Lectures on Brane World Scenarios IV

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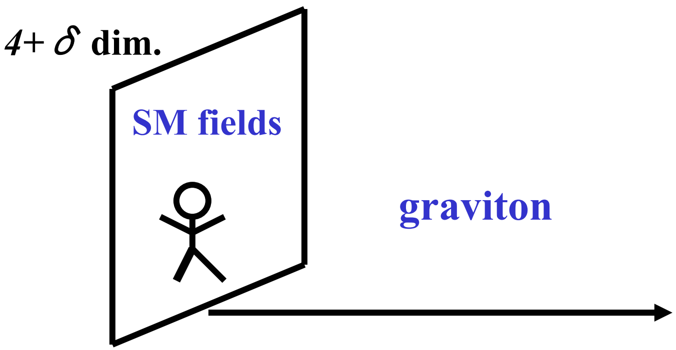
Brane World Scenarios as phenomenological model beyond the SM

Basic picture



δ (compact) extra-dimensions

“3-brane”

i) We are confined on “3-brane”

ii) Graviton lives in the bulk

We discussed two typical Scenarios:

Large (flat) Extra Dimensions (Arkani-Hamed-Dimopoulos-Dvali, ’98)

Warped (small) Extra Dimensions (Randall-Sundrum, ’99)

→ provide an alternative solution to the gauge hierarchy problem

without SUSY, Technicolor etc.

New property → $\boxed{\text{Geometry}}$

Geometrical meaning → why EW scale is so small?

$$M_W \ll M_4 \sim 10^{19}\text{GeV}$$

Large extra-dimension scenario:

$$M_{4+\delta} \sim \mathcal{O}(\text{TeV}) \sim M_W$$

$$M_{4+\delta} = \left(\frac{M_4}{V_\delta}\right)^{\frac{1}{2+\delta}}$$ ← dilution by large extra-dimensional volume

Warped extra-dimension scenario:

$$M_4 \sim M_5 \sim 10\kappa$$ ← Mild hierarchy

$$M_W = M_4 \times e^{-\pi\kappa r_0}$$ ← suppression by ``warp'' factor

Today's topics:

(I) Introduction to extended models & their applications

Especially, in the light of the new property ``GEOMETRY''

There are some interesting proposed models beyond the SM

in the geometrical point of view

→ new solutions or new interpretations

to problems in the SM

SUSY models

etc.

(II) Radius stabilization problem

Introduction to extended models & their applications

Example I: New interpretation of tiny neutrino masses

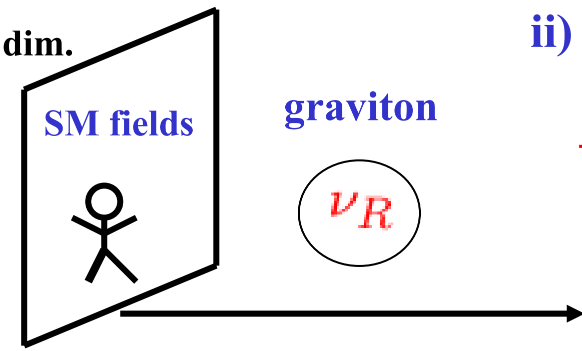
a) **Dilution by large extra-dimensional volume**

Dvali & Smirnov,
Nucl. Phys. B563:63,1999

Slight extension of basic picture

i) We are confined on "3-brane"
ii) Graviton lives in the bulk

\+ ν_R in the bulk

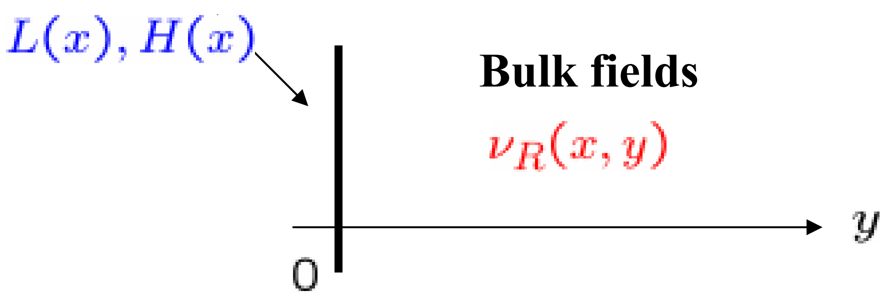


Note that ν_R is a singlet under the SM gauge group

Brane fields

5 dim. model

with orbifolding on S^1/Z_2



$$\nu_R(x,y) = \frac{1}{\sqrt{\pi R}}\nu_R^{(0)}(x) + \sum_{n=1}^{\infty} \frac{2}{\sqrt{\pi R}}\nu_R^{(n)}(x)\cos\left(\frac{ny}{R}\right)$$

$$\mathcal{L}_Y^5 = \frac{\mathcal{O}(1)}{\sqrt{M_5}}\,\bar{L}(x)H(x)\nu_R(x,y)\,\delta(y)$$

