Ground Motion

Power Spectrum P(f)   [m^2/Hz]

$f < 0.1$ Hz   diffusive   "ATL"

$P(f) = k / f^2$, $k = 1 \sim 100\ \mathrm{nm}^2\ \mathrm{Hz}$

$f > 0.1$ Hz   elastic   "waves"

$P(f) \propto 1 / f^4$

ocean swell   0.2Hz
cultural noises   1~100Hz
crustal resonances   3Hz
μ earthquake

Example of ATL

$\sigma^2 = \langle \Delta y \rangle^2 = A\ T\ L$

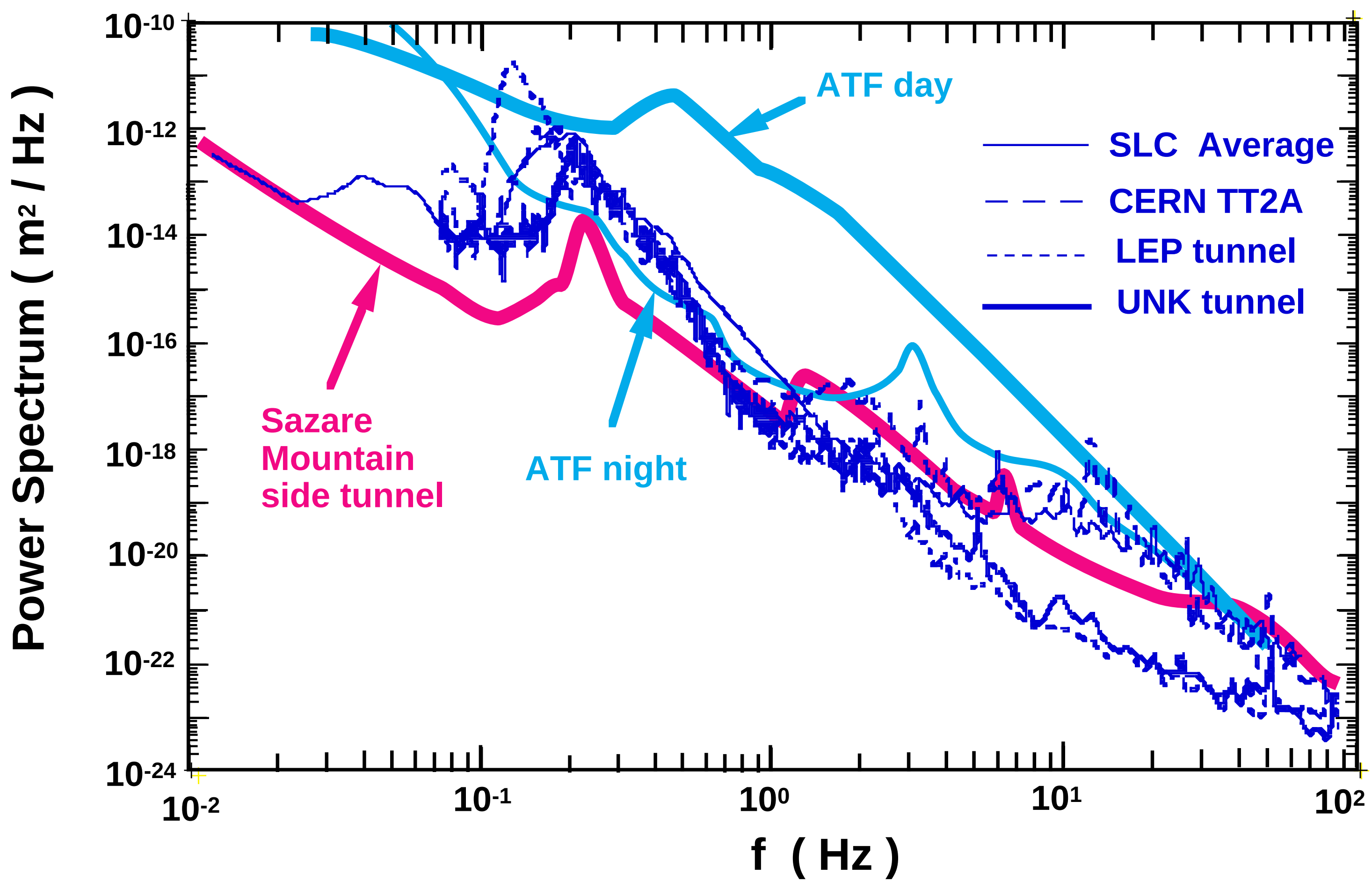
$T = \langle \Delta y \rangle^2 / A\ L$

for QC1,

L is a distance betwen 2 QC1's, L= 4~8.4m,

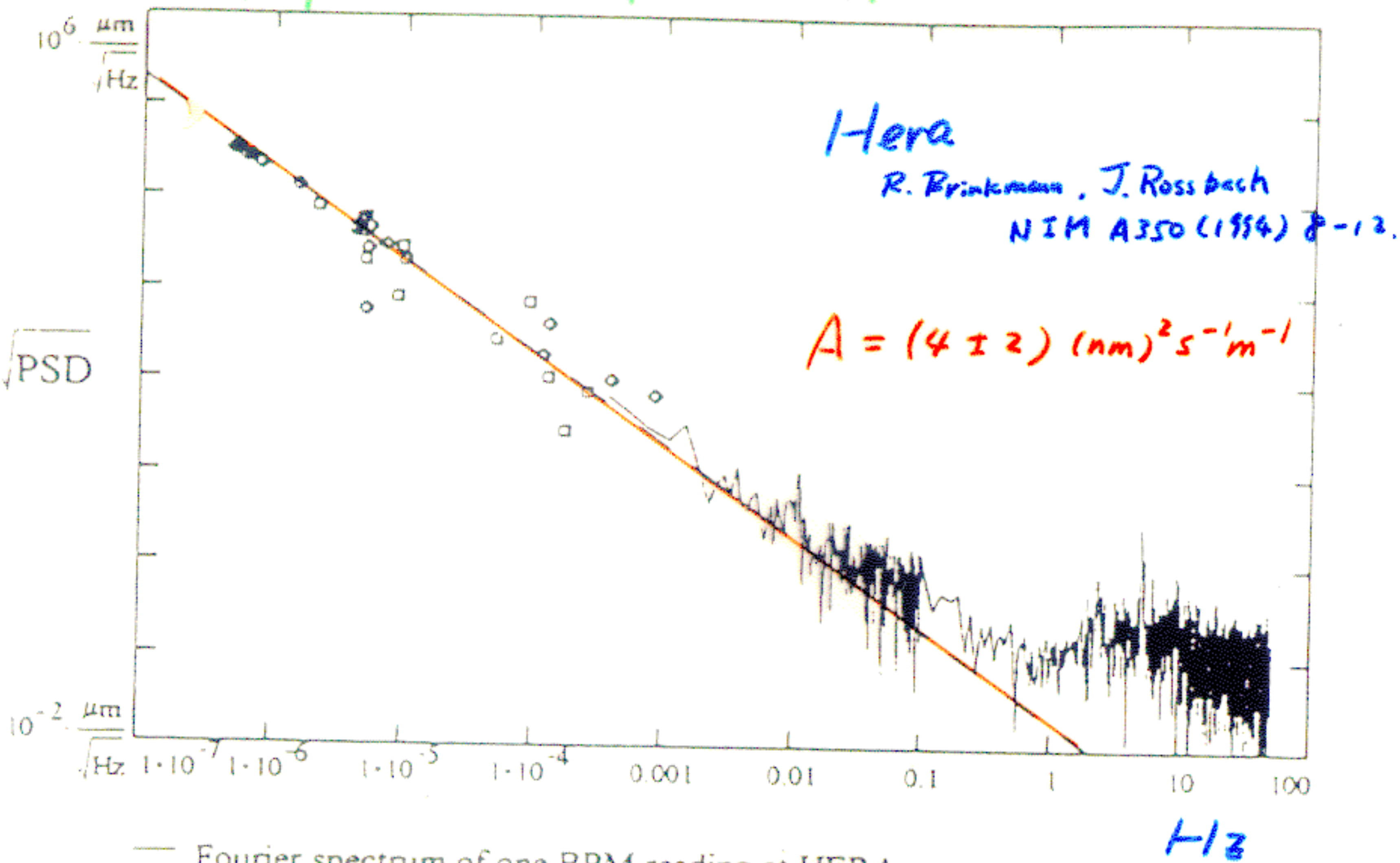
$\langle \Delta y \rangle^2 = 1\ \mathrm{nm}^2$, and $A = 40\ \mathrm{nm}^2$/m/sec at KEK,

