Scaling of Luminosity on RF Frequency

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Luminosity

The luminosity of a linear collider is basically determined by four factors:

$$\mathcal{L} = H_D \cdot \frac{N}{\sigma_x^*} \cdot \frac{1}{\sigma_y^*} \cdot N m_b f_{\mathrm{rep}} \ , \tag{1}$$

where

N:	particles/bunch,
m_b:	bunches/pulse,
f_{rep}:	repetition frequency,
H_D:	enhancement due to disruption.

- H_D and $1/\sigma_y^*$ are nearly independent of frequency.
- N/σ_x^* is limited by Beamstrahlung, so also independent of frequency[1].
- Then only $N m_b f_{\mathrm{rep}}$ is the problem which depends on rf.

In other words, the energy efficiency of the acceleration:

$$\eta \equiv \frac{N m_b f_{\mathrm{rep}} E}{P_{\mathrm{rf}}} \ , \tag{2}$$

where P_{rf} is the rf power on the accelerating structure, determines the luminosity for a given power, assuming the efficiency from wall-plug to rf does not strongly depend on the frequency.

[1] When N is too small, N/σ_x^* does not reach the limit since σ_x^* cannot be focused small enough.

Basic Quantities on Acceleration

Two basic quantities Q and C are determined by the rf-frequency ω, the linac length L, and a factor a/λ:

$$Q = \frac{c_q(a/\lambda)}{\sqrt{\omega}} \ . \tag{3}$$

$$C \equiv \frac{Q}{R\omega} = \frac{c_c(a/\lambda)}{L\omega^2} \ . \tag{4}$$

Another parameter to determine the acceleration is the attenuation parameter:

$$\tau = \frac{\omega\tau_f}{2Q} \ , \tag{5}$$

where τ_f is the filling time.

Avobe parameters first determines the average beam loading voltage of the linac as

$$V_l = c_l(\tau, m_b\tau_b)\frac{Nm_be}{2C} = \frac{ec_l}{2c_c}Nm_bL\omega^2 \ , \tag{6}$$

where τ_b is the bunch separation. The rf power fed to the linac is given as

$$P_{\mathrm{rf}} = c_a(\tau)\frac{(E+V_l)^2}{R}\tau_{\mathrm{rf}}f_{\mathrm{rep}} \ , \tag{7}$$

where τ_{rf} is the length of the rf pulse, *i.e.*,

$$\tau_{\mathrm{rf}} = \tau_f + m_b\tau_b \ . \tag{8}$$

Efficiency

Substituting Eqs. (3), (4), (6), and (7) rewrites the efficiency Eq. (2) as

$$\eta = \frac{2c_q}{ec_l c_a} \frac{EV_l}{(E+V_l)^2} \frac{1}{\tau_{\mathrm{rf}} \omega^{3/2}} \ . \tag{9}$$

The above expression has the broad maximum at $E = V_l$,

$$\eta \leq \frac{c_q}{2ec_l c_a} \frac{1}{\tau_{\mathrm{rf}} \omega^{3/2}} \ . \tag{10}$$

Since $\omega^{-1} \lesssim \tau_{\mathrm{rf}} \lesssim \omega^{-3/2}$, **the efficiency is same or even higher in a lower frequency**, **if an enough beam loading ratio $V_l/E \sim 1$ is achieved**.

The beam loading ratio is written as

$$\frac{V_l}{E} = \frac{ec_l}{2c_c E} N m_b L \omega^2 \ . \tag{11}$$

Thus for a low frequency, the loading reduces as ω^2, it is necessary to increase the charges/pulse ($= Nm_b$) and the linac length L. If both Nm_b and L are limited, the efficiency will be degraded.