$$\rightarrow \mathcal{L}_{eff} = \int_0^{\pi R} dy \mathcal{L}_5 \rightarrow \frac{\mathcal{O}(1)}{\sqrt{M_5 \pi R}}\bar{L}(x)\langle H\rangle \nu_R^{(0)} + \ldots$$

$$\boxed{m_\nu \sim \frac{1}{M_5 \pi R} M_W \ll M_W}$$

since $1/R \ll M_5$

in large extra-dimension model

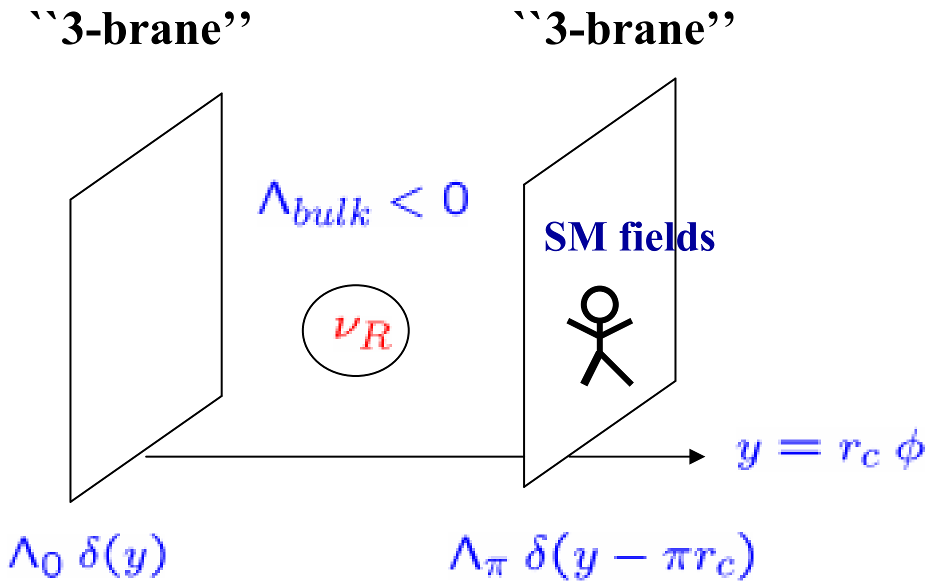
Volume suppression effect

b) Wave function overlapping: bulk ν_R in warped geometry

Extension of Randall-Sundrum model with bulk ν_R

Grossman & Neubert,
Phys. Lett. B474: 361, 2000

slice of AdS$_5$



Background metric:

$$\boxed{ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2}$$

Massive fermion in the bulk

$$S_5 = \int d^4x \int_0^{\pi} d\phi \left[e^{-3\sigma} \left(\frac{i}{2} \bar{\Psi} \gamma^{\mu} \partial_{\mu} \Psi + h.c. \right) - e^{-4\sigma} m \epsilon(\phi) \left(\bar{\Psi} \Psi \right) \right]$$

$$\Psi_{R,L} = \frac{1 \pm \gamma_5}{2} \Psi \qquad \begin{cases} \textbf{Odd :} & \Psi_L(x,\phi) = -\Psi_L(x,-\phi) \\ \textbf{Even:} & \Psi_R(x,\phi) = +\Psi_R(x,-\phi) \end{cases}$$

KK mode decomposition & solving E.O.M.

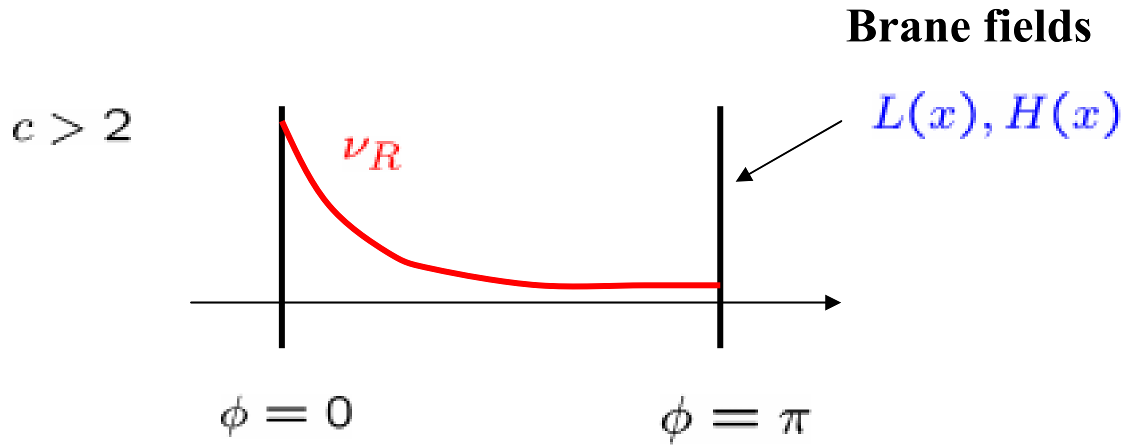
Massless mode: $\Psi_R(x,\phi) \rightarrow \sqrt{\frac{(1+2c)\kappa}{1-\omega^{(1+2c)}}}\, e^{(2-c)\kappa r_c \phi}\, \psi^{(0)}(x)$

$$\begin{cases} \omega = e^{-\kappa r_c \pi} & \textbf{: warp factor} \\ c = \dfrac{m}{\kappa} & \end{cases}$$