T= 6.25 ~ 2.98msec



Power Spectrum (m2 / Hz)
f (Hz)
ATF day
ATF night
Sazare
Mountain
side tunnel
SLC Average
CERN TT2A
LEP tunnel
UNK tunnel

Experimental verification of ATL



— Fourier spectrum of one BPM reading at HERAe

○ HERA rms electron orbit motion after certain time intervals

□ HERA rms proton orbit motion after certain time intervals

— spectrum density scaling as expected by ATL rule

$L^{e}_{FODO} = 23.5\ m$

$L^{p}_{FODO} = 47\ m$

$L \equiv L_{FODO}$

1-D power spectrum

$$\alpha^2 = \langle x^2 \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt \equiv \langle x(t)^2 \rangle$$

$$P(f) \equiv \lim_{T\to\infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} x(t)\, e^{-i\omega t} dt \right|^2$$

$$f = \frac{\omega}{2\pi}$$

$$\alpha^2 = \int_{-\infty}^{\infty} P(f)\, df$$

$\because)$

$$P(f) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, e^{-i\omega t} dt \int_{-T/2}^{T/2} x(t')\, e^{i\omega t'} dt'$$

$$= \lim_{T\to\infty} \frac{1}{T} \iint x(t) x(t')\, e^{-i\omega(t-t')}\, dt\, dt'$$

$\tau = t - t'$ $\qquad$ $t = \tau + t'$

$d\tau = dt$

$$= \lim_{T\to\infty} \frac{1}{T} \iint x(t'+\tau) x(t')\, e^{-i\omega\tau}\, dt'\, d\tau$$

$$= \int \langle x(t'+\tau) x(t') \rangle\, e^{-i\omega\tau} d\tau$$

$$\langle x(t+\tau) x(t) \rangle = \int P(f)\, e^{i\omega\tau}\, df$$

if $\tau = 0$,

$$\alpha^2 = \langle x(t)^2 \rangle = \int P(f)\, df$$

relative motion at different times

$$\Delta x^2 = \langle (x(t) - x(t'))^2 \rangle = \langle x(t)^2 - 2x(t)x(t') + x(t')^2 \rangle$$

$$\Delta x^2 = 2\left(\langle x(t)^2 \rangle - \langle x(t)x(t') \rangle\right)$$

$$= 2\left(\int P(f)\,df - \int P(f)\,e^{i\omega\tau}\,df\right)$$

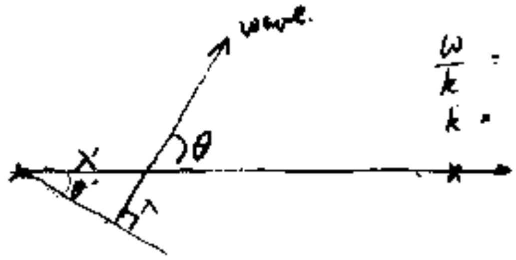
$$= 2\int P(f)\,(1 - e^{i\omega\tau})\,df$$

$$\Delta x^2 = 2\int P(f)\,(1 - \cos\omega\tau)\,df$$

If a transverse wave between 2 points,

$$\tau = \frac{L}{v}$$

$$\omega\tau = \frac{L}{v}\omega = \frac{2\pi L f}{v} = \frac{2\pi L}{\lambda} = kL$$



$$\frac{\omega}{k} = \frac{L}{\tau} = v,$$
$$k = \frac{\omega}{v}$$

$$\cos\omega\tau = \cos kL$$

$$\cos k'L = \cos(kL\sin\theta')$$

$$\theta' = 0 \sim \frac{\pi}{2}$$

$$\lambda' \sin\theta' = \lambda$$

$$\lambda' = \frac{\lambda}{\sin\theta'}$$

$$k' = \frac{2\pi}{\lambda'} = k\sin\theta'$$

$$\left(\begin{array}{l} \theta = \frac{\pi}{2} - \theta' \\ \theta' = \theta + \frac{\pi}{2} \\ \sin\theta' = \cos\theta \\ \theta' = 0 \sim \frac{\pi}{2} \\ \theta = \frac{\pi}{2} \sim 0 \end{array}\right)$$

note: $\int_0^{\frac{\pi}{2}} \cos(\alpha\sin x)\,dx = \frac{\pi}{2} J_0(\alpha)$

$$\Delta x^2 = 2\int P(\omega)\,(1 - J_0(kL))\,\frac{d\omega}{2\pi}$$

$$\left\langle \int_0^{\frac{\pi}{2}} \cos(kL\sin\theta)\,d\theta \right\rangle = \underline{\frac{\pi}{2} J_0(kL)} = J_0(kL)$$

$k : 0 \sim k_0$

$$k = k_0 \sin\theta$$

$$dk = k_0 \cos\theta \, d\theta$$

$$d\theta = \frac{dk}{k_0 \cos\theta} \qquad \sin\theta = \frac{k}{k_0}$$

$$d\theta = \frac{dk}{\sqrt{k_0^2 - k^2}} \qquad \cos\theta = \sqrt{1 - \sin^2\theta} = \frac{\sqrt{k_0^2 - k^2}}{k_0}$$

$$J_0(k_0 L) = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos(k_0 L \sin\theta)\, d\theta = \int_0^{k_0} \frac{4 \cos(kL)\, dk}{\sqrt{k_0^2 - k^2}\; 2\pi} = \int_{-k_0}^{k_0} \frac{2\cos(kL)}{\sqrt{k_0^2 - k^2}} \frac{dk}{2\pi}$$

$$k_0 = \frac{\omega}{V(1)}$$

relative motion between 2 points

$$P(\omega,L) = \lim_{T\to\infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} \left[x_1(t) - x_2(t)\right] e^{-i\omega t}\,dt \right|^2$$

$$= \lim_{T\to\infty} \frac{1}{T} \iint_{-T/2}^{T/2} \left[x_1(t) - x_2(t)\right]\left[x_1^*(t') - x_2^*(t')\right] e^{-i\omega(t-t')}\,dt\,dt'$$

$$[\ \ ][\ \ ] = x_1(t)x_1^*(t') + x_2(t)x_2^*(t') - x_1(t)x_2^*(t') - x_2(t)x_1^*(t')$$

$$t = t' + \tau$$

$$= \int_{-\infty}^{\infty} \Big(\langle x_1(t'+\tau)x_1(t')\rangle + \langle x_2(t'+\tau)x_2(t')\rangle - \langle x_1(t'+\tau)x_2(t')\rangle - \langle x_2(t'+\tau)x_1(t')\rangle \Big)$$

$$\times e^{-i\omega\tau}\,d\tau$$

$$\langle (x_1 - x_2)^2 \rangle = \int_{-\infty}^{\infty} P(\omega,L)\,\frac{d\omega}{2\pi} \qquad \because \tau = 0$$

$$x_1 = x(t, s+L)$$
$$x_2 = x(t, s)$$

$$P(\omega,L) = P_1(\omega) + P_2(\omega) - P_{12}(\omega,L) - P_{21}(\omega,L)$$
$$= 2P(\omega)\left\{1 - \underbrace{R_e\left[N_{12}(\omega,L)\right]}_{\text{correlation or coherence}}\right\}$$

note: $$P_{12}(f) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x_1(t) e^{-i\omega t}\,dt \int_{-T/2}^{T/2} x_2^*(t') e^{i\omega t'}\,dt'$$

$$= \int_{-\infty}^{\infty} \langle x_1(t'+\tau)\,x_2(t')\rangle e^{-i\omega\tau}\,d\tau$$

$$P_{12}(f) = P_{21}^*(f)$$

2-D Power spectrum

generalizing. 2-D power spectrum.

