Interaction of High Energy Electrons and Photons with Intense Electromagnetic Wave: Linear Collider Applications

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Abstract

The process of radiation from high-energy electrons and that of electron-positron pair creation by high-energy photons in an intense laser field are studied in the parameter region where the multi-photon interaction is significant. The complete pictures of the behavior of the radiation probability/intensity and probability of the pair creation as functions of two basic parameters are obtained. Spectra of radiation and created particles are also found. Applications for linear colliders are discussed.

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1 Introduction

Projects of electron-positron linear colliders of the energy region of the order of TeV are being discussed now in several laboratories. There are proposals (see review [1] and references therein) to convert such accelerators into γ-γ colliders, which may be useful for physics investigation in TeV region.

The most effective way of converting a high-energy ($\gtrsim 100$ GeV) electron beam into a γ beam is the inverse Compton scattering by a laser beam, namely to irradiate the electron beam by an intense laser beam and to make use of the back-scattered high-energy photons. The maximum energy of the radiated photon $\omega_{\max}$, i.e., the energy of the photon radiated in the direction of the initial electron is ($\hbar = c = 1$, metrics $a^\mu b_\mu \equiv a^0 b^0 - \mathbf{a}\cdot\mathbf{b}$ is used) given by

$$\omega_{\max} = \varepsilon \frac{\lambda}{1+\lambda}, \qquad \lambda = \frac{2q^\mu p_\mu}{m^2} = \frac{4\varepsilon\omega_0}{m^2}, \tag{1.1}$$

where ε (m) is the energy (mass) of the initial electron, ω_0 is the energy of the initial laser photon, $p^\mu (q^\mu)$ is the 4-momentum of the initial electron (photon).

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A large λ or, equivalently, a short laser wavelength is preferred in order to get the photon energy as high as possible from the given electron energy ε. However, if the energy of the radiated photons is too high, a significant fraction of the photons will be lost by the electron-positron pair creation through the interaction with the same laser beam, since the interaction region between the electron and the laser beam has to be long enough for converting most electrons into photons. Therefore, it is common that the wavelength of the laser is chosen in a such way that the maximum energy of the radiated photons is below the threshold of the pair creation. The threshold of such a process is given by

$$\eta \equiv \frac{k^\mu q_\mu}{2m^2} = \frac{\omega\omega_0}{m^2} = 1, \tag{1.2}$$

where k^μ is the 4-momentum of the photon interacting with the laser beam.

The relations (1.1) and (1.2) comes from the picture in which the laser beam is an emsemble of incoherent photons and the incident electron (or photon) interacts with one of the laser photons. This picture applies only when the laser field is low, which in general contradicts with the requirement of a high e-γ conversion rate. Even with a low intensity laser, of course, one can in principle make the conversion rate arbitrarily high by making the electron-laser intraction length long. Because of the effect of the depth of focus, however, this inevitably makes the laser spot size large and, therefore, calls for an enormous laser power. Thus, in the actual situation of the e-γ conversion, the laser field has to be extremely intense in order to have a high conversion rate. In this paper we shall discuss the situation when the laser field is so intense that the simultaneous interaction with many laser photons cannot be ignored. The theory of radiation and pair creation in an intense electromagnetic wave has been developed in series of papers [2]-[4], [6], [7], [9]-[11].

We shall describe the plane wave of the laser by the potential

$$A_\mu = a_{1\mu}\cos\varphi + a_{2\mu}\sin\varphi, \tag{1.3}$$

where $\varphi = q^\mu x_\mu = q^0 x^0 - \mathbf{q}\cdot\mathbf{x}$ and

$$q^\mu q_\mu = 0, \qquad q_\mu a_1^\mu = q_\mu a_2^\mu = a_1^\mu a_{2\mu} = 0. \tag{1.4}$$

For a circularly polarized electromagnetic wave which we consider below, $a_1^\mu a_{1\mu} = a_2^\mu a_{2\mu} \equiv a^\mu a_\mu$. It is often convenient to adopt a special coordinate system and a gauge with which $A_0 = 0$ and the vector $\mathbf{q}$ points along the third axis, i.e., $q^0 = q^3 = \omega_0$, while the electromagnetic field of the wave lies in the plane perpendicular to the third axis.

The state of the incident electron is described by the constant 4-momentum p_μ ($p^\mu p_\mu = m^2$). Because of the wiggling motion in the laser field, the average kinetic momentum ("quasi-momentum") is not equal to p^μ but is given by $l_\mu \equiv p_\mu - e^2a^2q_\mu/(2q^\nu p_\nu)$. Its square is not m^2 but is $l^\mu l_\mu = m_*^2$ where

$$m_* = m\sqrt{1+\xi^2} \tag{1.5}$$

is called the effective mass in the wave field.

The intensity of the field is characterized by the parameter

$$\xi^2 = e^2(-a^\mu a_\mu)/m^2, \qquad \xi = eE/(m\omega_0) \tag{1.6}$$

where E is the strength of the electric field of the wave. In the case of low intensities $\xi^2 \ll 1$, the perturbation theory is applicable and the first term of the expansion in powers of ξ^2 gives the characteristics of the interaction between the incident particle and one laser photon. In the case $\xi^2 \gg 1$, many laser photons interact with the incident particle simultaneously and the limit $\xi \to \infty$ corresponds to the interaction of the particle with a constant field.

Thus, the radiation process in the plane wave field is characterized by the two parameters: the energy parameter λ defined in eq.(1.1) and the intensity parameter ξ defined in eq.(1.6). When ξ is small, the first term of the expansion with respect to ξ^2 (one-photon process) corresponds to the ordinary Compton scattering. When ξ is large, the process can better be described by the parameter

$$\chi = \xi\lambda/2 = 2eE\varepsilon/m^3 = \frac{e}{m^3}\sqrt{\mid F^{\mu\nu}p_\nu \mid^2} \tag{1.7}$$

rather than λ, where $F^{\mu\nu}$ is the external electromagnetic field tensor. The limit of $\xi \to \infty$ is the synchrotron radiation in a constant field. (When $\chi \ll 1$, this is the well-known synchrotron radiation in storage rings.) One can use any two parameters of λ, ξ and χ for the description of the radiation.

On the other hand, the pair creation process is characterized by η defined in eq.(1.2) and ξ. In this case the first term of the ξ^2 expansion corresponds to the Breit-Wheeler process $\gamma + \gamma \to \mathrm{e}^+ + \mathrm{e}^-$. Similarly, as in the case of radiation, the parameter

$$\kappa = 2\xi\eta = 2eE\omega/m^3 = \frac{e}{m^3}\sqrt{\mid F^{\mu\nu}k_\nu \mid^2} \tag{1.8}$$

is better than η, when ξ is large.

We shall discuss the radiation in the next section and the pair creation in Sec.3. An application the the e-γ converter in γ-γ collider will be described in Sec.4.

2 Radiation

The radiation by an electron in a field of intense electromagnetic wave has been studied using the exact wave function of the electron in the wave field. For a circularly polarized wave the total probability of radiation has a relatively simple form [2, 3]:

$$W(\lambda,\xi) = \frac{\alpha m^2}{2\varepsilon}\sum_{n=1}^{\infty}\int_0^{u_n}\frac{du}{(1+u)^2}\left[-2J_n^2(z) + \xi^2 f(u)\left(J_{n-1}^2(z) + J_{n+1}^2(z) - 2J_n^2(z)\right)\right], \tag{2.1}$$

where $J_n(z)$ is the Bessel function, $\alpha = e^2/\hbar c$ the fine structure constant and

$$u_n = \frac{n\lambda}{1+\xi^2}, \quad f(u) = 1 + \frac{u^2}{2(1+u)}, \quad z = \frac{2\xi}{\lambda}\sqrt{1+\xi^2}\sqrt{u(u_n-u)}. \tag{2.2}$$

Here the parameters λ and ξ are defined in (1.1) and (1.6), ε is the zeroth component of the kinetic momentum (quasi-momentum) of the particle in the wave field.

The n-th term in the sum in eq.(2.1) describes the interaction of the n photons of the wave with electron. The value u_n is a just kinematic parameter of this interaction with

the effective mass m_* of the electron in the wave field taken into account. So, u_1 describes the Compton effect, u_2 describes the interaction of two photons of the wave with electron which can emit harder photons comparing with the case of one photon interaction, etc. Substituting into the kinematic relation (1.1) we see that harder photons are radiated with the increase of n.

Another representation of the total probability of radiation in a circularly polarized wave was derived in the frame of an operator approach to QED in an external field [4]. For unpolarized electrons it has the form [4]:

$$W(\lambda,\xi) = -\frac{\alpha m^2}{\pi\varepsilon}\,\mathrm{Im}\int\limits_0^\infty \frac{dt}{t}\int\limits_0^\infty \frac{du}{(1+u)^2}\left[\Big(1+2f(u)\xi^2\sin^2 t\Big)\exp(-iu\phi)-\exp(-2iut/\lambda)\right] \tag{2.3}$$

where

$$\phi = \frac{2t}{\lambda}\left[1+\xi^2\left(1-\frac{\sin^2 t}{t^2}\right)\right]. \tag{2.4}$$

Utilizing the generating function for the Bessel function and the power series expansion of the square of the Bessel function, one can reproduce the formula (2.1) from the representation (2.3). In practice, eq.(2.3) is better when ξ is large. It's worth noting that the total probability of radiation in an elliptically polarized monochromatic plane wave can also be represented by a double integral, but with the integrand containing the Bessel functions (see[4]).

The integral over u in eq.(2.3) can be evaluated analytically. As a result we obtain an expression for the total probability of radiation from an electron moving in the circularly polarized electromagnetic wave in the form of a single integral:

$$W(\lambda,\xi) = -\frac{e^2m^2}{\pi\varepsilon}\,\mathrm{Im}\int\limits_0^\infty \frac{dt}{t}\left[\Big(1+2\xi^2\sin^2 t\Big)P(\phi)+\xi^2\sin^2 t\,T(\phi)-P(\phi_0)\right] \tag{2.5}$$

where

$$\begin{aligned}
P(\phi) &= \int\limits_0^\infty \frac{du\exp(-iu\phi)}{(1+u)^2} = i\phi\exp(i\phi)\,\mathrm{Ei}(-i\phi)+1,\\
T(\phi) &= \int\limits_0^\infty \frac{u^2du\exp(-iu\phi)}{(1+u)^3} = -\left[1+2i\phi+\frac{(i\phi)^2}{2}\right]\exp(i\phi)\,\mathrm{Ei}(-i\phi)-\frac{3+i\phi}{2},\\
\phi_0 &= \frac{2t}{\lambda}(1+\xi^2).
\end{aligned} \tag{2.6}$$

Here $\mathrm{Ei}(z)$ is the exponential integral defined as

$$\mathrm{Ei}(-z) = \int\limits_{-\infty}^{-z}\frac{e^t}{t}dt. \qquad \left(\begin{array}{l} z\neq \text{negative real.}\\ \text{The integration path is parallel to the real axis.}\end{array}\right) \tag{2.7}$$

There are a number of different representations of this function (see e. g. [5]).

When $\xi \ll 1$, the integrand in eq.(2.3) may be expanded in powers of ξ^2. Retaining only the terms $\propto \xi^2$, we obtain

$$W_{10}(\lambda,\xi) = \frac{2\xi^2\alpha m^2}{\pi\varepsilon}\int\limits_0^\infty \frac{du}{(1+u)^2}\left[f(u)s_1 - \frac{u}{\lambda}s_2\right]. \tag{2.8}$$

Here s_1 and s_2 are integrals defined by

$$\begin{aligned} s_1 &= \int\limits_0^\infty \frac{dt}{t}\sin\frac{2ut}{\lambda}\sin^2 t = \frac{\pi}{4}\vartheta\left(1-\frac{u}{\lambda}\right), \\ s_2 &= \int\limits_0^\infty \frac{dt}{t^2}\cos\frac{2ut}{\lambda}\sin^2 t = \frac{\pi}{2}\left(1-\frac{u}{\lambda}\right)\vartheta\left(1-\frac{u}{\lambda}\right), \end{aligned} \tag{2.9}$$

where $\vartheta(x)$ is the Heaviside function (i.e., 0 if $x<0$ and 1 if $x>0$), which determines the kinematic boundary for the variable u.