If **c > 2** → **localized around** $\phi = 0$ **brane**

c < 2 → **localized around** $\phi = \pi$ **brane**

Identify $\psi^{(0)}(x) \to \nu_R$



$$\mathcal{L}_Y = \sqrt{-g_{vis}}\,\frac{\mathcal{O}(1)}{\sqrt{M_5}}\,\bar{L}(x)H(x)\,\Psi_R(x,\phi=\pi)$$

$$\sqrt{-g_{vis}} = \omega^4$$

Canonical normalization: $L \to L\omega^{-3/2}$, $H \to H\omega^{-1}$

$$\Psi_R(x,\pi) \to \sqrt{\frac{(1+2c)\kappa}{1-\omega^{(1+2c)}}}\,\omega^{c-2}\,\nu_R(x)$$

$$\mathcal{L}_{eff} \sim \langle H \rangle \, \omega^{c-1/2} \sim M_W \, \omega^{c-1/2}$$

Randall-Sundrum model: $\omega \sim \dfrac{M_W}{M_4} \rightarrow m_\nu \sim M_W \times \left(\dfrac{M_W}{M_4}\right)^{c-1/2}$

Result from small overlapping of wave functions

Compare to well-known see-saw mechanism:

$$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \rightarrow m_\nu \sim m_D \left(\frac{m_D}{M_R}\right)$$

Same relation for c = 3/2

c) **Wave function overlapping: domain wall fermions**

Arkani-Hamed & Schmaltz,

Phys. Rev. D61: 033005,2000

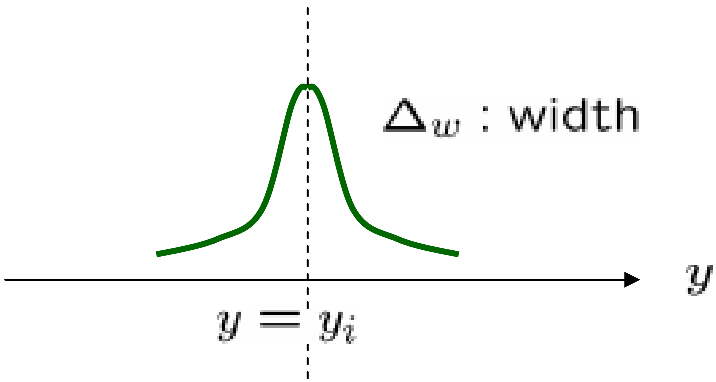
Fermion mass hierarchy problems in the SM

$$\begin{cases} m_u \ll m_c \ll m_t \\ m_d \ll m_s \ll m_b \\ m_e \ll m_\mu \ll m_\tau \end{cases}$$

$$\frac{m_e}{m_t} = \frac{Y_e}{Y_t} \sim 10^{-5}$$

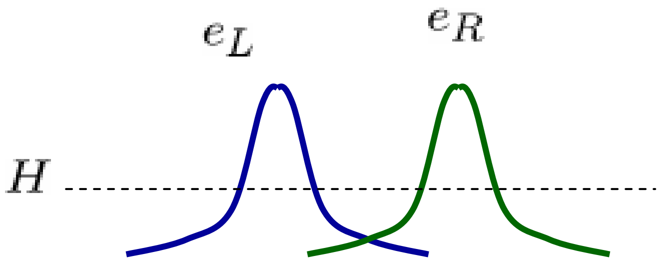
Suppose: SM chiral fermions in the bulk & localized around each points

$$\Psi(x,y)_i \sim e^{-\left(\frac{y-y_i}{\Delta_w}\right)^2} \times \psi(x)$$



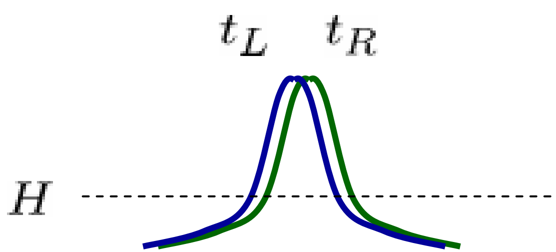
Geometrical interpretation of fermion mass hierarchy

Small electron Yukawa coupling → small overlapping of wave funcs.



$$Y_e \sim e^{-\left(\frac{y_i - y_j}{\Delta w}\right)^2}$$

Large top Yukawa coupling → well overlapping of wave funcs.