$$P(\omega, k) = \lim_{T\to\infty} \lim_{L\to\infty} \frac{1}{T}\frac{1}{L} \left| \int_{-T/2}^{T/2} \int_{-L/2}^{L/2} x(t,s)\, e^{-i\omega t} e^{-iks}\, dt\, ds \right|^2$$

$$x(t,s)\, e^{-i\omega t} e^{-iks} \quad x(t',s')\; e^{i\omega t'} e^{i\omega s'}$$

$$x(t',s')\, x(t'+\tau,\, s'+L) \quad e^{-i\omega\tau} e^{-ikL}$$

$$P(\omega,k) = \iint_{-\infty}^{\infty} \langle\, x(t,s)\, x(t+\tau,\, s+L) \,\rangle\; e^{-i\omega\tau} e^{-ikL}\, \frac{d\omega}{2\pi}\frac{dk}{2\pi}$$

$$a^2 \equiv \langle\, x(t,s)^2 \,\rangle$$

$$a^2 = \iint_{-\infty}^{\infty} P(\omega,k)\, \frac{d\omega\, dk}{(2\pi)^2} \qquad \because\ \tau = L = 0$$

$(G-b)^2 - a^2 b^2 - 2ab$ $\quad (x(T,S+L) - x(0,S+L)) - (x(T,S) - x(0,S))$

Relative motion "general" T & L

5

$$\alpha^2(T,L) = \langle [X(T,S+L) - X(T,S)]^2 \rangle$$
$$= \langle [x(T,S+L) - x(0,S+L) - x(T,S) + x(0,S)]^2 \rangle$$

$$[\quad] = x(T,S+L)^2 + x(0,S+L)^2 + x(T,S)^2 + x(0,S)^2$$
$$- 2x(T,S+L)\,x(0,S+L) - 2x(T,S)\,x(0,S)$$
$$- 2\,(x(T,S+L)\,x(T,S) - x(T,S+L)\,x(0,S) - x(0,S+L)\,x(T,S)$$
$$+ x(0,S+L)\,x(0,S))$$

$$= \iint \left(4 - 4e^{+i\omega T} - 4e^{ikL} + 4e^{i\omega T}e^{ikL}\right)$$
$$\times P(\omega,k)\,\frac{d\omega\,dk}{(2\pi)^2}$$

$$(\quad) = 4\,(1 - \cos\omega T - \cos kL + \cos\omega T\cos kL)$$

$$= 4\,(1-\cos\omega T)(1-\cos kL)$$

$$\alpha^2(T,L) = \iint_{-\infty}^{\infty} P(\omega,k)\,2(1-\cos\omega T)\,2(1-\cos kL)\,\frac{d\omega\,dk}{(2\pi)^2}$$

(NLC)

$$P(\omega,L) = \int_{-\infty}^{\infty} P(\omega,k)\,2\,(1-\cos kL)\,\frac{dk}{2\pi} \equiv 2P(\omega)\,R(\omega,L)$$

$$\Rightarrow \quad = \begin{cases} AL/\omega^2 & \text{if } 0<\omega<\omega_0 \\ B/\omega^4 & \text{if } \omega_0<\omega<\infty \end{cases} \qquad = 2P(\omega)\int \mu(\omega,k)(1-\cos kL)\times\frac{dk}{2\pi}$$

$$\omega_0 \equiv (B/AL)^{1/2}$$

Empirically

"uncorrelated motion"

$$\boxed{P(\omega,k) = \frac{A}{\omega^2 k^2}\left(1-\cos(L_0 k)\right] + D(\omega)\,U(\omega,k)}$$

elastic waves

$$L_0 = \frac{B}{A\omega^2}$$

$$U(\omega,k) = \begin{cases} \dfrac{2}{\sqrt{k_{max}^2 - k^2}} & \text{if } |k| \le k_{max} \\ 0 & \text{if } |k| > k_{max} \end{cases}$$

$$\int U(\omega,k)\,\frac{dk}{2\pi} = 1.$$

$$D(\omega) = \frac{a_i}{1 + \left[d_i(\omega-\omega_i)/\omega_i\right]^4}$$

resonance at ω_i and $1/\omega^4$ at $\omega \gg \omega_i$

$$P(\omega,k) = \int_0^\infty \cos(kL)\,[\underbrace{P(\omega,L=\infty)}_{=2P(\omega)} - P(\omega,L)]\,dL$$

[NLC]

note: $R(\omega,L) = 1 - J_0(k_0(\omega)L) \longrightarrow (k_0L/2)^2$

at $L\to 0$ $\qquad L\to 0$

$\therefore\ \propto L^2$ for waves

$\propto L$ for ATL $\qquad (k = \frac{\omega}{v})$

[NLC]

$$R(\omega,L) = 1 - \text{correlation} = 1 - J_0(2\pi f\Delta z/v) = \int \mu(\omega,k)(1-\cos kL)\frac{dk}{2\pi}$$

v を求める.

$$\Delta L/L_i = \frac{\langle\Delta^2\rangle_{f_0}}{4a_i^2} = \frac{1}{2\pi a_i^2}\int_{f_0}^{\infty} P(f)\int_0^{k_0}\frac{G(k)}{\sqrt{k_0^2-k^2}}\,dk\,df$$

$k_0 = 2\pi f/v(f)$

$\because\ P(\omega,k) \equiv P(\omega)\mu(\omega,k)$

$\mu(\omega,k) = \frac{4}{\sqrt{k_0^2-k^2}}$

correlation の測定 $\longrightarrow$ v_i (wave) の決定.

Tesla: c for waves + ATL

NLC: $v(\omega)$ ... fitted at $5 < f < 12$ Hz.

Relative motion

$$\alpha^2(L) = \langle (x_1 - x_2)^2 \rangle = \int_{-\infty}^{\infty} \rho(\omega, L) \, \frac{d\omega}{2\pi}$$

$$\alpha^2(T,L) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(\omega, k) \, 2(1-\cos\omega T) \, 2(1-\cos kL) \frac{d\omega dk}{(2\pi)^2}$$

$$= \int_{-\infty}^{\infty} \rho(\omega, L) \, 2(1-\cos\omega T) \frac{d\omega}{2\pi}$$

$$\rho(\omega, L) = P(\omega) \, 2 \left\{ 1 - \underbrace{Re[N_{12}(\omega, L)]}_{\text{correlation}} \right\}$$

$$\alpha^2(T, L) = ATL \quad \text{for} \quad \rho(\omega, L) = \frac{AL}{\omega^2}.$$

Relation to NLC-model,

$$\rho(\omega, L) = 2P(\omega) R(\omega, L)$$

$$R(\omega, L) = \int_0^{\infty} \mu(\omega, k) (1-\cos kL) \frac{dk}{2\pi}$$

$$= 1 - \underbrace{J_0(\omega L / v(\omega))}$$ waves.