Substituting eq.(2.9) into eq.(2.8) and evaluating the elementary integrals over u, we obtain the total probability to the order ξ^2

$$\begin{aligned} W_{10}(\lambda,\xi) = &\quad \frac{\xi^2\alpha m^2}{2\varepsilon}\int\limits_0^\lambda \frac{du}{(1+u)^2}\left[f(u) - \frac{2u}{\lambda}\left(1-\frac{u}{\lambda}\right)\right] \\ = &\quad \frac{\xi^2\alpha m^2}{4\varepsilon}\left[\ln(1+\lambda)\left(1-\frac{4}{\lambda}-\frac{8}{\lambda^2}\right)+\frac{1}{2}\left(1-\frac{1}{(1+\lambda)^2}\right)+\frac{8}{\lambda}\right] \end{aligned} \tag{2.10}$$

which is related to the total cross section of the Compton scattering (the Klein-Nishina formula) σ_c by

$$\sigma_c = \frac{8\pi\alpha\varepsilon}{\xi^2 m^4\lambda}W_{10}(\lambda,\xi) \tag{2.11}$$

The same procedure but with terms $\propto \xi^4$ taken into account results in the following changes.

(1) In eq.(2.10) λ should be substituted by u_1 which means the electron mass m is substituted by the effective mass m_* defined in eq.(1.5).

(2) There is an intensity correction to the Klein-Nishina formula:

$$W_{11}(\lambda,\xi) = \frac{\xi^4\alpha m^2}{4\varepsilon}\left[\ln(1+u_1)\left(\frac{8}{u_1}+\frac{8}{u_1^2}-\frac{24}{u_1^3}-\frac{16}{u_1^4}\right) -2-\frac{44}{3u_1}+\frac{16}{u_1^2}+\frac{16}{u_1^3}+\frac{2}{1+u_1}\right] \tag{2.12}$$

The probability of interaction of one laser photon with the electron is: $W_1 = W_{10} + W_{11} + W_{12} + \ldots = W_{10} + W_{11} + O(\xi^6)$.

(3) The interaction of two laser photons comes in (with the kinematic boundary given by $u < u_2$). The probability of this process is $W_2 = W_{20} + W_{21} + \ldots = W_{20} + O(\xi^6)$ with

$$W_{20}(\lambda,\xi) = \frac{\xi^4\alpha m^2}{4\varepsilon}\left[2+\frac{8}{3u_2}-\frac{64}{u_2^2}-\frac{64}{u_2^3}-\frac{2}{1+u_2} -\ln(1+u_2)\left(\frac{4}{u_2}-\frac{24}{u_2^2}-\frac{96}{u_2^3}-\frac{64}{u_2^4}\right)\right] \tag{2.13}$$

Explicit formulae to the order ξ^4 were obtained in [2, 3].

In the case of the high intensity wave $\xi^2 \gg 1$, the main contribution to the integral over t in eq.(2.3) comes from the region $t \ll 1$. Expanding the integrand in powers of t, we obtain, in the limit $\xi \to \infty$,

$$W_\infty(\chi) = -\frac{\alpha m^2}{2\pi\varepsilon}\,\mathrm{Im}\int\limits_{-\infty}^{\infty}\frac{dy}{y-i0}\int\limits_0^\infty\frac{du}{(1+u)^2}\left\{\left(1+2f(u)y^2\right)\exp\left[-i\frac{u}{\chi}\left(y+\frac{y^3}{3}\right)\right]\right\} \quad (2.14)$$

where we changed the variable to $y = \xi t$ and took into account that in this limit only the product of parameters $\lambda\xi$ appears in the formula, so that we used the strong field parameter χ defined in eq.(1.7).[1] Taking into account the known integrals

$$\begin{aligned}\int\limits_{-\infty}^{\infty}\frac{dy}{y-i0}\exp\left[-i\frac{u}{\chi}\left(y+\frac{y^3}{3}\right)\right] &= \frac{2i}{\sqrt{3}}\int\limits_{2u/(3\chi)}^{\infty}K_{1/3}(z)dz,\\ \int\limits_{-\infty}^{\infty}dy\,y\exp\left[-i\frac{u}{\chi}\left(y+\frac{y^3}{3}\right)\right] &= -\frac{2i}{\sqrt{3}}K_{2/3}\left(\frac{2u}{3\chi}\right),\end{aligned} \quad (2.15)$$

and substituting these integrals into eq.(2.14), we obtain

$$W_\infty(\chi) = \frac{\alpha m^2}{\pi\varepsilon\sqrt{3}}\int\limits_0^\infty\frac{du}{(1+u)^2}\left[2f(u)K_{2/3}\left(\frac{2u}{3\chi}\right)-\int\limits_{2u/(3\chi)}^{\infty}K_{1/3}(z)dz\right]. \quad (2.16)$$

This is the well-known expression for the probability of radiation in a constant external field in the quasiclassical approximation.

Now we take advantage of the general theory of radiation at quasiperiodic motion developed in [6] (see, also, [7]). The results of this theory coincide with theory of radiation in the plane wave field in the region where their application is overlapping. The theory of radiation at quasiperiodic motion has essential advantages since within it wider types of motion has been considered (radiation in circularly polarized wave coincides with radiation at helical motion), the differential probabilities, and the polarization of radiation, as well as spin effects, may be calculated. Using this theory one can see that the integrand as a function of u in eqs.(2.1), (2.3), (2.10) and (2.16) gives spectrum of radiated photons with the energy ω:

$$\omega = \varepsilon\frac{u}{1+u} \quad (2.17)$$

If one keeps the next terms of expansion in power of t of the integrand (2.3), one can evaluate $1/\xi^2$ correction to the high intensity limit:

$$dW(u,\chi,\xi) = dW_\infty(u,\chi) + \frac{dW_1(u,\chi)}{\xi^2}, \quad (2.18)$$

[1] The convention $-i0$ in eq.(2.14) indicates that the pole of the integrand is located at $y = +i0$, i.e., infinitesimally above the real axis. The contribution of the last term, which is identical to $\exp(-iuy/\chi)$, in the square brackets in eq.(2.3) vanishes as one can see by closing the contour with the lower semi-circle of infinite radius in the complex y-plane.

where $dW_\infty(u,\chi)$ is given by the integrand in (2.16) and

$$dW_1(u,\chi) = \frac{2\alpha m^2}{15\sqrt{3}\pi\varepsilon}\frac{du}{(1+u)^2}\left\{\left[f(u)+\frac{2}{3}\right]K_{2/3}(\eta) - \left[f(u)\left(\frac{14}{9\eta}-\eta\right)+\frac{\eta}{2}\right]K_{1/3}(\eta)\right\} \tag{2.19}$$

where $\eta = \dfrac{2u}{3\chi}$. This expression describes the correction to the probability spectrum. Unfortunately, it does not give the correction to the total probability (the expression (2.19) integrated over u) because of the infrared divergence.

Fig.2.1

Fig.2.2

At intermediate values of the parameter ξ, the total probability of radiation W should be calculated numerically using the representations (2.1) or (2.3). In Fig.2.1 the total probability W is plotted as a function of the parameter λ for different values of the parameter ξ. One can see that for $\xi = 0.3$ the total probability is still very close to that of the Compton scattering (one-photon interaction) and only at $\xi \sim 1$ the intensity effects become sizable. In Fig.2.2 the total probability W is shown in a 3D graph as a function of the two parameters ξ and χ. This graph gives a complete picture of the behaviour of W. One can see that W is a very smooth function of the two parameters. The cross section of this graph along $\chi = 1$ has already been given in [3].

Now we consider the total intensity of the radiation. Multiplying the differential probability by ω, we obtain the intensity of radiation (compare [7]):

$$\begin{aligned} dI(u,\lambda,\xi) = \; & -\alpha m^2 \frac{u\,du}{(1+u)^3}\left\{\mathrm{Im}\,\frac{1}{\pi}\int_0^\infty \frac{dt}{t}\left(1+2\xi^2 f(u)\sin^2 t\right)\exp\left(\frac{-2iut(1+\xi^2)}{\lambda}\right)\right. \\ & \left.\times\left[\exp\left(\frac{2iu\xi^2}{\lambda}\frac{\sin^2 t}{t}\right) - 1 - \frac{2iu\xi^2}{\lambda}\frac{\sin^2 t}{t}\right] + B\right\} \end{aligned} \tag{2.20}$$

where B is defined by

$$\begin{aligned} B = \; & \frac{\xi^2}{2}\left\{\left[f(u) - \frac{2u}{\lambda}\left(1+2\xi^2 f(u)\right)\left(1-\frac{u}{u_1}\right)\right]\vartheta\left(1-\frac{u}{u_1}\right)\right. \\ & \left. + f(u)\frac{\xi^2 u}{\lambda}\left(1-\frac{u}{u_2}\right)\vartheta\left(1-\frac{u}{u_2}\right)\right\} \end{aligned} \tag{2.21}$$

Here, we have introduced the subtraction term (the last term in the square brackets in eq.(2.20)) and the term B is its counterpart. Thus, the contribution of the subtraction term just cancels B. They are introduced for a better convergence at large t for numerical integrations.

After integrating eq.(2.20) with (2.21) over u, we obtain the total intensity of radiation

in the form of a single integral:

$$
\begin{aligned}
I(\lambda,\xi) = \quad & -\alpha m^2 \left\{ \mathrm{Im}\,\frac{1}{\pi}\int\limits_0^\infty \frac{dt}{t}\left(1+2\xi^2\sin^2 t\right)\left[R(\phi)-R(\phi_0)-\frac{2i\xi^2}{\lambda}\frac{\sin^2 t}{t}T(\phi_0)\right]\right. \\
& \left. +\xi^2\sin^2 t\left[S(\phi)-S(\phi_0)-\frac{2i\xi^2}{\lambda}\frac{\sin^2 t}{t}N(\phi_0)\right]+A\right\}
\end{aligned}
\tag{2.22}
$$

where

$$
\begin{aligned}
R(\phi) &= \int\limits_0^\infty \frac{u\,du\exp(-iu\phi)}{(1+u)^3} = i\phi(1+i\phi/2)\exp(i\phi)\,\mathrm{Ei}(-i\phi)+(1+i\phi)/2; \\
S(\phi) &= \int\limits_0^\infty \frac{u^3\,du\exp(-iu\phi)}{(1+u)^4} \\
&= -\left[1+3i\phi+\frac{3(i\phi)^2}{2}+\frac{(i\phi)^3}{6}\right]\exp(i\phi)\,\mathrm{Ei}(-i\phi)-\frac{11+8i\phi+(i\phi)^2}{6}, \\
N(\phi) &= \int\limits_0^\infty \frac{u^4\,du\exp(-iu\phi)}{(1+u)^4} \\
&= \left[4+6i\phi+2(i\phi)^2+\frac{(i\phi)^3}{6}\right]\exp(i\phi)\,\mathrm{Ei}(-i\phi)+\frac{1}{6}\left(\frac{6}{i\phi}+26+11i\phi+(i\phi)^2\right),
\end{aligned}
\tag{2.23}
$$

and the function $T(\phi)$ is defined in eq.(2.6). The term A comes from the subtraction counterpart B and has the form:

$$
\begin{aligned}
A = \quad & \xi^2\left(a_{10}+a_{11}\xi^2\right)+a_{20}\xi^4, \\
a_{10} = \quad & \frac{1}{2}\left[\left(\frac{1}{2}-\frac{2}{u_1}-\frac{6}{u_1^2}\right)\ln(1+u_1)-\frac{5}{12}+\frac{6}{u_1}-\frac{1}{2(1+u_1)}-\frac{1}{4(1+u_1)^2}+\frac{1}{6(1+u_1)^3}\right], \\
a_{11} = \quad & -\frac{1}{u_1}\left[\frac{35}{6}+\frac{u_1}{2}+\frac{4}{3(1+u_1)}-\frac{1}{6(1+u_1)^2}-\left(3+\frac{7}{u_1}\right)\ln(1+u_1)\right], \\
a_{20} = \quad & \frac{1}{u_2}\left[\frac{10}{3}+\frac{u_2}{2}+\frac{5}{6(1+u_2)}-\frac{1}{6(1+u_2)^2}-2\left(1+\frac{2}{u_2}\right)\ln(1+u_2)\right].
\end{aligned}
\tag{2.24}
$$

The term with a_{10} can be obtained from the probability of the Compton effect (the integrand in eq.(2.10)) by multiplying the photon energy (2.17) and integrating over u. At $u_1 \ll 1$ it gives the well-known expression for the classical radiation intensity: $I = \frac{2}{3}\alpha m^2\chi^2$, where χ is given by eq.(1.7). The term with a_{11} is the field-intensity correction to the intensity of the radiation of the Compton effect. The term with a_{20} is the intensity of the radiation connected with the absorption of two photons.