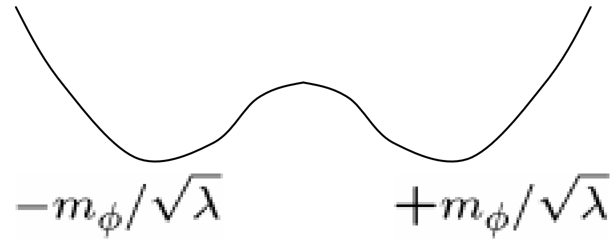
$$Y_t \sim e^{-\left(\frac{\epsilon}{\Delta w}\right)^2} \sim 1$$

Domain wall fermion

Rubakov & Shaposhnikov,

Phys. Lett. 125B, 136

Double well potential

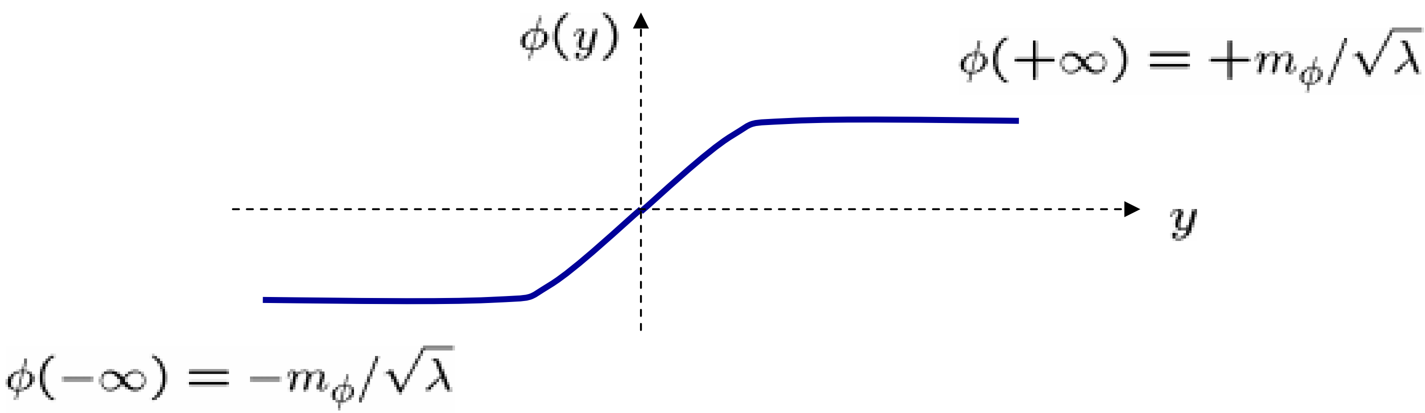


Real scalar in flat 5D:

$$\mathcal{L}_5 = \frac{1}{2}\partial_M\phi\partial^M\phi - \left(\frac{m_\phi^4}{2\lambda} - m_\phi^2\phi^2 + \frac{\lambda}{2}\phi^4\right)$$

Kink solution: non-trivial configuration in the 5th direction

$$\phi_{sol}(y) = \frac{m_\phi}{\sqrt{\lambda}}\tanh(m_\phi y)$$



Bulk fermion with Yukawa coupling to the background kink

Hisano & N. Okada,
PRD 61, 106003, 2000

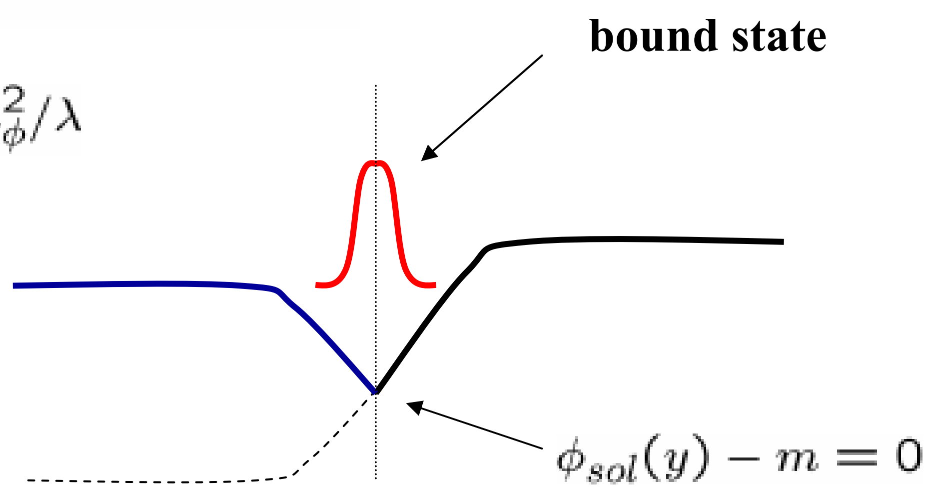
$$\mathcal{L}_5 = \bar{\Psi}(x,y)\left[i\gamma^\mu \partial_\mu - \gamma_5 \partial_y + \phi_{sol}(y) - m\right]\Psi(x,y)$$

