame of the $Z_H$ boson. Note that $E_h^*$ is given as $(M_{Z_h}^2 + M_{Z_H}^2 - M_{4l}^2)/(2M_{Z_H}).$

Figure 3(a) shows the energy distribution of the reconstructed Higgs bosons with remaining backgrounds. The background events are subtracted from Fig. 3(a), assuming that the background distribution can be understand completely. Then, the endpoints, $E_{\text{max}}$ and $E_{\text{min}}$, have been estimated by fitting the distribution with a line shape determined by a high statistics signal sample. The fit resulted in $m_{A_H}$ and $m_{Z_H}$ to be 83.2 ± 13.3 GeV and 366.0 ± 16.0 GeV, respectively, which should be compared to their true values: 81.85 GeV and 368.2 GeV. Figure 3(b) shows the probability contours for the masses of $A_H$ and $Z_H$.

Since the masses of the heavy gauge bosons are from the vacuum expectation value ($f$), $f$ can be determined by fitting the energy distribution of the reconstructed Higgs bosons. Then, $f$ was determined to be $f = 576.0 \pm 25.0$ GeV.

Figure 3: (a) Energy distribution of the reconstructed Higgs bosons with remaining backgrounds after the mass cut. (b) Probability contours corresponding to 1- and 2-$\sigma$ deviations from the best fit point in the $A_H$ and $Z_H$ mass-plane. The shaded area shows the unphysical region of $m_{A_H} + m_{Z_H} > 500$ GeV.
Table 3: Signal and background processes considered in the $W^+_H W^-_H$ analysis.

<table>
<thead>
<tr>
<th>Process</th>
<th>cross sec. [fb]</th>
<th># of events</th>
<th># of events after all cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^+_H W^-_H \rightarrow A_H A_H qqqq$</td>
<td>106.5</td>
<td>53,258</td>
<td>37,560</td>
</tr>
<tr>
<td>$W^+_H W^-_H \rightarrow qqqq$</td>
<td>1773.5</td>
<td>886,770</td>
<td>306</td>
</tr>
<tr>
<td>$e^+ e^- W^+ W^- \rightarrow e^+ e^- qqqq$</td>
<td>464.9</td>
<td>232,442</td>
<td>23</td>
</tr>
<tr>
<td>$\nu e W^+ W^- \rightarrow \nu e qqqq$</td>
<td>25.5</td>
<td>12,770</td>
<td>3,696</td>
</tr>
<tr>
<td>$Z_H \rightarrow A_H A_H hh$</td>
<td>99.5</td>
<td>49,757</td>
<td>3,351</td>
</tr>
<tr>
<td>$\nu \bar{\nu} W^+ W^- \rightarrow \nu \bar{\nu} qqqq$</td>
<td>6.5</td>
<td>3,227</td>
<td>1,486</td>
</tr>
</tbody>
</table>

4.b $e^+ e^- \rightarrow W^+_H W^-_H$ at 1 TeV

$W^+_H W^-_H$ production has large cross section (277 fb) at ILC with $\sqrt{s} = 1$ TeV. Since $W^+_H$ decays into $A_H$ and $W^\pm$ with the 100% branching ratio, analysis procedure depends on the $W$ decay modes. In this analysis, we have used 4-jet final states from hadronic decays of two $W$ bosons, $W^+_H W^-_H \rightarrow A_H A_H qqqq$. Signal and background processes considered in the analysis are summarized in Table 3.

All events have been reconstructed as 4-jet events by adjusting the cut on y-values. In order to identify the two $W$ bosons from $W^\pm$ decays, two jet-pairs have been selected so as to minimize a $\chi^2$ function,

$$\chi^2 = (\text{rec} M_{W1} - \text{tr} M_W)^2/\sigma_{M_W}^2 + (\text{rec} M_{W2} - \text{tr} M_W)^2/\sigma_{M_W}^2,$$

(4.74)

where $\text{rec} M_{W1(2)}$ is the invariant mass of the first (second) 2-jet system paired as a $W$ candidate, $M_W$ is the true $W$ mass (80.4 GeV), and $\sigma_{M_W}$ is the resolution for the $W$ mass (4 GeV). We required $\chi^2 < 26$ to obtain well-reconstructed events. Since $A_H$ bosons escape from detection resulting in a missing momentum, the missing transverse momentum ($\text{miss}_p_T$) of the signal peaks at around 175 GeV. We have thus selected events with $\text{miss}_p_T$ above 84 GeV. Then, the reconstructed $W$ energy is required to be between 0 GeV to 500 GeV. The numbers of events after the selection cuts are shown in Table 3. The number of remaining background events is much smaller than that of the signal.

As in the case of the $A_H Z_H$ production, the masses of $A_H$ and $W_H$ bosons can be determined from the edges of the $W$ energy distribution. Figure 4(a) shows the energy distribution of the reconstructed $W$ bosons. After subtracting the backgrounds from Fig.4(a), the distribution has been fitted with a line shape function. The fitted masses of $A_H$ and $W_H$ bosons are 82.29 $\pm$ 1.10 GeV and 367.8 $\pm$ 0.8 GeV, respectively, which are to be compared to their input values: 81.85 GeV and 368.2 GeV. Figure 4(b) shows the probability contours for the masses of $A_H$ and $W_H$ at 1 TeV. The mass resolution improves dramatically at $\sqrt{s} = 1$ TeV, compared to that at $\sqrt{s} = 500$ GeV. Then, $f = 579.7 \pm 1.1$ GeV was obtained by fitting the energy distribution of the reconstructed $W$ bosons.

5 Summary

The Littlest Higgs Model with T-parity is one of the attractive candidates of physics beyond the Standard Model since it solves both the little hierarchy and dark matter problems simultaneously. One of the important predictions of the model is the existence of new heavy gauge bosons, where they acquire mass terms through the breaking of global symmetry necessarily
imposed on the model. The determination of the masses are, hence, quite important to test the model.

We have performed Monte Carlo simulations in order to estimate measurement accuracy of the masses of the heavy gauge bosons at ILC. At ILC with $\sqrt{s} = 500$ GeV, it is possible to produce $A_H$ and $Z_H$ bosons. Here, we can observe the excess by $A_HZ_H$ events in the Higgs energy distribution with the statistical significance of 3.7-sigma. Furthermore, the masses of these bosons can be determined with accuracies of 16.2% for $A_H$ and 4.3% for $Z_H$. Once ILC energy reaches $\sqrt{s} = 1$ TeV, the process $e^+e^- \rightarrow W_H^+W_H^-$ opens. Since the cross section of the process is large, the masses of $W_H$ and $A_H$ can be determined as accurately as 1.3% and 0.2%, respectively. Then, the vacuum expectation value, $f$, can be determined with accuracy of 4.3% at $\sqrt{s} = 500$ GeV and 0.2% at 1 TeV.

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