In the case of high intensity wave $\xi^2 \gg 1$, the main contribution to the integral over t in eq.(2.20) comes from the region $t \ll 1$. In this case the subtraction procedure is not needed, i.e., one should omit B and the last term in the square brackets in eq.(2.20). Expanding the integrand in powers of t and following the derivation of (2.16) from eq.(2.3),

we obtain, in the limit $\xi \to \infty$,

$$dI_\infty(u,\chi) = \frac{\alpha m^2}{\pi\sqrt{3}} \frac{u du}{(1+u)^3} \left[2f(u) K_{2/3}\left(\frac{2u}{3\chi}\right) - \int\limits_{2u/(3\chi)}^{\infty} K_{1/3}(z) dz \right] \tag{2.25}$$

Naturally, this expression is identical to the integrand of eq.(2.16) multiplyed by the photon energy (2.17).

If one keeps the next terms of the expansion in power of t of the integrand (2.20), one finds the $1/\xi^2$ correction to high intensity limit:

$$dI(u,\chi,\xi) = dI_\infty(u,\chi) + \frac{dI_1(u,\chi)}{\xi^2}, \tag{2.26}$$

with

$$dI_1(u,\chi) = \omega dW_1(u,\chi), \tag{2.27}$$

where dW_1 is given by eq.(2.19) and ω by eq.(2.17). The formula for the total intensity upto $1/\xi^2$ is then given by integrating over u (see also [8]):

$$I(\chi,\xi) = I_\infty(\chi) + \frac{I_1(\chi)}{\xi^2}. \tag{2.28}$$

Fig.2.3

At intermediate values of the parameter ξ, the total intensity of radiation has been calculated numerically using the representation (2.22). In Fig.2.3 the total intensity I is shown as a function of the parameter λ for different ξ. Comparing with Fig.2.1, one can see that the field-intensity effects for I sets on at lower ξ than for W.

Fig.2.4

Fig.2.5

It is evident that the spectral distribution of the intensity (power spectrum) is of the great importance. In Fig.2.4 the power spectrum is presented for various ξ and λ. Here the abscissa in each graph is $x = \omega/\varepsilon$, ω being the energy of the radiated photon and ε the initial electron energy. The vertical scale of each graph is in relative units. The lowest row of graphs ($\xi = 0.25$) is very close to the radiation intensity for the Compton effect (one-photon interaction), where the maximum photon energy is given by eq.(1.1) with the effective mass of the electron taken into account. These graphs are very similar to those for the radiation in undulators with a harmonic motion. In "quantum undulator" $\lambda \gg 1$, the photons are mainly radiated with energy $\omega \to \varepsilon$ ($x \to 1$). Actually, this is also true for higher rows of graphs, but with increased ξ, the interactions of two, three, etc photons become more and more important (peaks lying to the right of the main (one-photon) peak). For $\xi = 2$, and especially for $\xi = 4$, one-photon interaction contributes only a small part of the intensity, whilst the radiation due to the multi-photon interaction (to the right of the one-photon peak) dominates. The form of the spectrum changes as

λ increases and a wide multi-photon (constant field) peak appears near the end of the spectrum (at $x = \omega/\varepsilon \to 1$ for $\lambda \gg 1$), with some ripples due to one, two, three, etc photon interaction. The same evolution can be seen in Fig.2.5, where, instead of the parameter λ which useful for $\xi \ll 1$, we introduced the parameter χ useful for transition to the constant field limit ($\xi \gg 1$) (see eqs.(2.14)-(2.19)). This is clear from the row for $\xi = 10$, if one compares it with the constant field calculations (see [7]). Spectra of the probability of radiation (formula (2.1)) for relatively small ξ and λ were calculated numerically in [12], unfortunately, with a poor accuracy.

3 Pair Creation

The electron-positron pair creation by a photon in a field of an intense electromagnetic wave has been studied using the exact wave function of the electron (positron) in the wave field. For a circularly polarized wave the total probability of pair creation takes a relatively simple form [3]:

$$W_e(\eta,\xi) = \frac{\alpha m^2}{4\omega} \sum_{\substack{n\geq 1 \\ u_n > 1}}^{\infty} \int_1^{u_n} \frac{du}{u\sqrt{u(u-1)}} \left[2J_n^2(z) + \xi^2(2u-1)\left(J_{n-1}^2(z) + J_{n+1}^2(z) - 2J_n^2(z) \right) \right], \tag{3.1}$$

where $J_n(z)$ is the Bessel function and

$$u_n = \frac{n\eta}{1+\xi^2}, \quad z = \frac{2\xi}{\eta}\sqrt{1+\xi^2}\sqrt{u(u_n-u)}. \tag{3.2}$$

Here the parameters η and ξ are defined in eqs.(1.2) and (1.6), and ω is the energy of the incident photon interacting with the laser beam.

The n-th term in the sum in eq.(3.1) describes the interaction of the n photons of the wave with the incident photon. The physical region of the n-photon pair-creation is given by the inequality $u_n \geq 1$ and the threshold by $u_n = 1$ which means $2nk^\mu q_\mu = 4m_*^2 = 4m^2(1+\xi^2)$. So, u_1 describes the kinematics of the pair creation by the incident photon with one laser photon, u_2 describes that with two laser photons (in this case the threshold is twice lower), etc.

Another representation of the total probability of pair creation by a photon in circularly polarized wave was derived in the frame of the operator approach to QED in an external field [9]. For an unpolarized incident photon, it has the form [9]:

$$W_e(\eta,\xi) = \frac{\alpha m^2}{2\pi\omega} \operatorname{Im} \int_0^{\infty} \frac{dt}{t} \int_1^{\infty} \frac{du}{u\sqrt{u(u-1)}} \left[\left(1 - 2\xi^2(2u-1)\sin^2 t \right) e^{-2iu\phi} - e^{-2iut/\eta} \right] \tag{3.3}$$

where

$$\phi = \frac{t}{\eta}\left[1 + \xi^2 \left(1 - \frac{\sin^2 t}{t^2} \right) \right]. \tag{3.4}$$

Utilizing the generating function of the Bessel functions and the power series expansion of the square of the Bessel function, one can obtain the formula (3.1) from the representation (3.3). The total probability of radiation in an elliptically polarized monochromatic plane

wave can also be represented by a double integral, but with integrand containing the Bessel functions (see[9]).

It's worth noting that the integrands (over u) in eqs.(3.1) and (3.3) (after integration over t) gives the spectrum of one of the particles of the created pair. For description of the spectrum the following variables are used:

$$u = \frac{1}{1-v^2} = \cosh^2 y = \frac{\omega^2}{4\epsilon\epsilon_f} = \frac{1}{4x(1-x)}, \quad x = \frac{\epsilon}{\omega}, \tag{3.5}$$

where ϵ and ϵ_f are the energies of the created electron and positron: $\epsilon + \epsilon_f = \omega$. The spectrum which is represented by eq.(3.1) or (3.3) can be obtained from the radiation spectrum given by eq.(2.1) or (2.3) using the substitution law:

$$\begin{aligned} &\omega^2 d\omega \to \epsilon^2 d\epsilon, \quad \omega \to -\omega, \quad \epsilon \to -\epsilon; \\ &u \to -\frac{\omega}{\epsilon_f}, \quad 1+u \to -\frac{\epsilon}{\epsilon_f}, \quad \frac{u\,du}{(1+u)^3} \to dx = \frac{dy}{2\cosh^2 y}. \end{aligned} \tag{3.6}$$

The integral over u in eq.(3.3) can be evaluated analytically. As a result we obtain an expression for the total probability of pair creation by a photon interacting with the circularly polarized electromagnetic wave in a form of a single integral [9]:

$$\begin{aligned} W_e(\lambda,\xi) = -\frac{e^2m^2}{2\omega}\Big\{ \mathrm{Im}\int_0^\infty \frac{dt}{t} e^{-i\phi}\Big[\left(1+2\xi^2\sin^2 t\right)\phi\left(H_0^{(2)}(\phi) + iH_1^{(2)}(\phi)\right) \\ -2i\xi^2\sin^2 t\, H_0^{(2)}(\phi)\Big] - 1\Big\}, \end{aligned} \tag{3.7}$$

where $H_{0,1}^{(2)}$ is the Hankel function. This representation is very convenient in numerical calculations for arbitrary values of the parameters η and ξ.

In the case $\xi^2 \gg 1$, just as in the radiation problem, many photons of the wave simultaneously interact with the incident photon and this corresponds to the interaction of the photon with a constant crossed field. This interaction can better be described by the parameter κ defined in eq.(1.8). One can use any two parameters of η, ξ and κ for the description of the process. Theoretically, the intensity effect is most interesting at $\xi \sim 1$.

When $\xi \ll 1$, the integrand in eq.(3.3) may be expanded in powers of ξ^2. Retaining only the terms $\propto \xi^2$, we obtain (according to the note above, we present the spectral form):

$$dW_e^{(10)}(\eta,\xi) = \frac{\xi^2\alpha m^2}{\pi\omega}\frac{du}{u\sqrt{u(u-1)}}\left[(2u-1)s_1 + \frac{u}{\eta}s_2\right], \tag{3.8}$$

where s_1 and s_2 are integrals over t defined in (2.9).

Substituting (2.9) into (3.8), we obtain the spectrum of one of the particles of the created pair:

$$dW_e^{(10)}(\eta,\xi) = \frac{\alpha m^2\xi^2}{4\omega}\frac{du}{u\sqrt{u(u-1)}}\left[(2u-1) + 2\frac{u}{\eta}\left(1-\frac{u}{\eta}\right)\right]\vartheta\left(1-\frac{u}{\eta}\right). \tag{3.9}$$

Evaluating the elementary integrals over u, we obtain the total probability of the pair creation upto $O(\xi^2)$:

$$W_e^{(10)}(\eta,\xi) = \frac{\xi^2\alpha m^2}{2\omega}\left[\left(2+\frac{2}{\eta}-\frac{1}{\eta^2}\right)L - \sqrt{1-\frac{1}{\eta}}\left(1+\frac{1}{\eta}\right)\right], \tag{3.10}$$

where $L = \ln(\sqrt{\eta}+\sqrt{\eta-1})$. The formulae (3.9) and (3.10) are related to the differential and the total cross section $d\sigma_e$ of the electron-positron pair creation by two photons (Breit-Wheeler formula) as

$$d\sigma_e = \frac{2\pi\alpha\omega}{\xi^2 m^4\eta} dW_e^{(10)}(\eta,\xi) \tag{3.11}$$

The same procedure but with terms $\propto \xi^4$ taken into account results in the following changes:

(1) In eqs.(3.9)-(3.11) η should be substituted by u_1 defined in (3.2) (the mass of the electron is substituted by the effective mass).

(2) There is a field-intensity correction to the Breit-Wheeler formula:

$$dW_e^{(11)}(\eta,\xi) = -\frac{\xi^4\alpha m^2}{2\omega}\frac{du}{u\sqrt{u(u-1)}}\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\left[4u-1+\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\right]\vartheta(1-\frac{u}{u_1}) \tag{3.12}$$

The probability of the interaction of one laser photon with the incident photon is $dW_e^{(1)} = dW_e^{(10)} + dW_e^{(11)} + \ldots$, etc. After integration over u we obtain the intensity correction to the one-photon probability:

$$W_e^{(11)}(\eta,\xi) = -\frac{\xi^4\alpha m^2}{4\omega}\left[\left(\frac{4}{u_1}-\frac{2}{u_1^2}-\frac{3}{u_1^3}+\frac{5}{4u_1^4}\right)L + \sqrt{1-\frac{1}{u_1}}\left(4-\frac{10}{3u_1}-\frac{13}{6u_1^2}+\frac{5}{4u_1^3}\right)\right], \tag{3.13}$$

(3) An interaction of two laser photons with the incident photon comes in (with the kinematic boundary defined by $u_2 \geq 1$). The probability of this process is

$$dW_e^{(20)}(\eta,\xi) = \frac{\xi^4\alpha m^2}{\omega}\frac{du}{u\sqrt{u(u-1)}}\frac{u}{u_2}\left(1-\frac{u}{u_2}\right)\left[2u-1+2\frac{u}{u_2}\left(1-\frac{u}{u_2}\right)\right]\vartheta\left(1-\frac{u}{u_2}\right). \tag{3.14}$$

After integration over u we obtain the total probability of the pair creation by the two laser photons :

$$W_e^{(20)}(\eta,\xi) = \frac{\xi^4\alpha m^2}{4\omega}\left[\left(\frac{6}{u_2^2}-\frac{12}{u_2^3}+\frac{5}{u_2^4}\right)L + \sqrt{1-\frac{1}{u_1}}\left(4+\frac{2}{3u_2}-\frac{26}{3u_2^2}+\frac{5}{u_1^3}\right)\right], \tag{3.15}$$

Explicit formulae in order ξ^4 were obtained in [3].