$$\Psi(x,y) = U_L(y)\psi_L(x) + U_R(y)\psi_R(x)$$

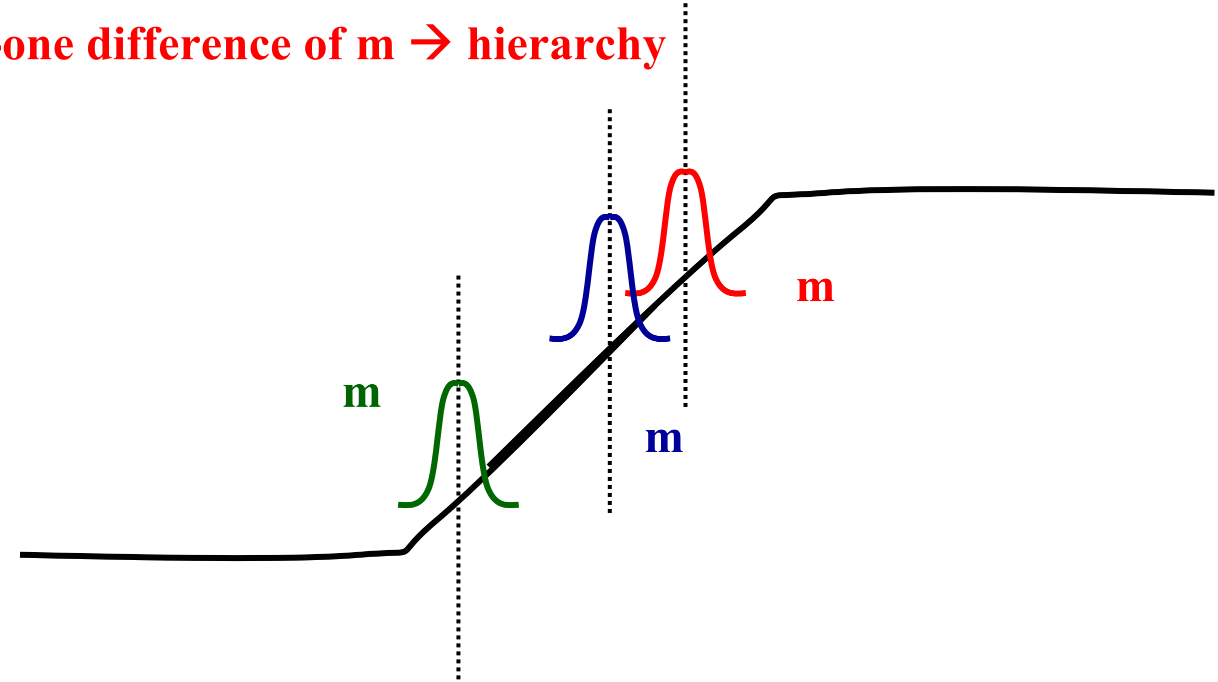
5D Dirac eqs. → Schrodinger equations

$$\frac{d^2}{dy^2}U(y) + (E + V(y))\,U(y) = 0$$

$$\begin{cases} E = m_n^2 - m_\phi^2/\lambda \\ V(y) = \end{cases}$$



Order-one difference of m → hierarchy



Avoid rapid proton decay

$$\mathcal{L}_{int} \sim \frac{1}{M_{4+\delta}} qqq\ell \qquad M_{4+\delta} = \mathcal{O}(1\,\mathrm{TeV}) \quad \rightarrow \textbf{Rapid proton decay}$$

Lepton walls **Quark walls**

Example II: application to SUSY models

Why introduce SUSY?

Extra-dimensional model → originally proposed SUSY alternative

Alternative motivation to consider SUSY brane world

Important issues in 4D SUSY models:

SUSY breaking & its mediation

→ simplest models is SUGRA model

→ generally suffer from SUSY FCNC&CP problem

Soft scalar masses: contact terms between hidden & visible sectors

$$\int d^4\theta \, c_{ij} \, \frac{Z^\dagger Z \, Q_i^\dagger Q_j}{M_4^2} \to c_{ij} m_{3/2}^2 \tilde{Q}_i{}^\dagger \tilde{Q}_j$$

In general, $c_{ij} \sim \mathcal{O}(1)$ is complex, not diagonal

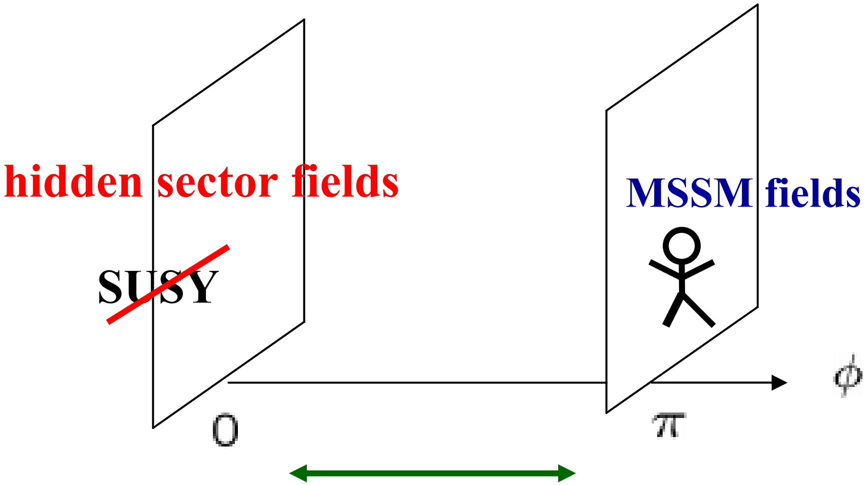
We want to kill the contact term!