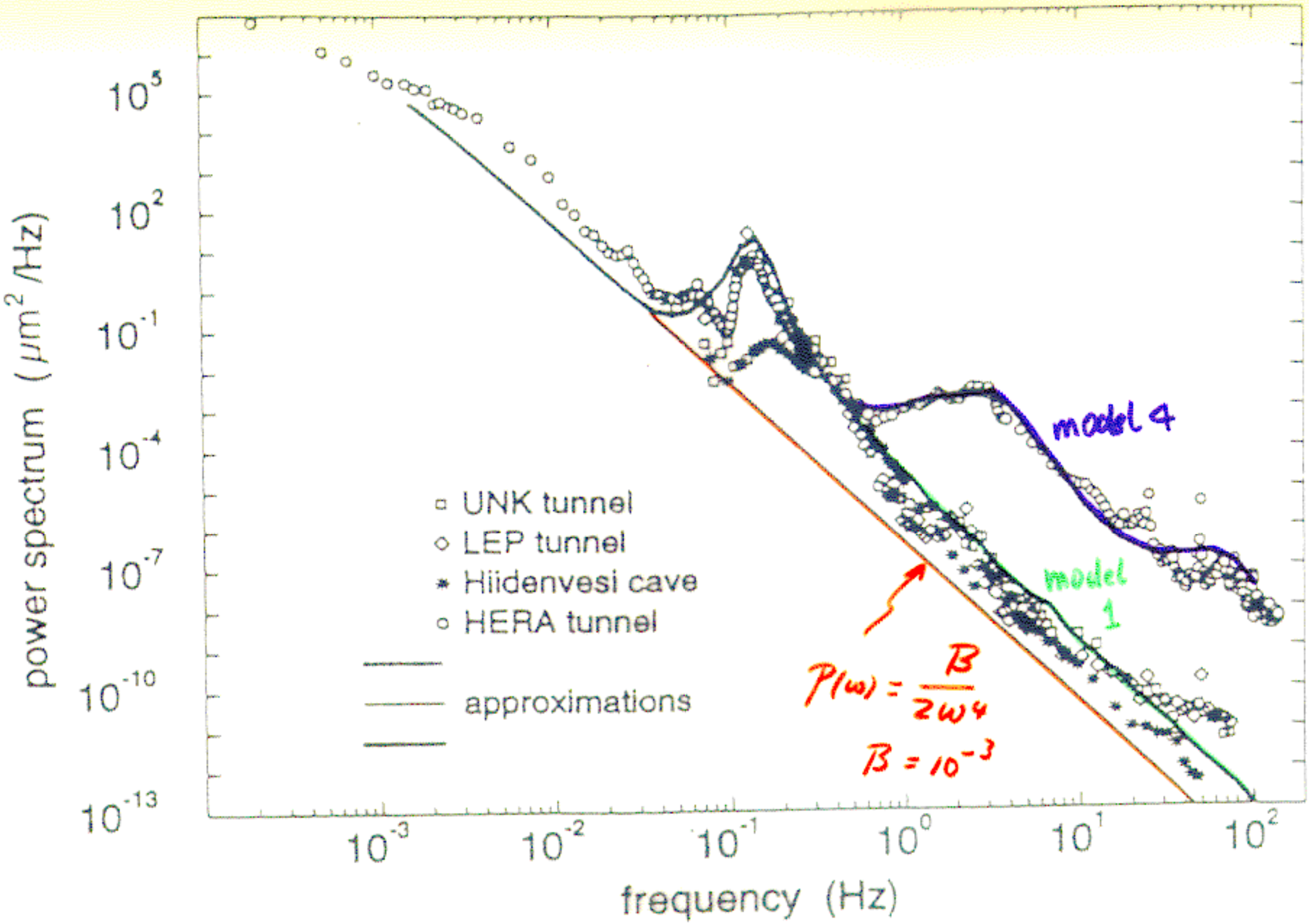
$$\mu(\omega, k) = \frac{4}{\sqrt{k_0^2 - k^2}} \quad \text{for } k < k_0$$

$$= 0 \quad \text{for } k \geq k_0$$

$$k_0 = \frac{\omega}{v(\omega)}$$

Models of ground motion at TESLA study

Model	A $\mu m^2 s^{-1} m^{-1}$	B $\mu m^2 s^{-3}$	a_i $\mu m^2 Hz$	$f_i = \omega_i / 2\pi$ Hz	v_i m/s	d_i
1.very conservative	10^{-4}	10^{-3}	a_1:10	0.14	1000	5
2. conservative	10^{-4}	10^{-6}	a_1:10	0.14	1000	5
3. good tunnel	10^{-6}	10^{-6}	a_1:10	0.14	1000	5
4. HERA	10^{-5}	10^{-3}	a_1:10	0.14	1000	5
(cuiltural noises)			a_2:10^{-3}	2.5	400	1.5
			a_3:10^{-7}	50	400	1.5



power spectrum (μm²/Hz)
10^5
10^2
10^-1
10^-4
10^-7
10^-10
10^-13
10^-3
10^-2
10^-1
10^0
10^1
10^2
frequency (Hz)
UNK tunnel
LEP tunnel
Hiidenvesi cave
HERA tunnel
approximations
model 4
model 1
P(ω) = B/2ω⁴
B = 10⁻³

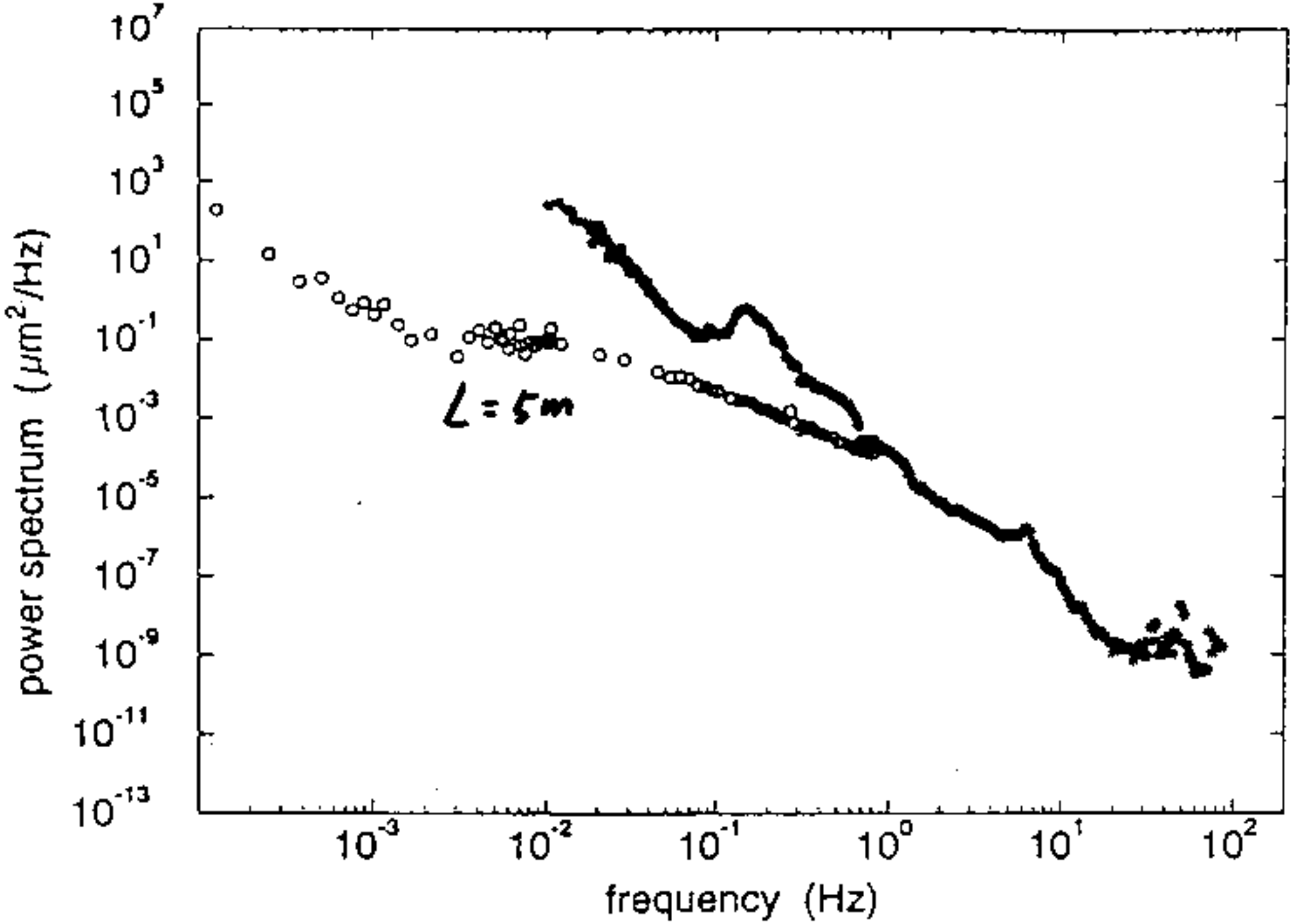


FIG. 4. Comparison of the spectrum of relative ground motion [at a distance of 5 m between probes (circles)] with the spectrum of absolute ground motion (stars), measured by Baklakov *et al.* in the laboratory building in Protvino [14].