In the case of high intensity wave $\xi^2 \gg 1$, the main contribution to the integral over t in eq.(3.3) comes from the region $t \ll 1$. Expanding the integrand in powers of t, we obtain the spectral distribution of created pairs:

$$dW_{e,\infty}(\kappa) = -\frac{\alpha m^2}{4\pi\omega} dv \,\mathrm{Im} \int\limits_{-\infty}^{\infty} \frac{dy}{y - i0} \left(1 - 2y^2 \frac{1+v^2}{1-v^2}\right) \exp\left[-i \frac{4}{\kappa(1-v^2)} \left(y + \frac{y^3}{3}\right)\right], \tag{3.16}$$

where we used the variable v (see eq.(3.5)) ($-1 \leq v \leq 1$), changed the integration variable to $y = \xi t$, and introduced the strong field parameter κ defined in (1.8) since in this limit only the product of parameters $\eta\xi$ appears. Using the integrals (2.15), we obtain

$$\begin{aligned} dW_{e,\infty}(\kappa) &= \frac{\alpha m^2}{2\pi\sqrt{3}\omega} dv \left[2 \frac{1+v^2}{1-v^2} K_{2/3}(\rho) + \int\limits_{\rho}^{\infty} K_{1/3}(z) dz\right] \\ &= \frac{\alpha m^2}{\pi\sqrt{3}\omega^2} d\epsilon \left[\frac{\epsilon^2 + \epsilon_f^2}{\epsilon\epsilon_f} K_{2/3}(\rho) + \int\limits_{\rho}^{\infty} K_{1/3}(z) dz\right] \end{aligned} \tag{3.17}$$

where $\rho = \dfrac{8}{3\kappa(1-v^2)} = \dfrac{2\omega^2}{3\kappa\epsilon\epsilon_f}$. This is the well-known expression for the spectral distribution of one of the created particles in a constant external field in quasiclassical approximation (see e.g. [7]).

Integrating eq.(3.17) we obtain the total probability of pair creation by the photon in a constant external field:

$$W_{e,\infty}(\kappa) = \frac{\alpha m^2}{3\sqrt{3}\pi\omega} \int\limits_0^1 dv \frac{9 - v^2}{1 - v^2} K_{2/3}(\rho). \tag{3.18}$$

(The integrand of this formula does not represent the spectrum because we have employed the integration by parts.)

If one keeps the next terms of the expansion in powers of t of the expression (3.3), which describes the spectrum of created particles, one finds the $1/\xi^2$ correction to the high intensity limit:

$$dW_e(\kappa, \xi) = dW_{e,\infty}(\kappa) + \frac{dW_e^1(\kappa)}{\xi^2}, \tag{3.19}$$

with

$$dW_e^1(\kappa) = -\frac{\alpha m^2}{15\sqrt{3}\pi\omega} dv \left\{\left(\frac{2}{3} - \frac{1+v^2}{1-v^2}\right) K_{2/3}(\rho) + \left[\frac{1+v^2}{1-v^2}\left(\frac{14}{9\rho} - \rho\right) - \frac{\rho}{2}\right] K_{1/3}(\rho)\right\} \tag{3.20}$$

where ρ is defined in (3.17). This expression describes the correction to the probability spectrum. The correction to the total probability (the expression (3.18) integrated over v) may be obtained in a straightforward manner.

Fig.3.1

For intermediate values of parameters κ and ξ, we calculated the total probability of pair creation numerically using the representations (3.1) and (3.3). In Fig.3.1 the dependence of the total probability of pair creation W_e (normalized by $\alpha m^2/\omega$) on the parameter η is given for various values of the parameter ξ. The multi-photon effects become apparent as ξ increases.

Fig.3.2

When one changes the incident photon energy ω with the fixed field strength E, the total probability is proportional to $W_e/(\kappa/\omega)$. The 3D graph Fig.3.2 gives the complete picture of the behavior of the total probability of pair creation $W_e/(\kappa\alpha m^2/\omega)$ under the fixed field strength as a function of the two parameters κ and ξ. It is seen that W_e depends on the two parameters essentially non-monotonically. For fixed $\kappa = 2\eta\xi$ there exists a number of maxima which become less and less pronounced with increasing ξ. Such dependence on ξ is associated with the fact that the process of pair creation by n photon absorption has the threshold

$$u_n = \frac{\eta n}{1+\xi^2} = \frac{\kappa n}{2\xi(1+\xi^2)} = 1. \tag{3.21}$$

As ξ increases, the minimal number of photons to be absorbed from the wave in order to generate a pair also increases. For $\kappa = 1$ and $n = 1$ solution of (3.21) is $\xi_1 = 0.424$, and for $n = 2$, $\xi_2 = 0.682$. If $\xi > \xi_1$, two or more photons have to be absorbed from the wave to create a pair. The threshold cusp is situated just at $\xi = \xi_1$ and the next cusp is situated at next threshold $\xi = \xi_2$. For relatively low ξ the probability is dominated by one-photon absorption . It attains the maximum and then decreases because of the reduction of the phase space.

When κ increases from $\kappa = 1$, the height of the maximum also increases. The value of $W_e/(\kappa/\omega)$ takes an absolute maximum at $\xi = 1.36$ and $\kappa = 9.3$ and at this point the total pair creation probability is given by

$$W_e = 0.1171\frac{2\alpha m E}{E_0}, \tag{3.22}$$

where E is the electric field defined in (1.5) and $E_0 = m^2/e = (m^2c^3/(e\hbar)) = 1.32 \times 10^{18}$V/m. Beyond this point the height of the maximum starts to decrease as κ increases. Some of the cross sections of the graph are already known: for $\kappa = 1$, $W_e(\kappa, \xi)$ is presented in [3], (see also [10] where origin of non-monotonic behavior is discussed); for $\xi \to \infty$, $W_e(\kappa)$ is presented in [7] and it has the maximum at $\kappa = 11.7$.

Fig.3.3

Fig.3.4

In Fig.3.3 the spectrum of the created positron (electron) is presented for various ξ and η. Here the abscissa is $x = \epsilon/\omega$, ω being the energy of the initial high energy photon and ϵ the energy of the created particle. The vartical scale of each graph is given in its own units.

In the lowest row of graphs ($\xi = 0.25$), the contribution of the n-photon process with the smallest n satisfying $u_n > 1$ (i.e., $n > (1+\xi^2)/\eta$) dominates. So, the graphs for $\eta > 1 + \xi^2 \approx 1.06$ are essentially the one-photon contribution and, therefore, are very similar to the energy distribution of the Breit-Wheeler formula. For $\eta = 0.2$ and $\eta = 0.5$ (below the threshold of one-photon reaction) we have multi-photon contributions only, the absolute value of which is much smaller than in the graphs for $\eta > 1$, although this plot does not show the absolute scale.

As ξ increases, the relative weight of the multi-photon contribution (peaks lying on sides of the main (one-photon) distribution) increases, causing a rise of the absolute value and widening of the distribution curve. . For $\xi = 2$, and especially for $\xi = 4$, one-photon interaction contributes only a small part of the probability, while the pair creation is dominated by the multi-photon interaction. The form of the spectrum changes as η increases and wide multi-photon (constant field) peaks appear near the low and high energy edges of the spectrum (at $x = \epsilon/\omega \to 0, 1$ for $\eta \gg 1$), with some ripples due to one, two, three, etc photon interaction. This tendency of creating low energy $e^{\pm}$ and high energy $e\mp$ for large η is seen for small ξ, too.

One can see the same evolution in Fig.3.4 where, instead of the parameter η, we introduced the parameter κ which is useful for transition to the constant field limit ($\xi \gg 1$) (see eqs.(3.16)-(3.20)). This is clear from the row for $\xi = 10$, if one compares it with constant field calculations (see [7]).

We considered above the radiation and pair creation in the field of circularly polarized intense wave. An analysis for the wave with other polarizations is much more complicated (the integrand in the double integrals of the type (2.3), (2.20) and (3.3) contains Bessel functions) but actually there is no essential difference between the linear polarization and general case of elliptic polarization (see,[4], [9]). General formulae for linear polarization of the wave were obtained also in [11], [10], but no analysis for arbitrary ξ was made. Nevertheless, one can make some qualitative remarks [10] for the pair creation process. When many photons $n \gg 1$ from circularly polarized wave are absorbed, the created pair acquires high angular momentum of order n which is a product of the relative momentum and the effective distance. The effective distance is the same order for any polarization, while the relative momentum is maximal for circularly polarized wave, since in the case of linear polarization the acquired angular momentum is distributed between 0 and n and, therefore, the relative momentum is lower. As a consequence, the energy distribution of the created particles in circularly polarized wave is shifted to the higher energies comparing with the case of linear polarization for the smae given parameters ξ and κ.

4 Application in Linear Colliders

Since several years ago, it has been proposed to convert an e^+e^- linear collider into a γ-γ or e-γ collider by using backward Compton scattering from high energy electron beam irradiated by a laser beam [1]. It has also been suggested that the high energy photons created in this way might be used to generate low-emittance positron beam by the pair creation in the same or another laser beam [13].

If the initial particle (electron or photon) travels in a laser field for a long time,

the radiated photons can create e^+e^- pairs, which radiate next-generation photons, etc., causing an electromagnetic cascade to develop. The formulae of the probability spectra given above ((2.1), (2.3) or (2.20), (3.1) and (3.3)) can be used as the kernels of the kinetic equations describing the evolution of the cascade. Here, we shall assume that the whole process takes place in the same (circularly polarized) laser field with constant strength ξ.

We shall ignore all the effects of the polarization of electrons and radiated photons. Regorously speaking, this is not physically self-consistent because, even if the initial electron beam is unpolarized, the radiated photons are definitely polarized, which affects the subsequent pair-creation process. The cascade electroproduction process in a constant external field was considered in [14], in which the importance of taking into account the polarization of radiated photons is demonstrated.

Let us denote the initial electron energy by ε_0 and the λ parameter for ε_0 by λ_0. In order to describe the whole process, we need to specify the total duration of the beam-laser interaction, which we denote by T (not to be confused with the function $T(\phi)$ in eq.(2.6)). Since the initial electron emits $W(\lambda_0, \xi)$ photons per unit time, the fraction of the initial electrons which survives until T is given by $\exp(-W(\lambda_0, \xi)T)$. We introduce the dimensionless parameter

$$\tau \equiv W(\lambda_0, \xi)T. \tag{4.1}$$

Thus, the cascade process can be described by three dimensionless parameters ξ, λ_0 and τ.

The number of photons generated by the initial electrons (normalized by the initial number of electrons) is given by $1 - \exp(-\tau)$ and its spectrum is close to the differential Compton cross section if ξ is small. Actually, the total number of photons is larger than $1 - \exp(-\tau)$ because low energy electrons after radiation can still emit photons (at even larger rate than the full energy electrons). Thus, unless $\tau \ll 1$, the soft photons are much more abundant than can be expected from the differential Compton cross section. These photons, however, should have much larger out-coming angles than $1/\gamma$, though in this paper we have not dicussed the angular distribution. They will spread out until they come to the γ-γ collision point and will not contribute much to the final γ-γ luminosity. Therefore, we shall separate the contribution of the photons radiated from full energy electrons, which we shall call 'primary photons'. The number of the primary photons is given by $1 - \exp(-\tau)$ if they are not lost by the pair-creation process.

When the field is weak ($\xi \ll 1$), the single photon process dominates and the photon energy spectrum is given by the differential Compton cross section. In practice, however, ξ can not be very small if one aims at high luminosity of γ-γ collision. The e-γ conversion rate can be raised by making the laser pulse longer but this is not quite efficient because of the effect of the depth of focus as stated in the introduction. With ξ large, the multi-photon contribution into radiation intensity become more and more essential, making the photon spectrum softer. Just in the region $\xi \sim 1$, the multi-photon contribution becomes comparable with one-photon one. This is clearly demonstrated in Figs. 4.1 and 4.2.

Fig.4.1a and 4.1b

Fig.4.2a and 4.2b

In Fig.4.1 the radiation probability spectrum is plotted as a function of the photon energy ω ($x = \omega/\varepsilon$) and ξ for $\lambda = 1$ (Fig.4.1a) and 5 (Fig.4.1b). Fig.4.2a and 2b are corresponding power spectrum $dI/dx = xdW/dx$. One can see that by $\xi = 1$ the one-photon contribution becomes relatively small. So, one has to choose some optimal value of $\xi < 1$ from this point of view.

When ξ is small, the maximum energy of the photon is $\varepsilon_0\lambda_0/(1+\lambda_0)$, which corresponds to the pair creation parameter $\eta = \lambda_0^2/4(1+\lambda_0)$. If this η is larger than 1; i.e., if $\lambda_0 > 2 + 2\sqrt{2} \approx 4.8$, the photon can create e^+e^- pairs . Since, above the threshold, the pair creation rate is basically the same order with the radiation rate, the gain of number of primary photons decreases when τ is too large (while the total number of photons increases monotonically). Thus, when $\lambda_0 > 4.8$, there is an optimum τ which makes the number of primary photons maximal. On the other hand, when $\lambda_0 < 4.8$, the number of photons increases monotonically and all the initial electrons are eventually converted into photons. Thus, the maximum conversion rate is unity.