<u>Sequestering scenario</u>

Randall & Sundrum,

Nucl.Phys.B557:79-118,1999

5D model compactified on S^1/Z_2



Spatial separation between hidden and visible sectors

$$\rightarrow \begin{cases} c_{ij} = 0 & \text{naive} \\ c_{ij} \propto \dfrac{1}{M_5 R} \ll 1 & \text{Volume suppression} \end{cases}$$

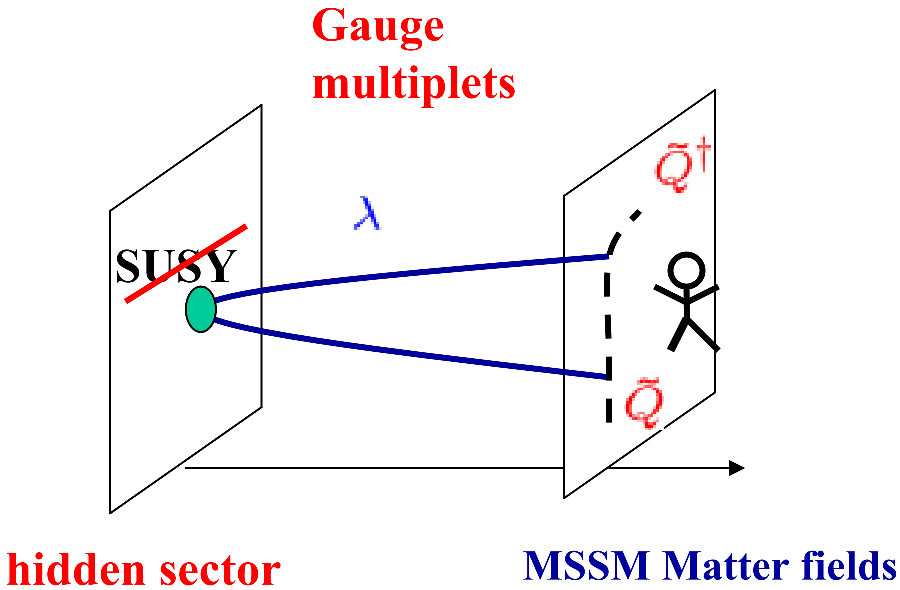
Geometrical interpretation of a special SUSY model

So called no-scale model

Boundary condition: $$M_{1/2} \neq 0$$ $$m_0 = 0$$

In 4D theory, a very special Kahler potential is necessary

5D interpretation: $\left\{ \begin{array}{l} \textbf{MSSM gauge multiplets live in the bulk} \\ \textbf{matter multiplets live on the brane} \end{array} \right.$



Bulk gaugino → directly couples to SUSY breaking
Scalar partners → through gaugino loop

→ $m_0 \ll M_{1/2}$

SUSY breaking → gaugino mass → scalar mass

So called Gaugino mediation

Kaplan, Kribs &Schmaltz,
PRD 62: 035010, 2000
Chacko et al.,
JHEP 0001,003, 2000

(II) Important issue on extra-dimension model

Radius stabilization

Realistic models → extra-dimensional radius should be stabilized

If the radius is destabilized → $V_{4+\delta} \to \infty$

$$M_4 = \sqrt{M_{4+\delta}^{2+\delta} V_\delta} \to \infty$$

If the radius is collapsing → $V_{4+\delta} \to 0$

$$M_4 \to 0$$

In SUSY models, radius stabilization is very difficult problem!

- Manifest SUSY → radius is undetermined
- SUSY is broken → radius destabilization or collapsing occurs

This is not the case in the SUSY Randall-Sundrum model

We can construct models in which

i) Supersymmetric radius stabilization is possible

ii) Destabilization of radius is not the problem

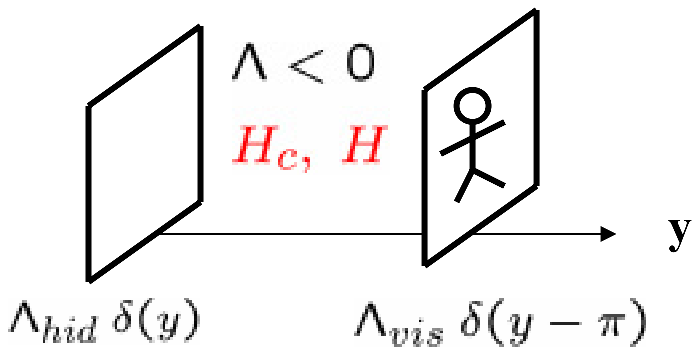
← This is basically because of the warped geometry