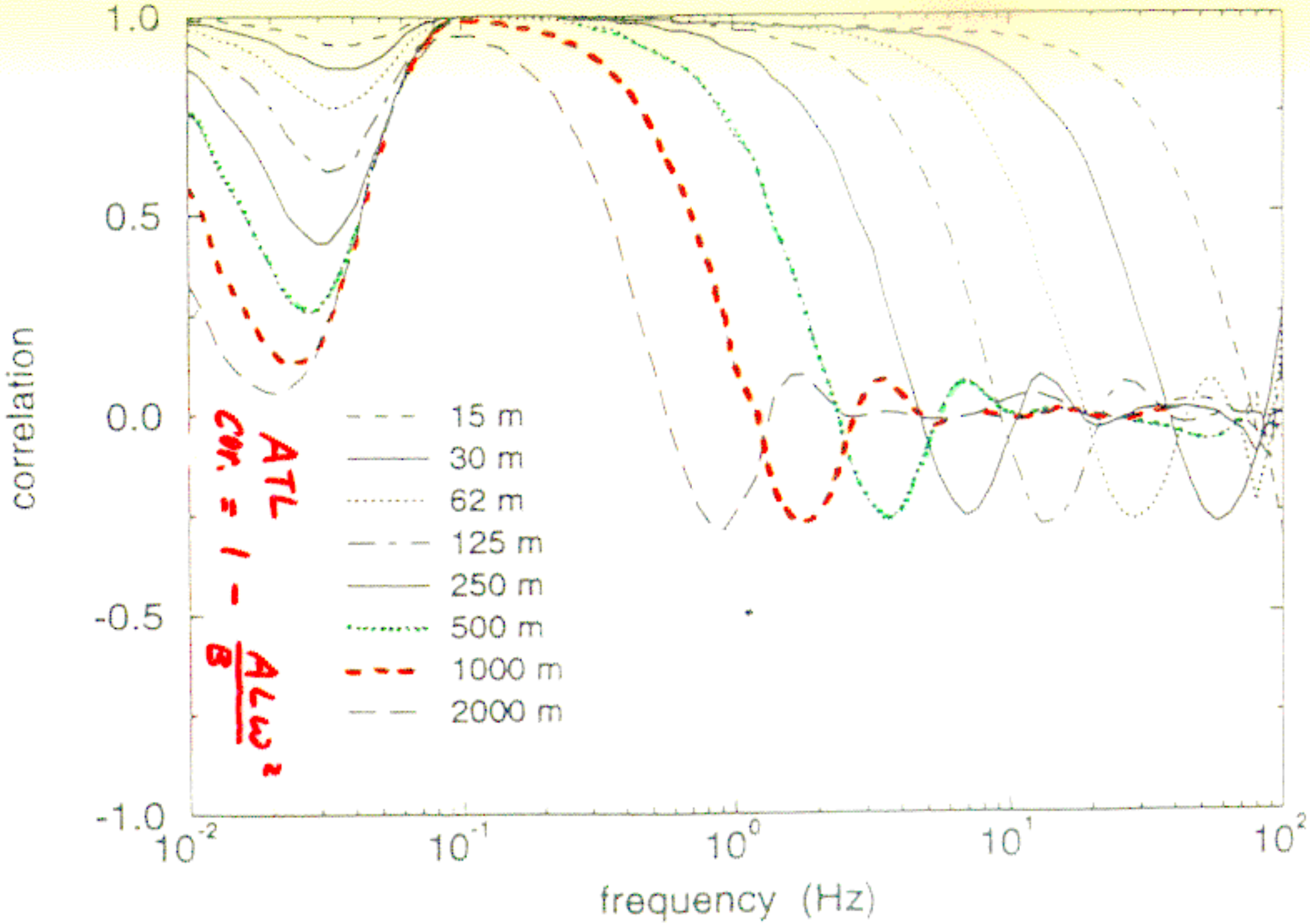


FIG. 6. Correlation spectra $N_{12}(\omega, L)$ calculated with the analytic model 1 of $P(\omega, k)$ for different distances between sensors.

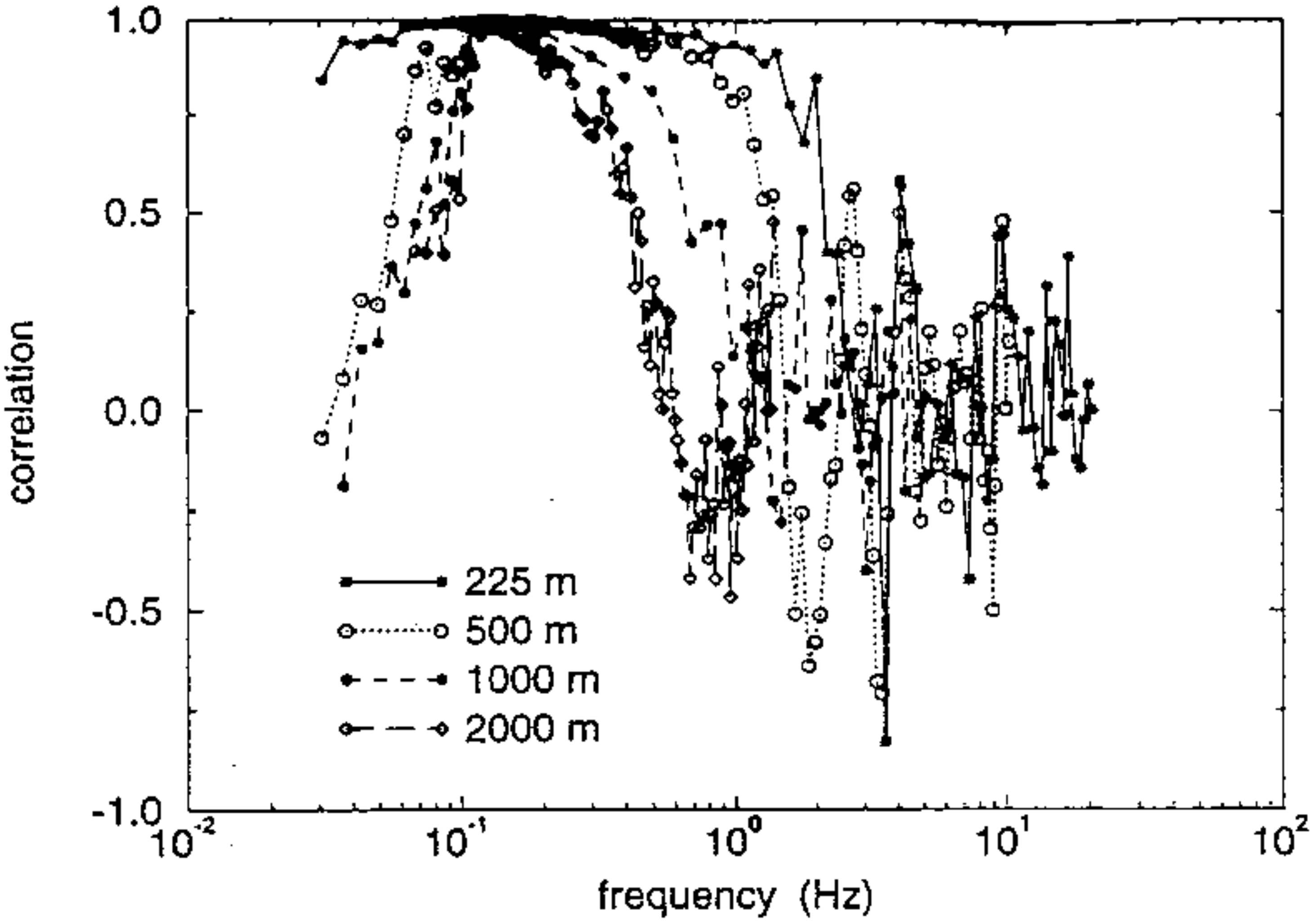


FIG. 3. Correlation spectra of ground motion measured at CERN in the LEP tunnel [7]. The distances between sensors were 225, 500, 1000, and 2000 m.