The above simple picture fails when ξ is not very small because of the multiple photon processes. Firstly, the radiation by multiple photons can generate photons with energies higher than $\varepsilon_0\lambda_0/(1+\lambda_0)$. The maximum energy with infinite number of photons is of course ε_0. With this effect only, the threshold of pair creation becomes $\lambda_0 = 4$. Secondly, even if $\lambda_0 < 4$, pairs can still be created by multiple photon absorption.

The maximum energy of the radiated photon via n-photon process is given by eq.(2.17) with u replaced by u_n defined in eq.(2.2); i.e.,

$$\omega = \varepsilon_0 \frac{n\lambda_0}{1+\xi^2+n\lambda_0}. \tag{4.2}$$

For this photon, the pair creation parameter η is given by

$$\eta = \frac{\lambda_0}{4}\frac{\omega}{\varepsilon_0} = \frac{\lambda_0}{4}\frac{n\lambda_0}{1+\xi^2+n\lambda_0}. \tag{4.3}$$

The threshold to create a pair via n'-photon absorbtion is $\eta = (1+\xi^2)/n'$, which is given by putting $u_n = 1$ in eq.(3.2). Therefore, the pair creation threshold via n-photon radiation and n'-photon pair creation is

$$\lambda_0 = \frac{2}{n'}(1+\sqrt{1+n/n'})(1+\xi^2). \tag{4.4}$$

With all these effects the pair creation takes place even for λ_0 considerably smaller than 4.8.

The evolution of the energy distribution of the electrons, positrons and photons, denoted by $f^-(\varepsilon,t)$, $f^+(\varepsilon,t)$, and $g(\omega,t)$, respectively, is described by the following rate equation.

$$\frac{df^\pm(\varepsilon)}{dt} = -W(\varepsilon)f^\pm(\varepsilon) + \int_\varepsilon^{\varepsilon_0} d\varepsilon_1 \left[\frac{dW(\varepsilon_1,\omega)}{d\omega}\right]_{\omega=\varepsilon_1-\varepsilon} f^\pm(\varepsilon_1) + \int_\varepsilon^{\varepsilon_0} d\omega \frac{dW_e(\omega,\varepsilon)}{d\varepsilon} g(\omega) \tag{4.5}$$

$$\frac{dg(\omega)}{dt} = -W_e(\omega)g(\omega) + \int_\omega^{\varepsilon_0} d\varepsilon_1 \frac{dW(\varepsilon_1,\omega)}{d\omega}[f^+(\varepsilon_1)+f^-(\varepsilon_1)]. \tag{4.6}$$

Here, $dW(\varepsilon,\omega)/d\omega$ is the probability of an electron of energy ε to emit a photon with energy ω in unit time, $dW_e(\omega,\varepsilon)/d\varepsilon$ is that of a photon with energy ω to create a pair of energy ε and $\omega-\varepsilon$, and $W(\varepsilon)$ and $W_e(\omega)$ are the total rates. (Since ξ is constant, we omitted it in the arguments of W and W_e.) The initial condition is $g = f^+ = 0$, and $f^-(\varepsilon,0) = \delta(\varepsilon-\varepsilon_0)$. The distribution of the primary photons, $g_0(\omega,t)$, can be obtained by taking into accout only f^- with $\varepsilon_1 = \varepsilon_0$ in eq.(4.6). For numerical integration of the set of equations (4.5), (4.6), one should divide the whole energy range (0,ε_0) into bins and compute the transition rates from a bin to a bin during a (small) time step. Then, these equations can be written in the form of multiplications of matrices in each time step. The matrix elements do not depend on time because we assume ξ is constant. The whole process scales with three independent parameters ξ, λ_0, and τ.

Fig.4.3

Figs.4.3 to 4.5 show the results of numerical integration of the differential equations (4.5) and (4.6). In Fig.4.3 the graph at the bottom is the value of τ when the number of primary photons $n(\gamma_0) = \int g_0(\omega)d\omega$ takes maximum and the two graphs above are the number of primary photons and that of positrons $n(e^+) = \int f^+(\varepsilon)d\varepsilon$ at that time for various values of ξ from 0.1 to 0.8. The horizontal axis is λ_0. The number of positrons includes all the positrons not only from primary photons.

One can see the small shift of the threshold of pair creation due to the effective mass (the factor $1+\xi^2$ in eq.(4.2) with n=n'=1) for the line $\xi = 0.2$ as indicated by an arrow. Above $\xi > 0.2$, however, the threshold is totally obscured due to the multiple photon effect. The effect is more significant for small λ_0. When $\lambda_0 \gtrsim 6.0$, there is no essential difference between different ξ's.

Fig.4.4

When λ_0 is small, the value of τ giving maximum gain is rather large. (It is infinity for $\xi \to 0$ because, without pair creation, the number of primary photons increase with time monotonically and the maximum gain becomes 1 at $\tau = \infty$, as stated before.) This is not quite practical because much smaller value of τ already gives a high gain close to the maximum. The time after is spent only for increasing the number of pairs (and wasting the laser power). Thus, in Fig.4.4 we have plotted the number of positrons against λ_0 for a more practical value $\tau = 1$. The one-photon thresholds are again indicated by arrows. We do not anyway want λ_0 much smaller than 4.8 because otherwise we need to construct higher energy electron linacs for given γ-γ energy. From practical considerations of the collider design, it seems ξ has to be 0.1 to 0.4 for the next generation linear colliders. Consequently, it is inevitable to have a few percent positrons (or at least a fraction of percent) compared with the initial number of electrons.

Fig.4.5

Fig.4.5 shows how many low energy photons are created, in the case $\xi = 0.3$ and $\lambda_0 = 4.80$, by the time $\tau = 0.25$, 0.5, 0.75 and 1.0. The spectrum of the primary photons (at $\tau = 1$) is shown by the dashed line for comparison. As τ increases, the spectrum

becomes soft rapidly. The ratio of the number of primary photons to the total number of photons, $n(\gamma_0)/n(\gamma)$ is approximately $(1-e^{-\tau})/\tau$ if we ignore the energy dependence of the total Compton cross section. Actually, it is a little smaller and is about 0.50 ($\tau = 1$) in the case above.

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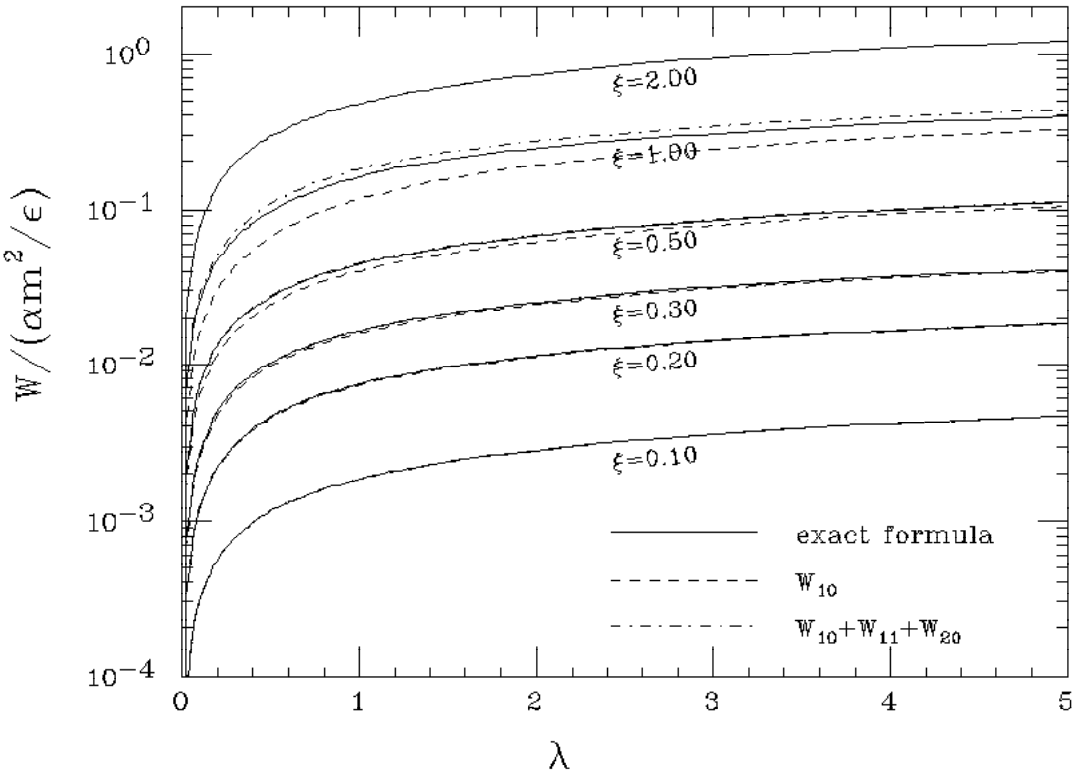

Fig.2.1 The total probability of radiation W for different values of the parameter ξ.

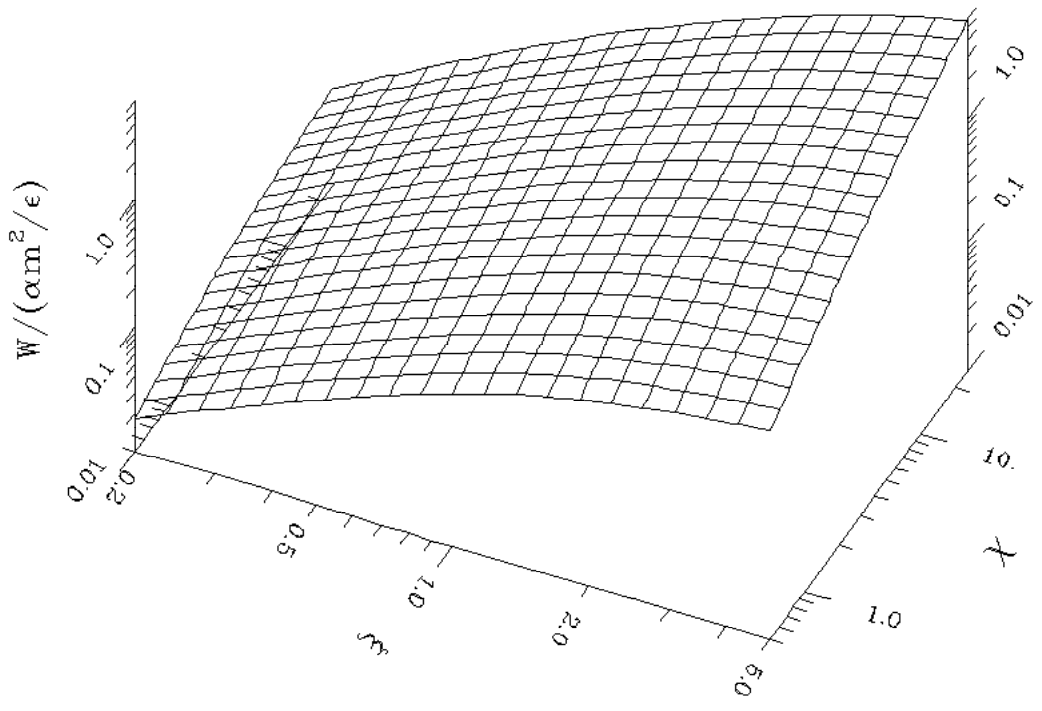

Fig.2.2 The total probability of radiation W in 3D graph as a function of the two parameters ξ and χ. This graph gives a complete picture of W behaviour for the parameter range considered.