(i) Simple radius stabilization model

N. Maru & N. Okada,

hep-th/0312148, to appear in PRD

SUSY Randall-Sundrum model with a bulk hypermultiplet



5-dim. Theory compactified on orbifold s^1/Z_2

(c: bulk mass)

$$\mathcal{L}_H = \int d^4\theta\, r_c \left(H_c^\dagger H_c + H^\dagger H\right)$$
$$+ \left[\int d^2\theta e^{-r_c\kappa|y|} H \left\{\left(-\partial_y + (c+\frac{1}{2}) r_c \kappa \epsilon(y)\right) H_c + e^{-r_c\kappa|y|} W_b\right\} + h.c.\right]$$

Odd : $H_c = \epsilon(y) f(x,|y|)$

Even: $H = g(x,|y|)$

$W_b = J_0\delta(y) - J_\pi\delta(y-\pi)$ ← **Source on each branes**

SUSY vacuum condition

$$\frac{\partial W}{\partial H} = 0$$

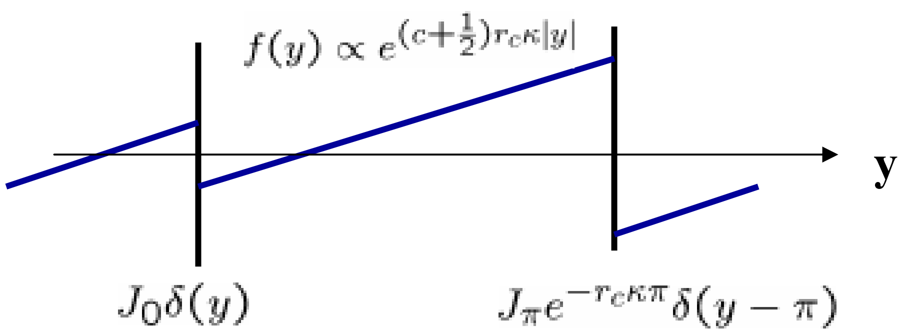
$$\rightarrow \quad \left[-\partial_y + (c+\frac{1}{2})r_c\kappa\epsilon(y)\right] H_c + e^{-r_c\kappa|y|}\left(J_0\delta(y) - J_\pi\delta(y-\pi)\right) = 0$$

$$\rightarrow \left[-\partial_y + (c+\frac{1}{2})r_c\kappa\right] f(y) = 0$$

with boundary conditions $\left\{ \begin{aligned} f(0) &= \frac{J_0}{2} \\ f(\pi) &= \frac{J_\pi}{2}e^{-r_c\kappa\pi} \end{aligned} \right.$

SUSY vacuum condition → $\boxed{J_0 - J_\pi e^{-(c+\frac{3}{2})r_c\kappa\pi} = 0}$

Radius is stabilized with appropriate sources on each branes and bulk mass c



(ii) Dynamical generation of alternative compactification in SUSY Randall-Sundrum model

N. Maru & N. Okada, in progress

Alternative to compactification

Randall-Sundrum, PRL 83 (1999) 4690

In order for models with extra dimensions to be realistic,

compactifications of extra dimensions is necessary

general requirement is finiteness of V_δ

$M_4^2 = M_{4+\delta}^{2+\delta} V\delta$ → **finite** M_4 **and** $M_{4+\delta}$

→ **finite** V_δ

In <u>warped</u> extra dimension scenario

$$V_5^{eff} = 2r_c \int_0^{\pi} d\phi \, e^{-2kr_c|y|} = \frac{1}{k}\left(1 - e^{-2kr_c\pi}\right)$$

$$\rightarrow \quad M_4^2 = \frac{M^3}{k}\left(1 - e^{-2kr_c\pi}\right)$$

Effective volume is finite even if $r_c \rightarrow \infty$

implies → **<u>Alternative compactification scenario</u>**

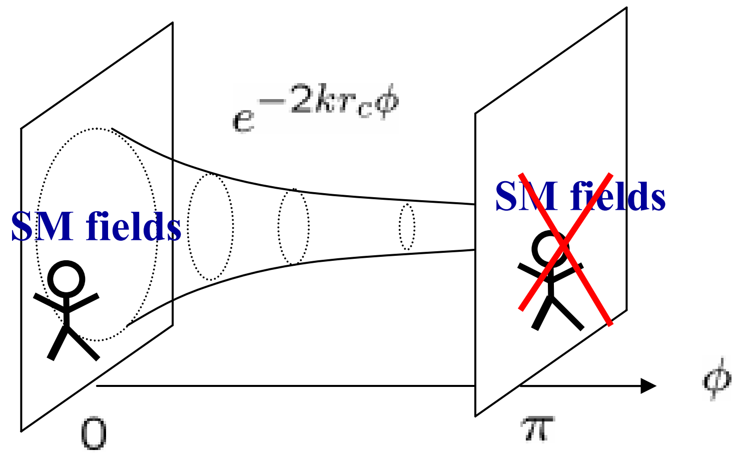
<u>What happen?</u>

$$r_c \rightarrow \infty \quad \begin{cases} m_{KK}^{(1)} \rightarrow 0 \\ m_{KK}^{(n+1)} / m_{KK}^{(n)} \rightarrow 1 \end{cases}$$