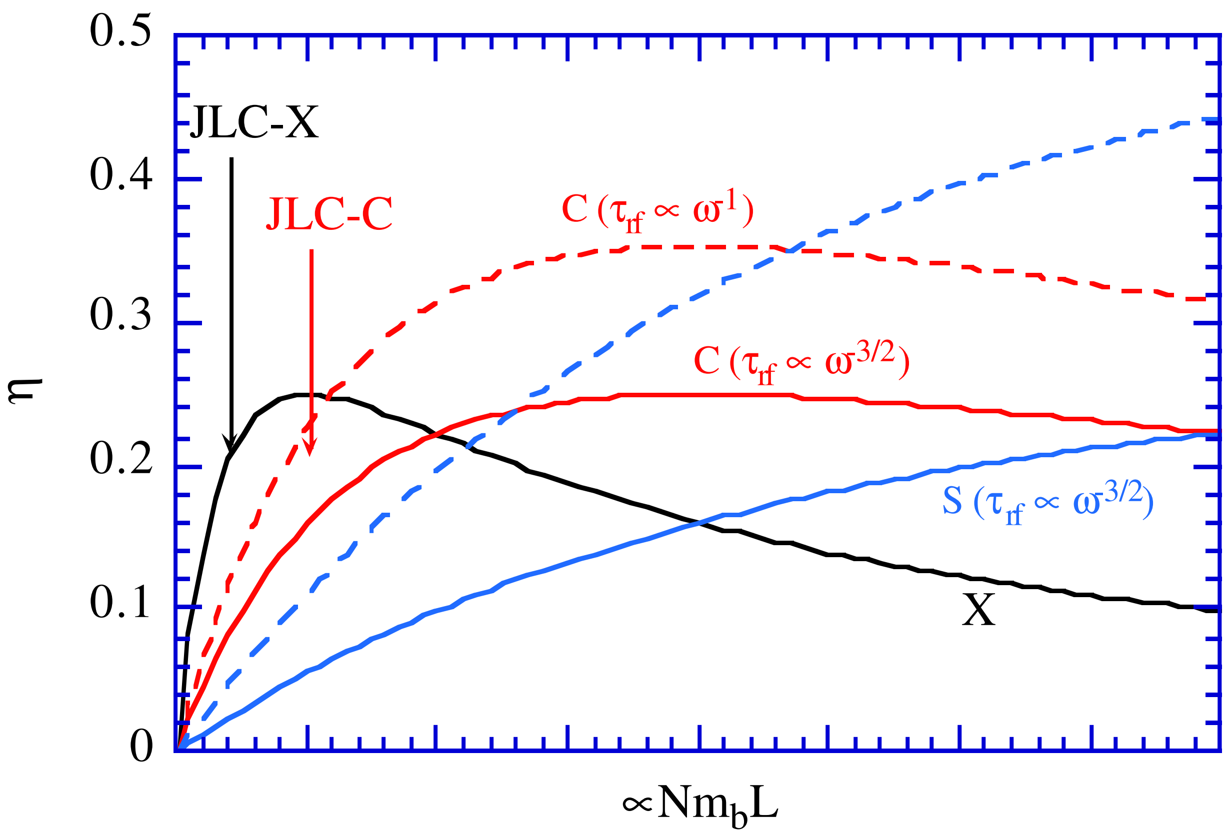

Figure 1: The acceleration efficiency as a function of Nm_bL (number of charges/pulse × the length of linac).

	JLC-C	JLC-X	NLC-X	
τ	0.53	0.59	0.59	
a/λ	0.135	0.165	0.18	
c_l	0.68	0.49	0.59	
$c_q(a/\lambda)$	1.3	1.3	1.3	$10^4\sqrt{\omega_S}$
$c_a(\tau)$	1.53	1.44	1.44	
c_q/c_lc_a	1.25	1.84	1.53	$10^4\sqrt{\omega_S}$
$1/\tau_{\rm rf}\omega^{3/2}$ (JLC-C=1)	1	0.74	0.77	
$c_q/c_lc_a\tau_{\rm rf}\omega^{3/2}$ (JLC-C=1)	1	1.09	0.94	

C-band

In the case of JLC-C, the loading ratio is enough high, $V_l/E \sim 0.28$, assuming $Nm_b = 8 \times 10^{11}$/pulse and $L/E = 22$ km/TeV. The factor $EV_l/(E+V_l)^2$ reaches 0.17 whereas the maximum is 0.25, so it has an enough high loading ratio.

	JLC-C	JLC-X	NLC-X	
E	1	1	1	TeV
L	22.2	17.3	17.0	km
$N \times m_b$	1.1×72	0.7×85	1.1×75	10^{10}
V_l/E	0.28	0.40	0.50	
$EV_l/(E+V_l)^2$	0.17	0.20	0.22	
τ_{rf}	0.48	0.23	0.22	μs
η (JLC-C=1)	1	1.1	1.1	

Then **there is no loss of efficiency in C compared to X**, if we involve the fact that $\tau_{\text{rf}} < \omega^{-3/2}$. Please note that **this situation is independent of energy, if we make $L \propto E$.**

Tolerances

The cell-to-cell (or structure-to-structure) alignment tolerance of the accelerating structure to preserve the vertical emittance against the single-bunchtransverse wake is written as

$$\Delta_y \approx \frac{cZ_0}{4\pi r_e N W' \sigma_z} \left(\frac{\sqrt{\gamma_1 \gamma_0} \varepsilon_y n_c}{2\beta_{y0} L} \right)^{1/2} , \tag{12}$$

where W' is the slope of the transverse wake per unit length, n_c the number of cells per unit length, and β_{y0} is the vertical β-function at the entrance of the linac. We have assumed $\beta_y(s) \propto \gamma(s)^{1/2}$. Since the transverse wake is given as (K. Yokoya)

$$W' \approx \frac{cZ_0}{\pi a^4} \times 1.550 \left(\frac{a/\lambda}{0.14} \right)^{0.179} , \tag{13}$$

the tolerance depends on the rf frequency as

$$\Delta_y \propto \frac{(a/\lambda)^{3.8}}{N\omega^{3.5}\sqrt{L}} . \tag{14}$$

	JLC-C	JLC-X	NLC-X	
a/λ	0.135	0.165	0.18	
L	22.2	17.3	17.0	km
N	1.1	0.7	1.1	10^{10}
Δ_y (JLC-C=1)	1	0.42	0.44	

Extendability

There are two ways to boost the luminosity:

$$\mathcal{L} = H_D \cdot \frac{N}{\sigma_x^*} \cdot \frac{1}{\sigma_y^*} \cdot N m_b f_{\text{rep}} \ , \tag{15}$$

for a given AC power:

$$\text{Higher luminosity} = \begin{cases} \text{(1) higher efficiency in RF System} \\ \qquad\qquad \neq \text{higher gradient} \\ \\ \text{(2)smaller } \sigma_y^* = \text{smaller } \varepsilon_y \end{cases}$$

(1) will be possible for any rf-frequency, but(2) requires even looser tolerances than what are needed now.

Then the really extendable machine must have the capability to accept an emittance which is significantly smaller than the present level, as well as the possibility to improve the efficiency in the rf system.

Thus you may have to

Upgrade to C-band

not necessarily right now, but in future.