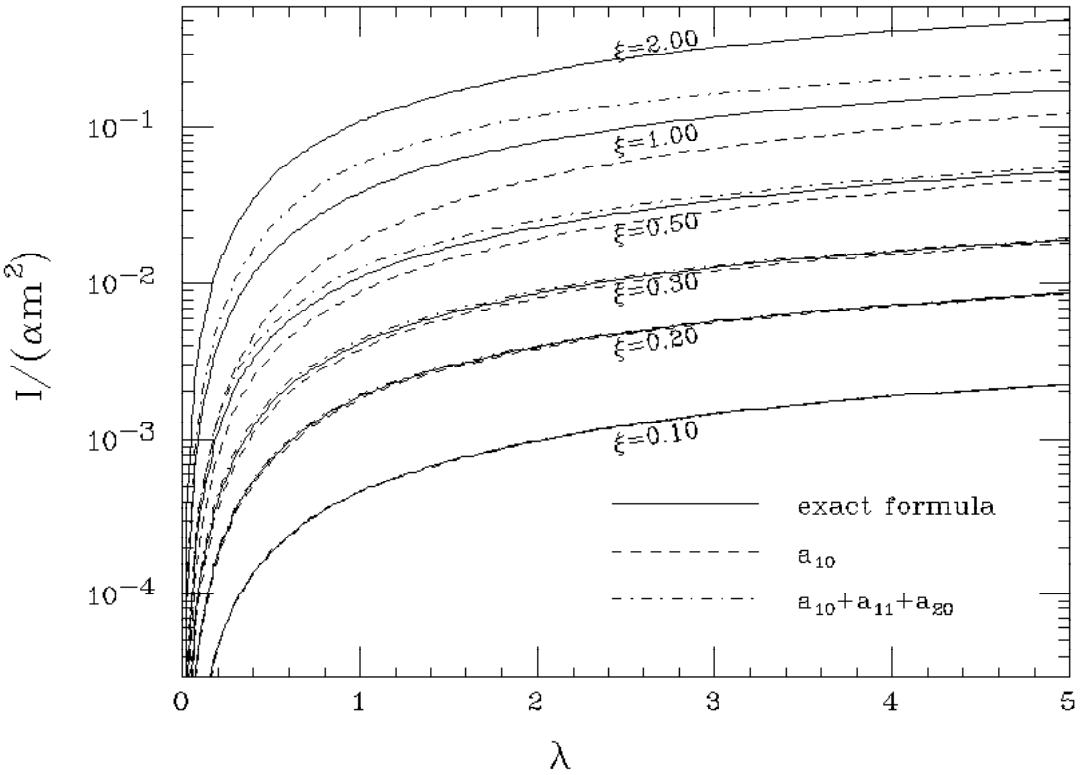

Fig.2.3 The total intensity of radiation I for different ξ. Comparing with Fig.2.1, one can see that intensity effects for I "turn on" at lower ξ than for W.

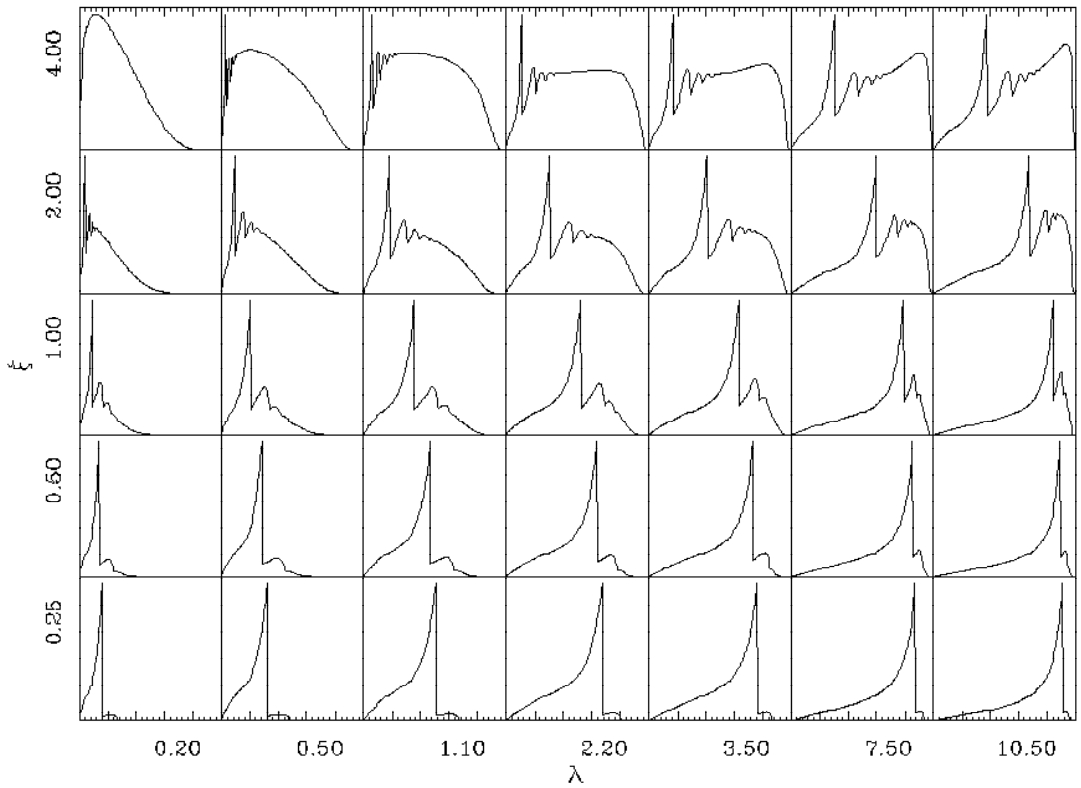

Fig.2.4 Power spectrum of radiation for various ξ and λ. Here, in each graph, the abscissa is $x = \omega/\varepsilon$, ω being the energy of the radiated photon and ε the initial energy of the particle. Each individual graph is given in relative units.

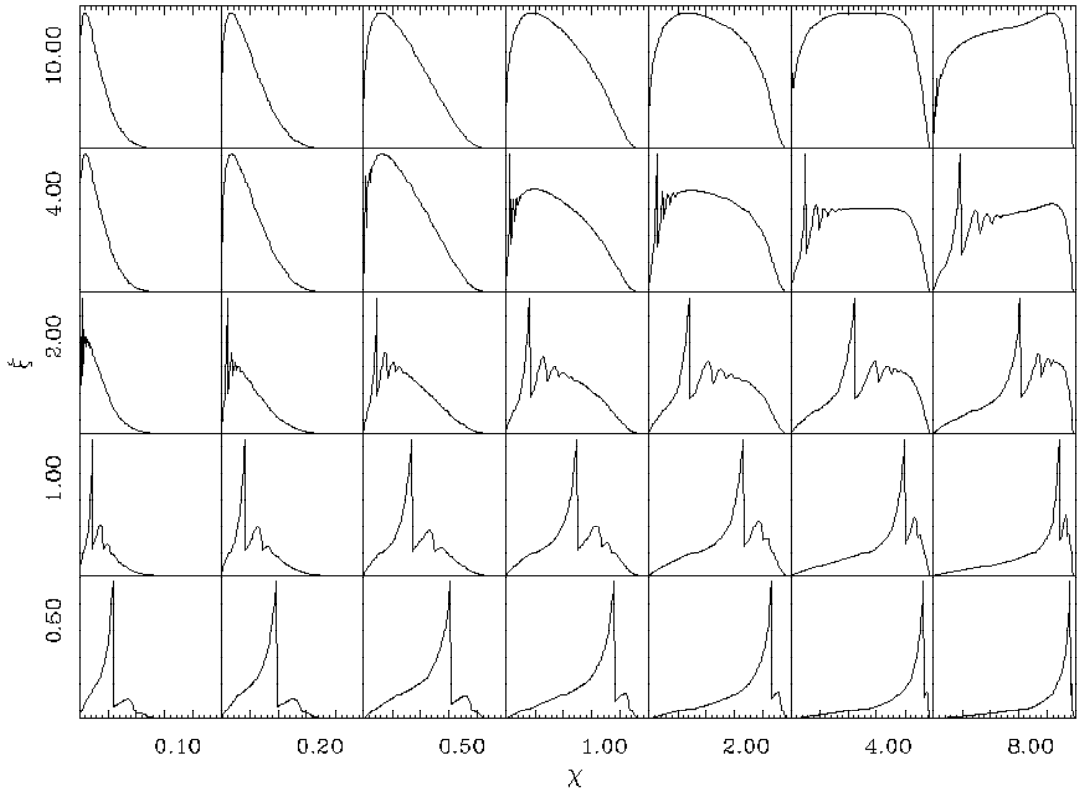

Fig.2.5 The same as Fig.2.4 but parameter χ is introduced instead of parameter λ.

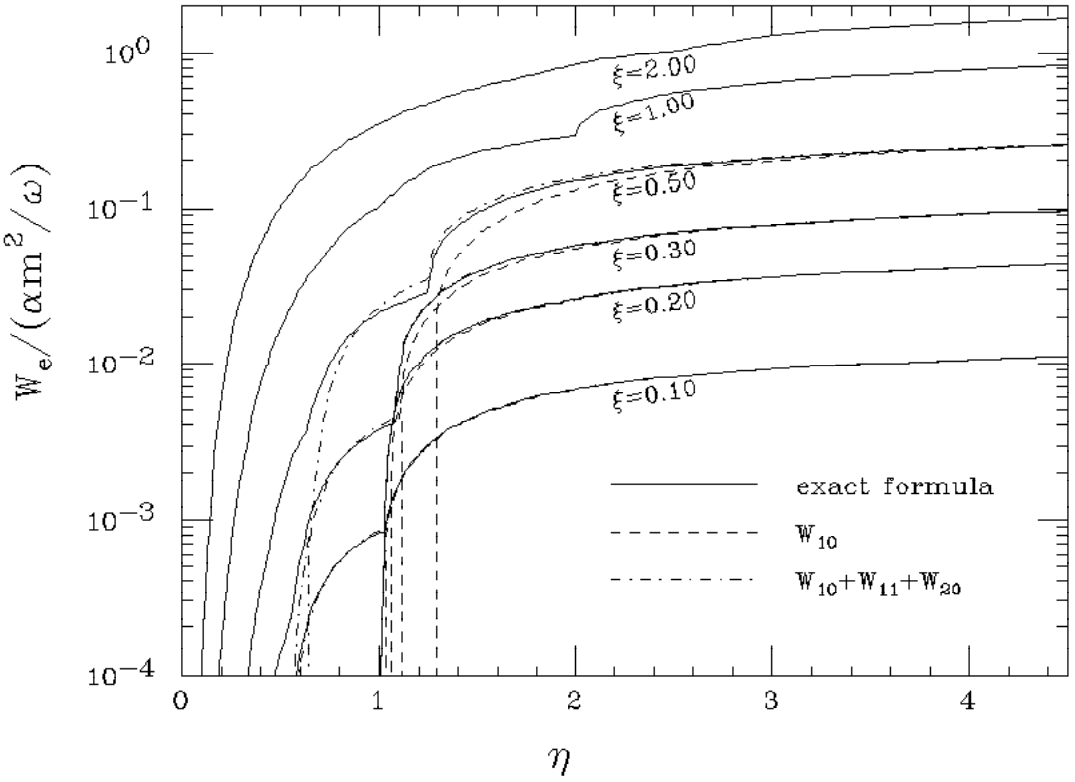

Fig.3.1 Dependence of the total probability of pair creation W_e on parameter η for different values of ξ.

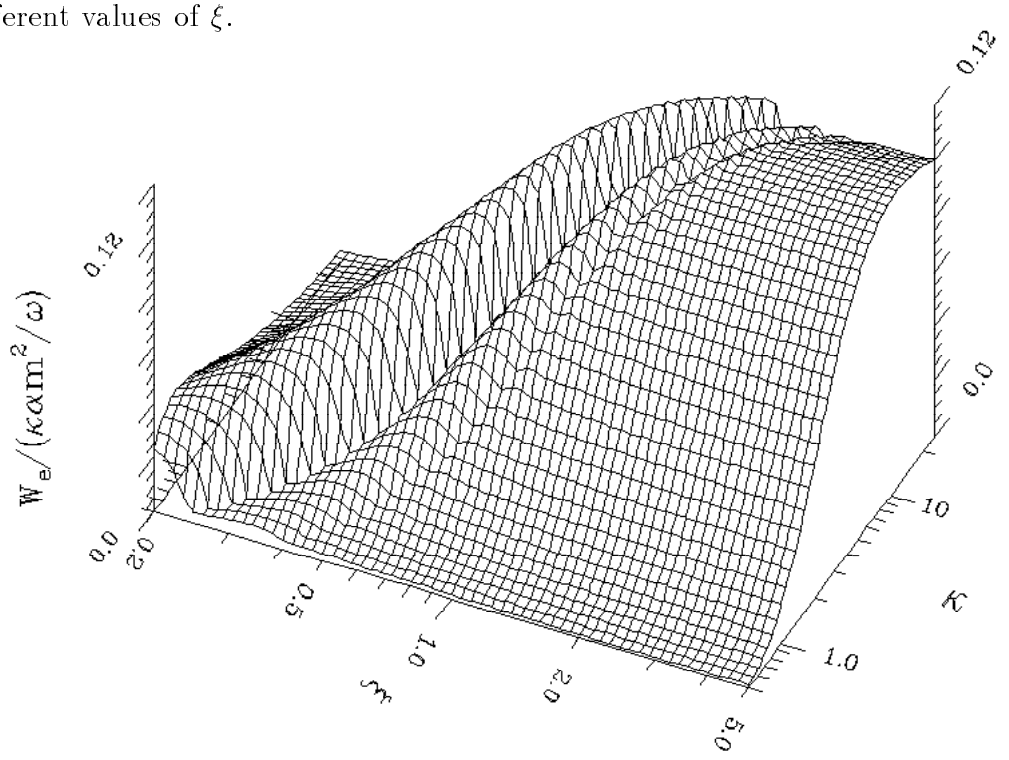

Fig.3.2 3D plot of the total probability of the pair creation W_e by photon for the fixed field of the wave, representing the complete picture of W_e behavior as a function of parameters ξ and χ.