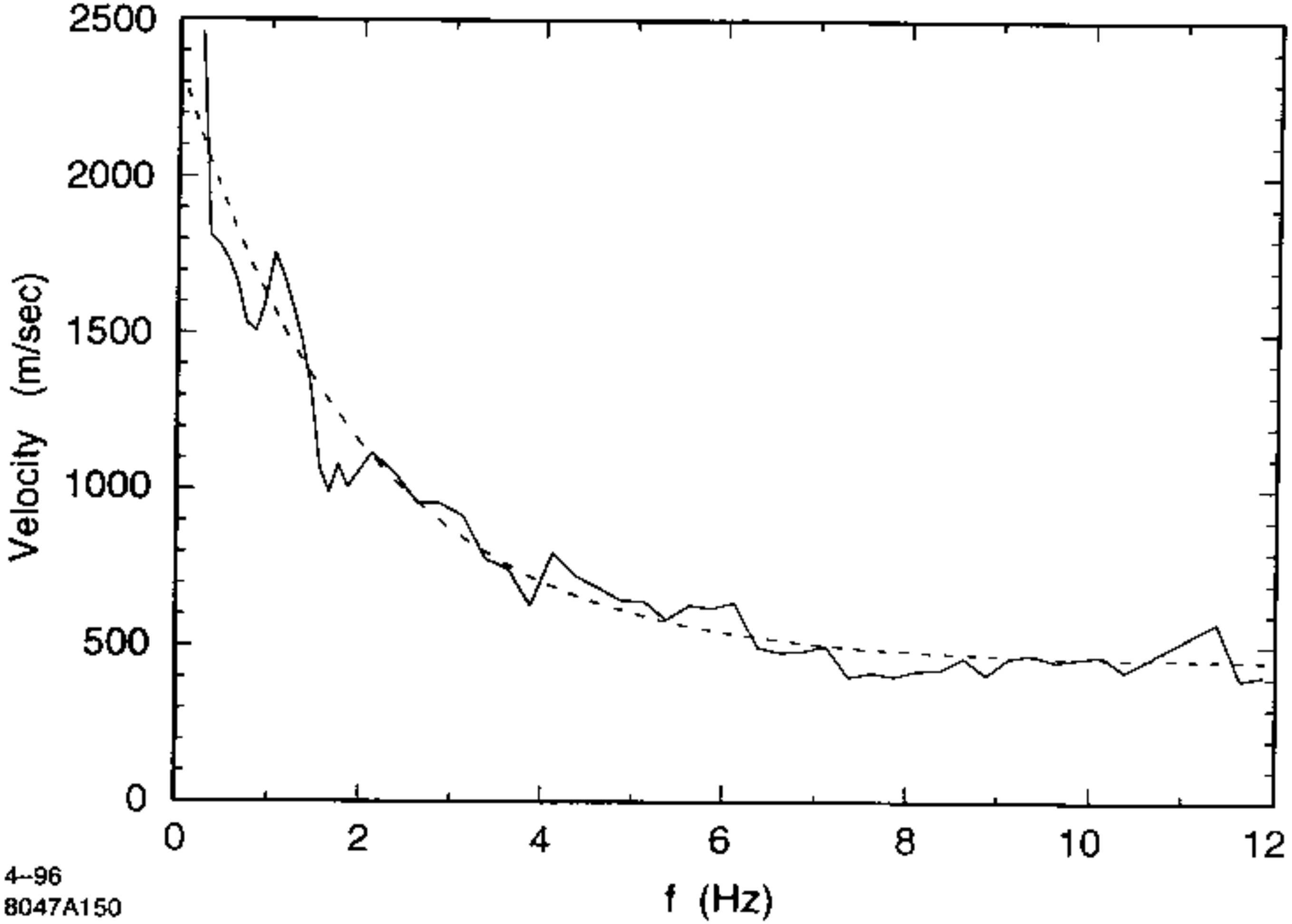


Figure C-18. *Velocity derived from fits to the correlation data (solid line). The dashed line is an empirical fit to the data:* $v = 450 + 1900\exp(-f/2.0)$.

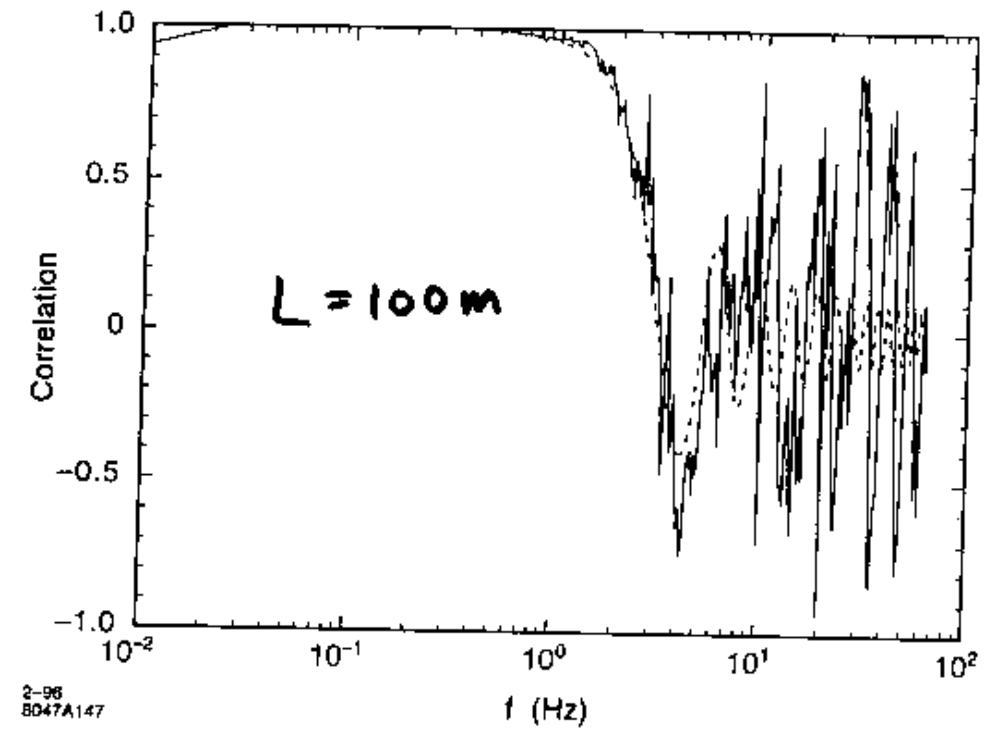


Figure C-15. *Correlation spectrum (solid line) measured with the seismometers separated by 100 m. The dashed line is described in the text.*