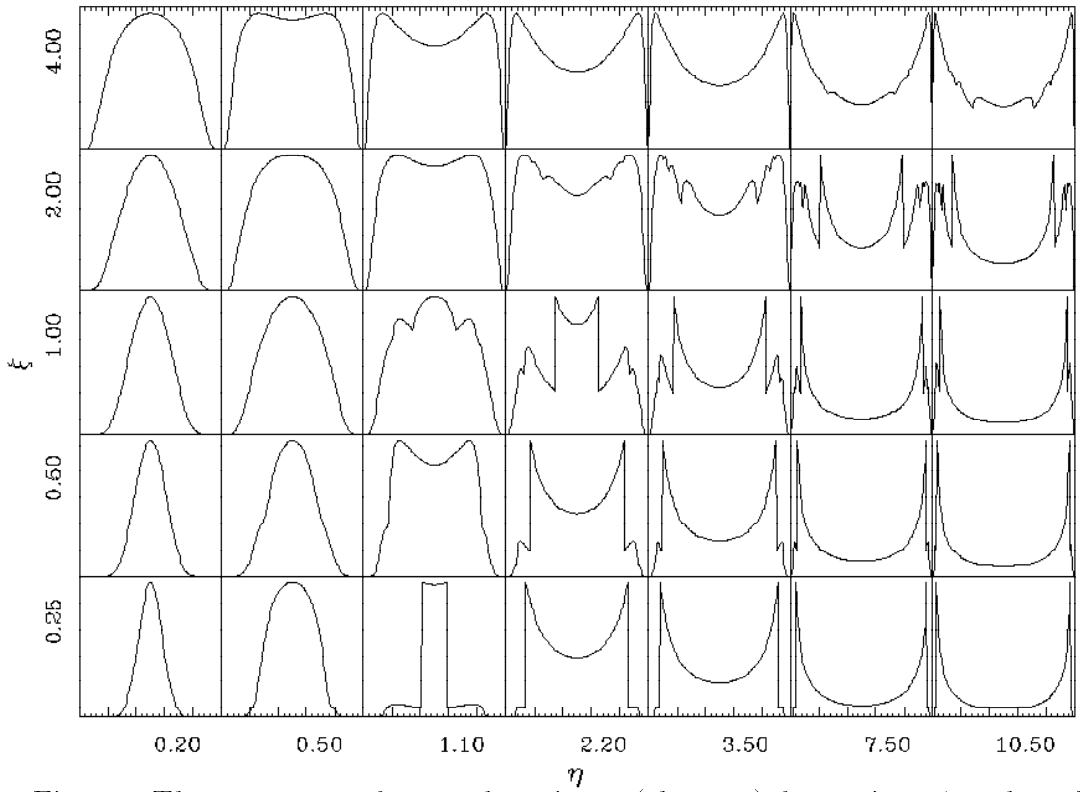

Fig.3.3 The spectrum of created positron (electron) for various ξ and η. Here the abscissa is $x = \epsilon/\omega$, ω being the energy of the initial high energy photon and ϵ the energy of the created particle. Each individual graph is given in relative units.

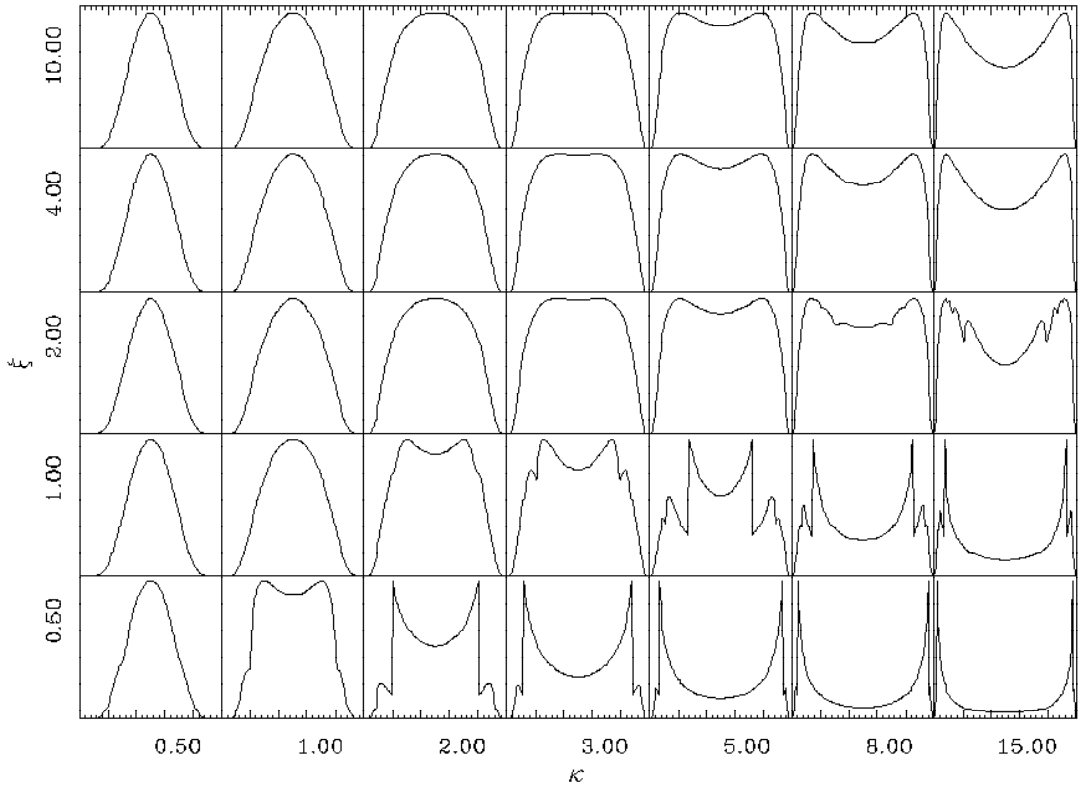

Fig.3.4 The same as in Fig.3.3 but parameter κ is introduced instead of parameter η.

Fig.4.1a

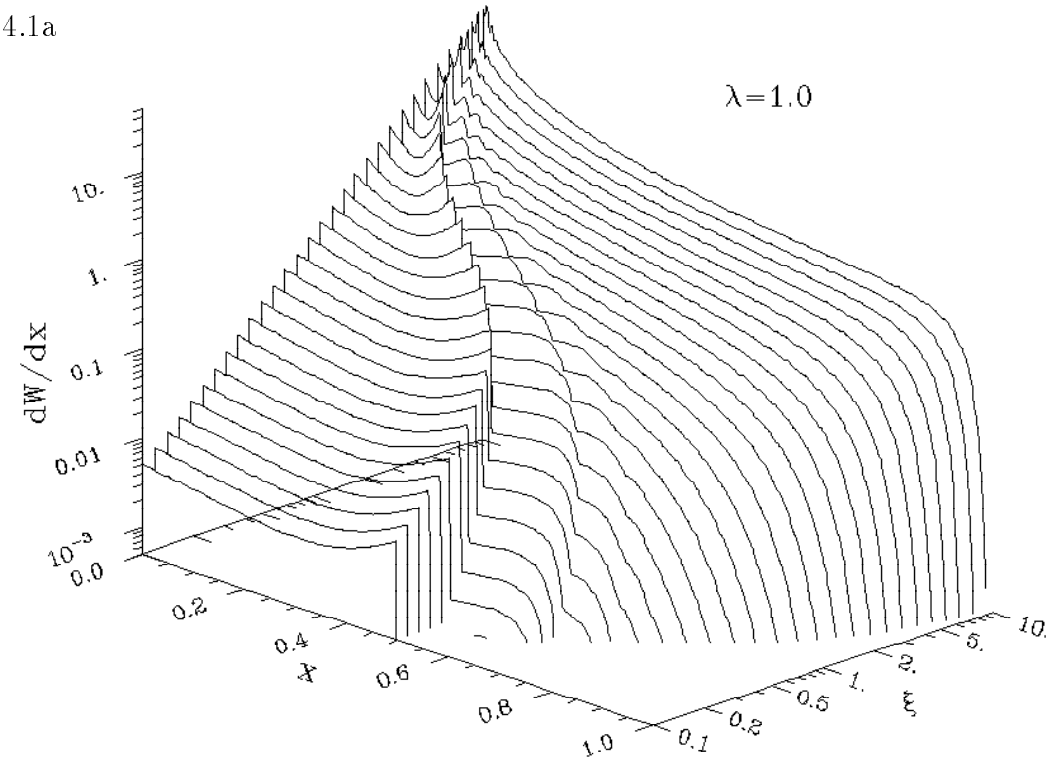

Fig.4.1b

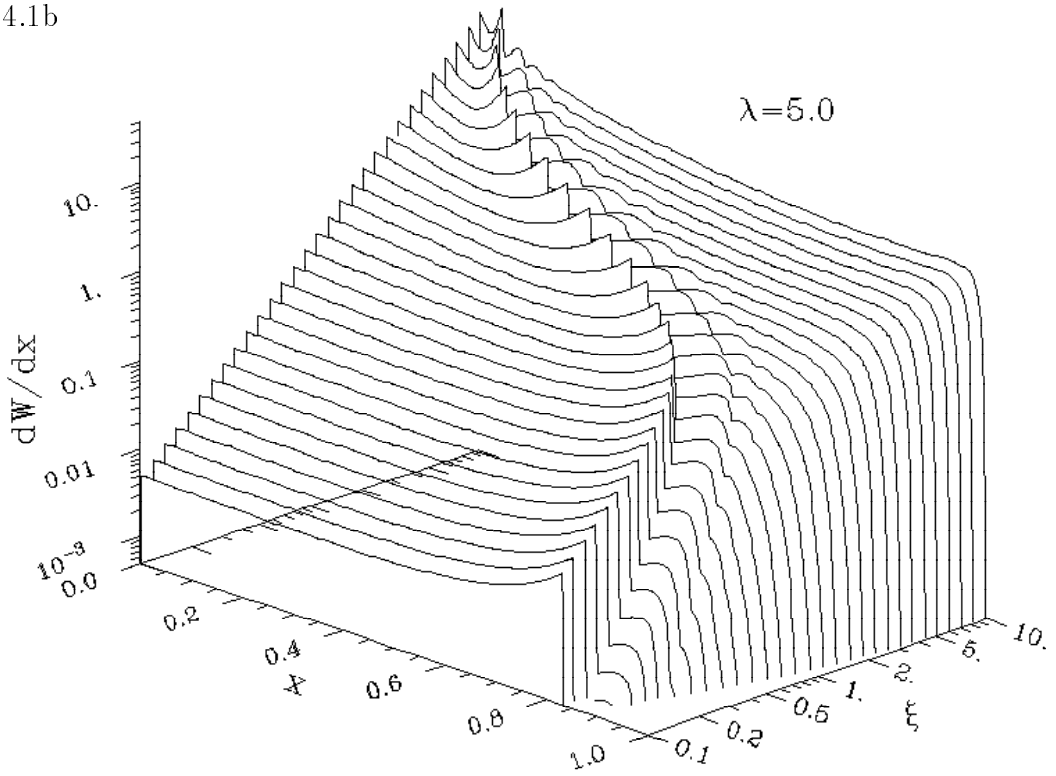

Fig.4.1 3D plot of the probability spectrum of radiation as a function of the photon energy ω ($x = \omega/\varepsilon$) and intensity parameter ξ for fixed $\lambda = 1$ and 5. The shift of the one-photon peak due to increase of ξ (increase of effective mass) is seen as well as increase of the contribution of multi-photon processes.