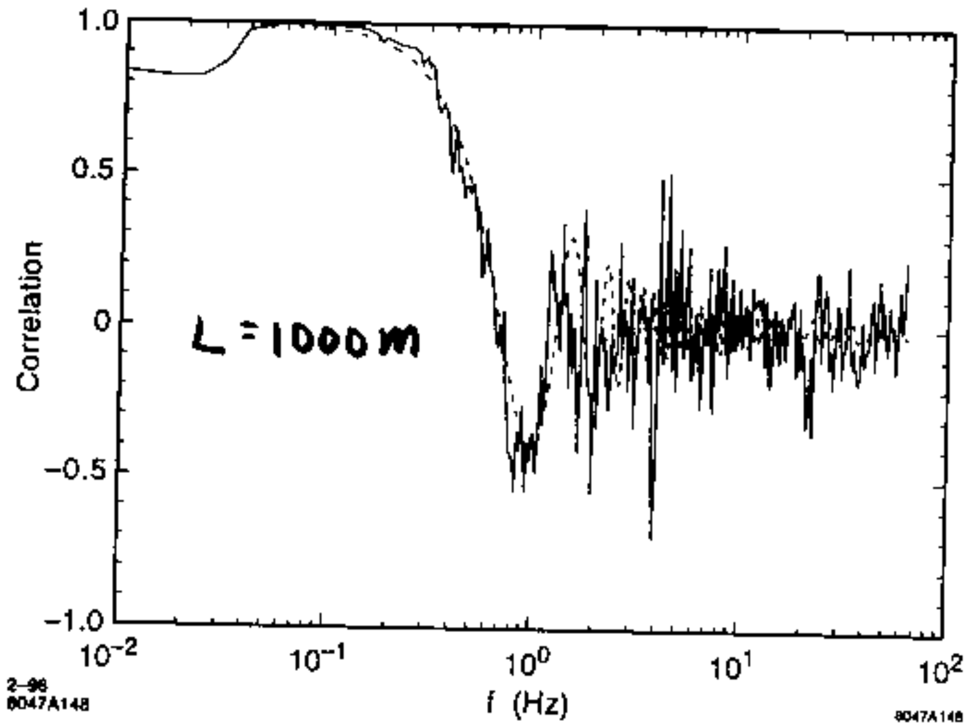
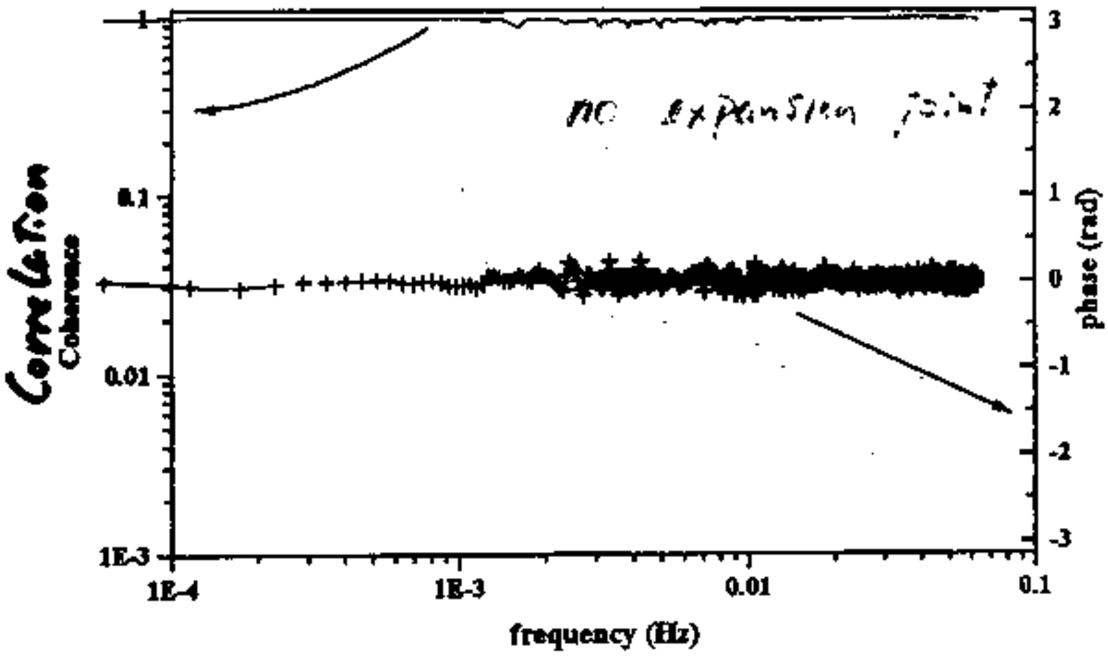


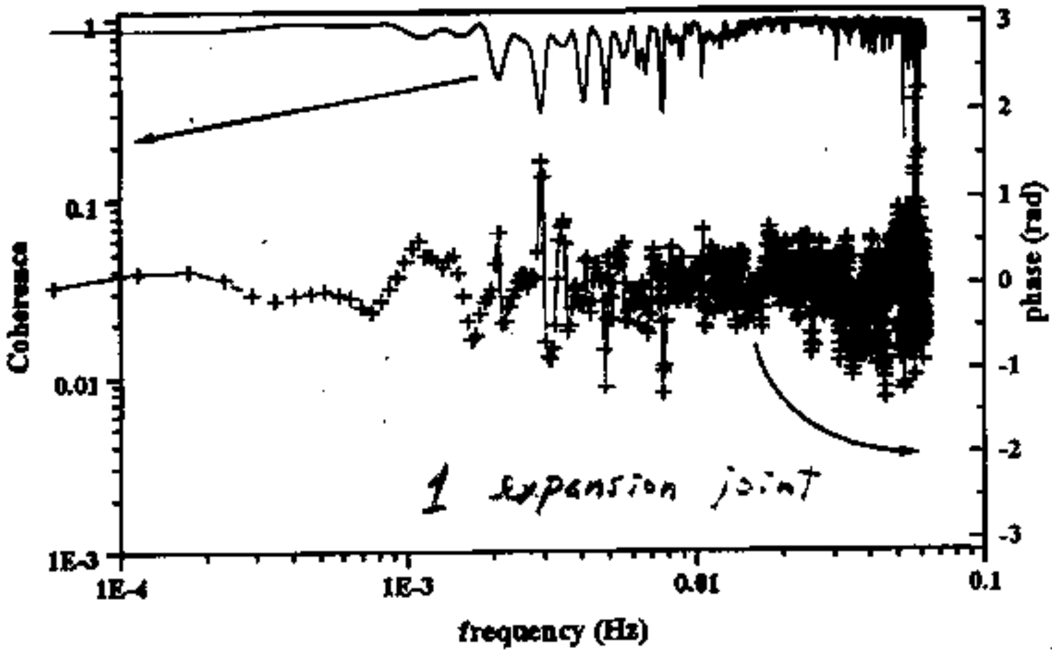
Figure C-16. *Correlation spectrum (solid line) measured with the seismometers separated by 1000 m. The dashed line is described in the text.*

- - - - - : "NLC" model

L = 50cm at KEK - tunnel.



gure 5: Coherence and phase difference between two points being no expansion joint.



igure 6: Coherence and phase difference between two points being one expansion joint.

istance and number of expansion joints. There is no difference between the two power spectra, ut the correlations show obvious differences as a function of numbers of the expansion joint.