Fig.4.2a

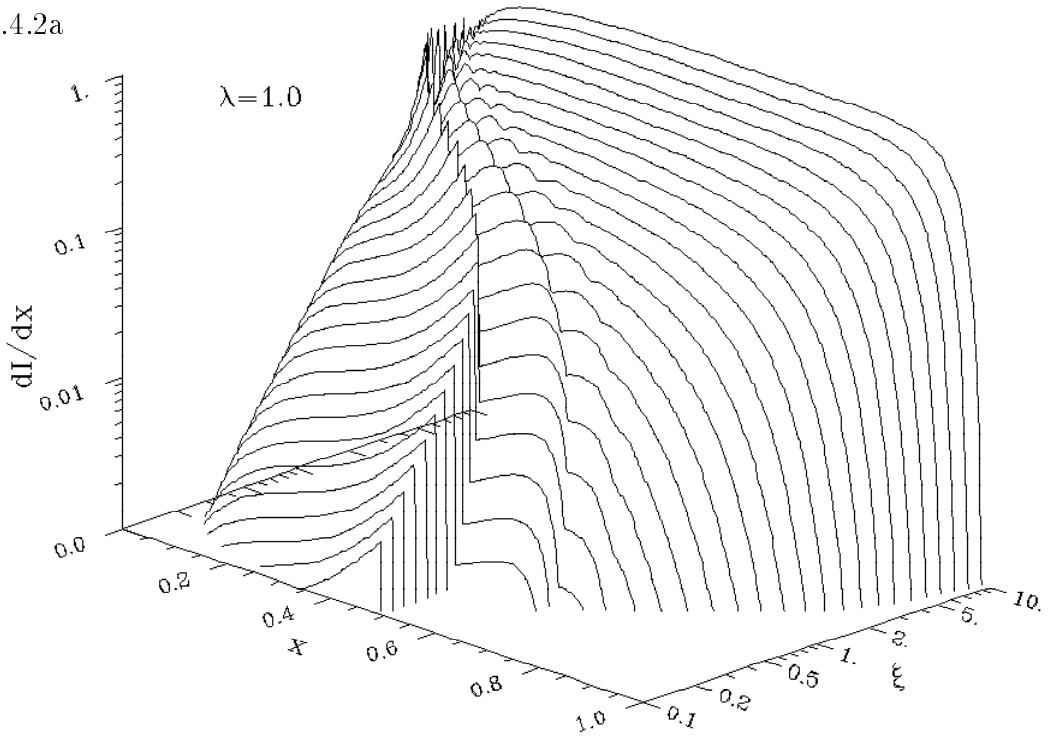

Fig.4.2b

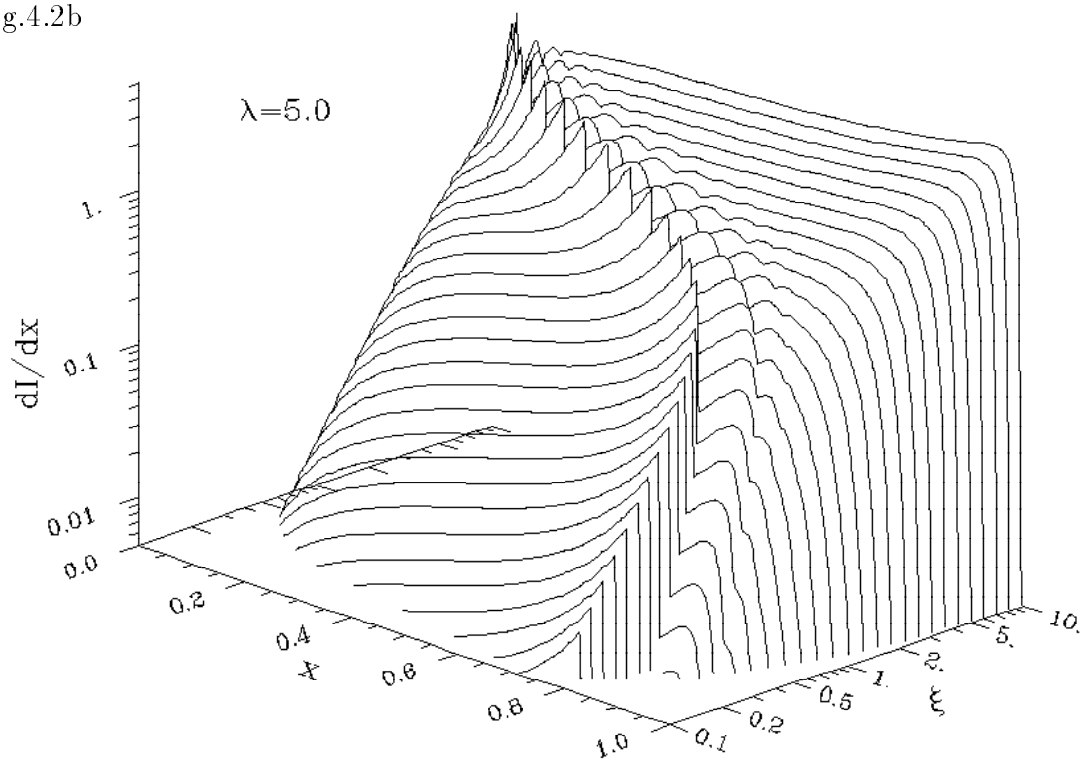

Fig.4.2 3D plot of the radiation power spectrum as a function of the photon energy ω ($x = \omega/\varepsilon$) and intensity parameter ξ for fixed $\lambda = 1$ and 5. The shift of the one-photon peak due to increase of ξ (increase of effective mass) is seen as well as increase of contribution of multi-photon processes. For $\xi > 1$ the contribution of one-photon process becomes relatively small and washing out with further ξ increase.

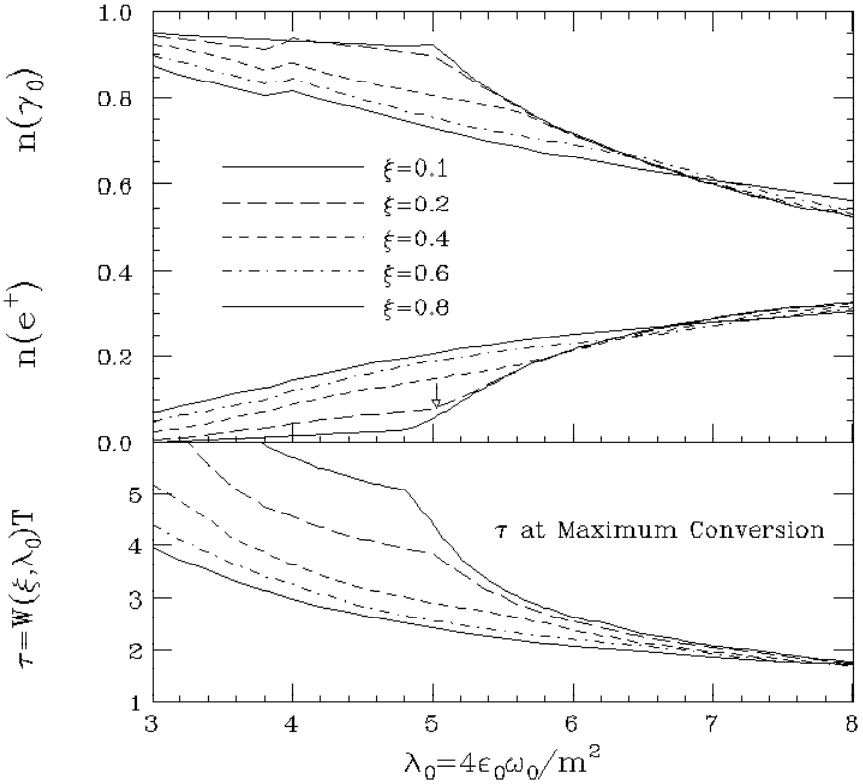

Fig.4.3 The number of photons and positrons in the cascade process of electron and laser beams when the yield of the primary photons takes maximum. The graph at the bottom shows when the yield takes maximum.

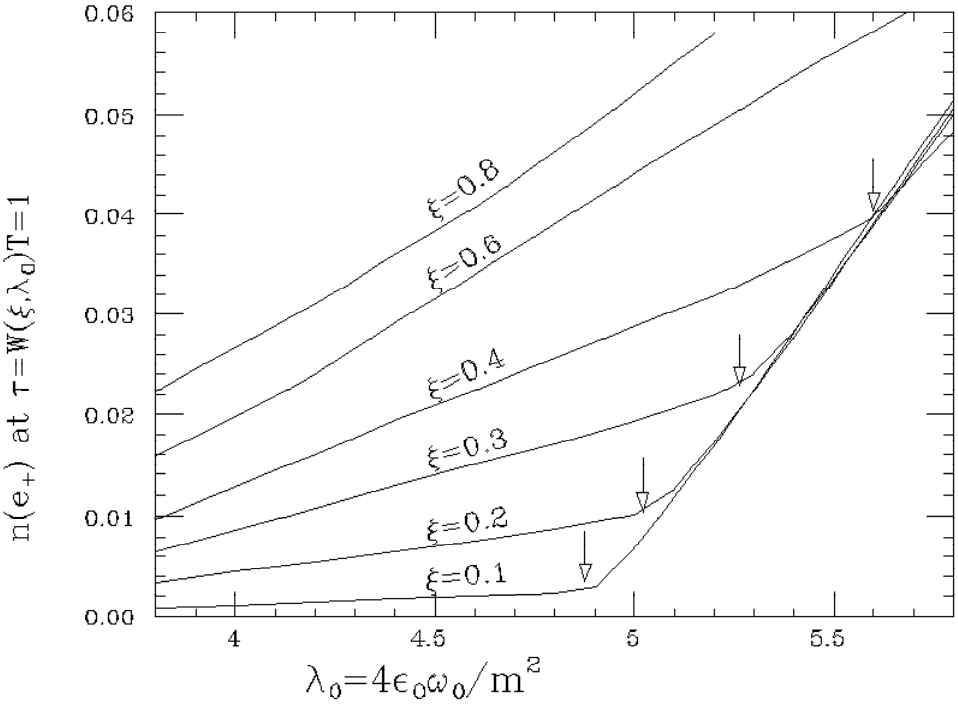

Fig.4.4 The number of positrons from the cascade process when τ=1.

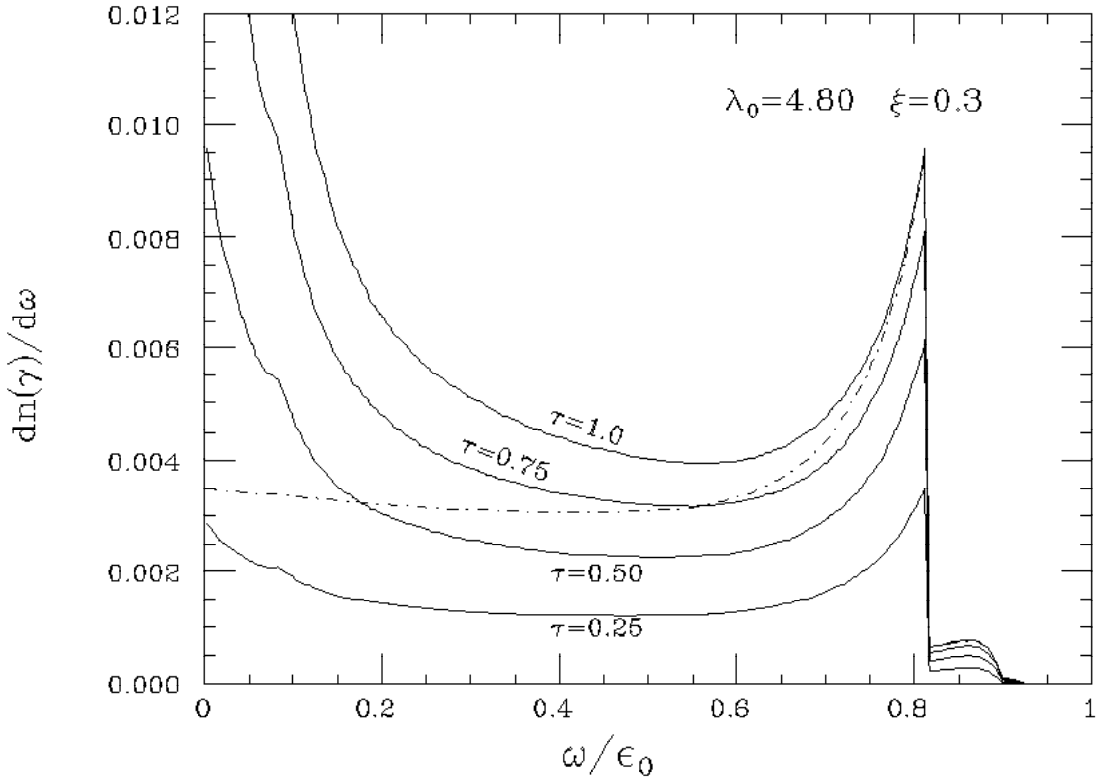

Fig.4.5 The photon spectrum at the time $\tau = 0.25, 0.50, 0.75$ and 1.0 for $\xi = 0.3$ and $\lambda_0 = 4.80$. The dashed line is the spectrum of the primary photons at $\tau = 1.0$.