ATF- tunnel

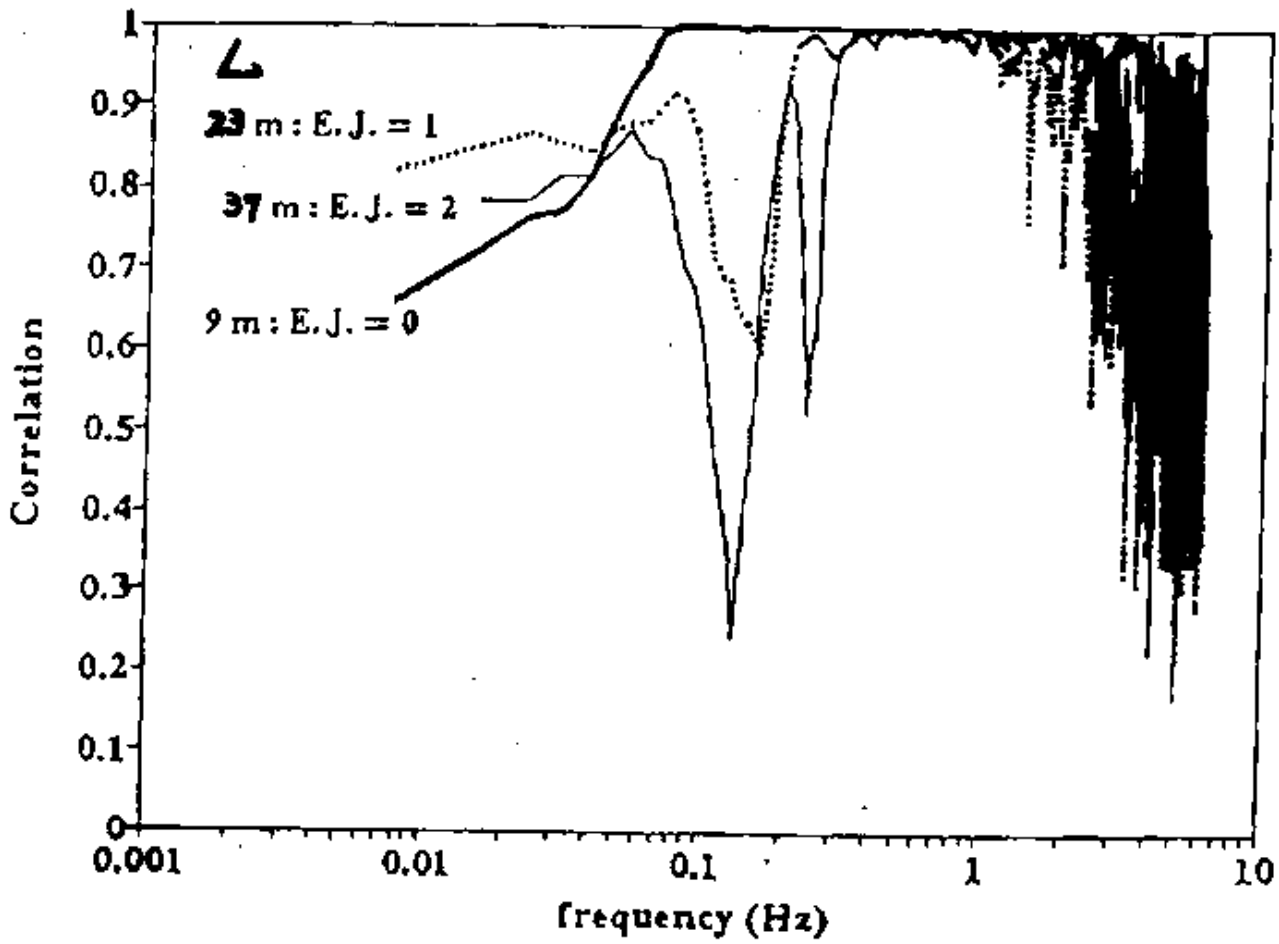


Fig. 2. Correlation between the same Fourier harmonics of the two different signals.

Beam offset at the IP

$x_i^\pm$: transverse position of focusing element i on the $e^\pm$ side at some later time T relative to some reference.

$$\langle [x_i^\pm(T) - x_j^\pm(T)]^2 \rangle = \iint_{-\infty}^{\infty} P(\omega,k)\, 2(1-\cos\omega T)\, 2(1-\cos[k(s_i^\pm - s_j^\pm)])\, \frac{d\omega dk}{(2\pi)^2}$$

$$x^+ - x^- = \sum_{i=0}^{N} a_i (x_i^+ - x_i^-)$$

$$\langle (x^+-x^-)^2 \rangle = \sum_{i=0}^{N}\sum_{j=0}^{N} a_i a_j \langle (x_i^+ - x_i^-)(x_j^+ - x_j^-) \rangle$$

$$= \sum_{i=0}^{N}\sum_{j=0}^{N} a_i a_j \left[\langle (x_i^+ - x_j^-)^2 \rangle - \langle (x_i^+ - x_j^+)^2 \rangle \right]$$

$$\because\ (x_1 - x_2)(x_3 - x_4) = \frac{1}{2}\left[(x_1-x_4)^2 + (x_2-x_3)^2 - (x_1-x_3)^2 - (x_2-x_4)^2\right]$$

$$\langle (x_i^+ - x_j^+)^2 \rangle = \langle (x_i^- - x_j^-)^2 \rangle,\ \langle (x_i^+ - x_j^-)^2 \rangle = \langle (x_i^- - x_j^+)^2 \rangle$$

for a pure ATL motion,

$$\langle (x^+-x^-)^2 \rangle = AT \sum_{i=0}^{N}\sum_{j=0}^{N} a_i a_j \left(|s_i^+ - s_j^-| - |s_i^+ - s_j^+| \right)$$

in general

$$\langle (x^+-x^-)^2 \rangle = \iint_{-\infty}^{\infty} P(\omega,k)\, 2(1-\cos\omega T)\, G(k)\, \frac{d\omega dk}{(2\pi)^2}$$

OPTICS $\times$ T.f.(?) feedback

$$G(k) = \sum_{i=0}^{N}\sum_{j=0}^{N} 2 a_i a_j \left\{ \cos[k(s_i^+ - s_j^-)] - \cos[k(s_i^+ - s_j^+)] \right\}$$

$s=0$ at IP, then $s_i^+ = -s_i^-$

$$G(k) = 4\left(\sum_{i=0}^{N} a_i \sin(k s_i^+) \right)^2$$

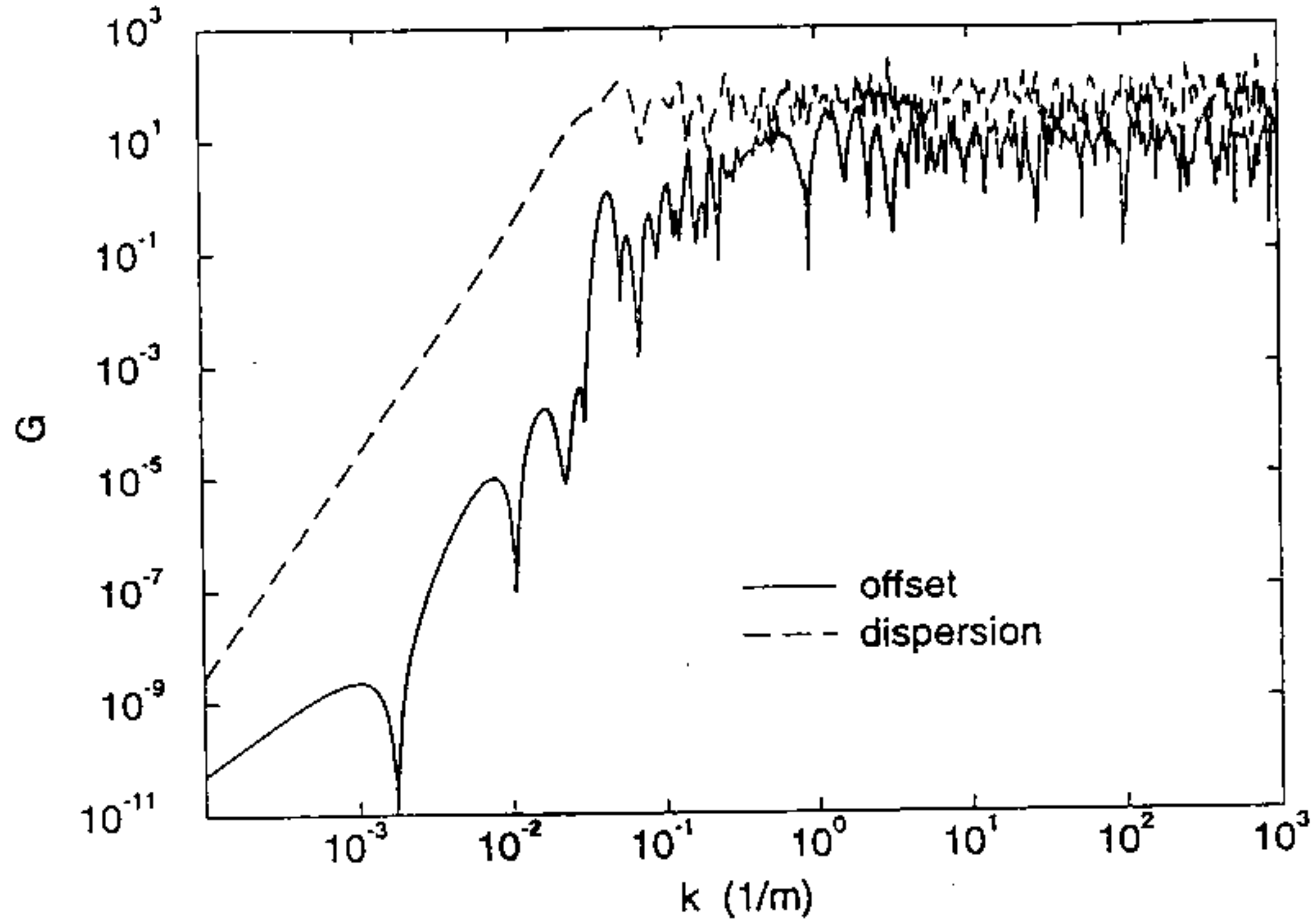


FIG. 10. Spectral response functions of the offset (G) and of dispersion (G_{2}) for the TESLA FFS