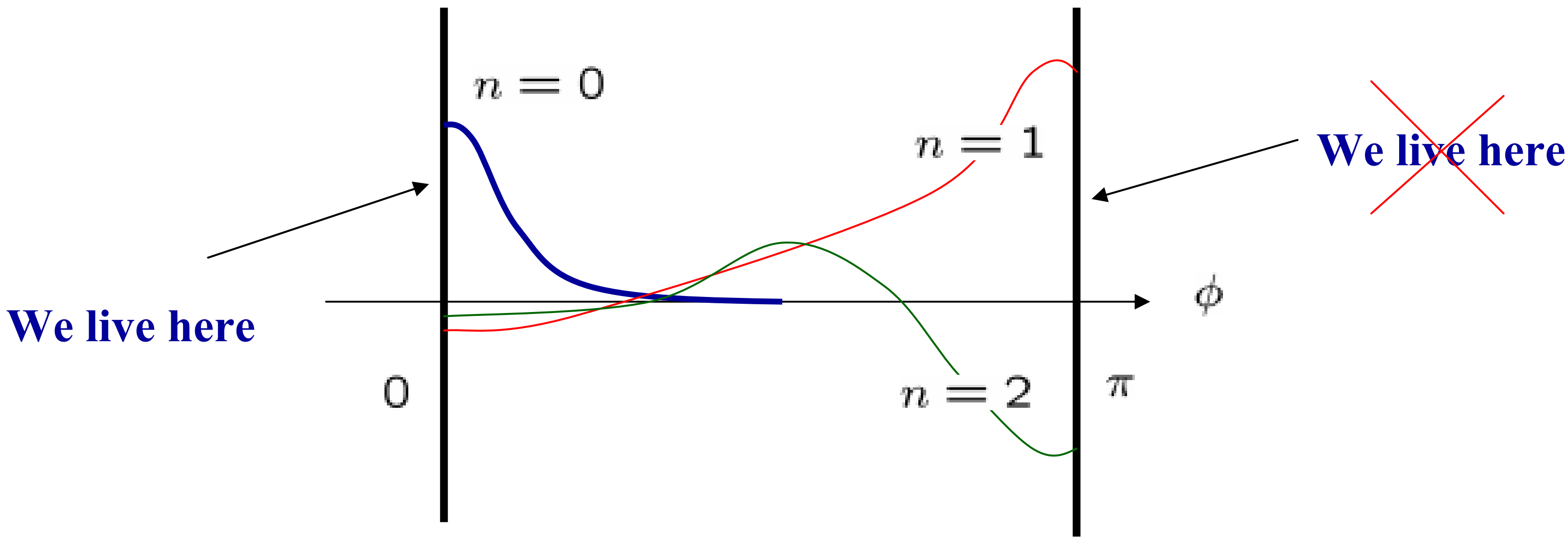
Continuous KK mode spectrum from 0 to infinity

→ Realistic model?

Changing the setup



KK mode configuration



$$\boxed{\begin{aligned}\mathcal{L}_{int} = & -\frac{1}{\bar{M}_4} G^{(0)}_{\mu\nu} T^{\mu\nu} \\ & -\frac{1}{\bar{M}_4 e^{+\kappa r_c \pi}} \sum_{n=1} G^{(n)}_{\mu\nu} T^{\mu\nu}\end{aligned}}$$

← **Strong: large overlap**

← **weak: small overlap**

Newton potential for continuum KK mode

$$\begin{aligned} V(r) & \sim G_N \frac{m_1 m_2}{r} + \int_0^\infty \frac{m dm}{k^2} G_N \frac{m_1 m_2 e^{-mr}}{r} \\ & = G_N \frac{m_1 m_2}{r} \left(1 + \frac{1}{r^2 k^2}\right) \end{aligned}$$

Gravity precision measurement → ***r k >1*** **for** ***r = 0.1 mm***

→ $k > 10^{-4}$ eV

$$M \sim \left(\bar{M}_4^2 k\right)^{\frac{1}{3}} \quad \rightarrow \quad M > 10^{7.7}\ \text{GeV}$$

Problems : i) gauge hierarchy problem

In alternative compactification model, we live on y=0 brane

→ need SUSY

ii) We can take infinite radius, but no mechanism as it should be

We propose a very simple SUSY Randall-Sundrum model

which can realize a dynamical generation of alternative compactification

Model Just put a constant superpotential at $\phi = \pi$

$$\mathcal{L}_5 = \int d^4\theta - 6M_5^3 \frac{T + T^\dagger}{2} e^{-(T+T^\dagger)\kappa\phi} + \left[\int d^2\theta e^{-3T\kappa\phi} W_\pi \delta(\phi - \pi) + h.c.\right]$$

Radion chiral multiplet: $T = r_c + ...$

$$\mathcal{L}_{eff} = \int_0^\pi d\phi\, \mathcal{L}_5$$
$$\rightarrow \int d^4\theta \left[-3\frac{M_5^3}{\kappa}\left(1 - e^{-(T+T^\dagger)\kappa\pi}\right)\right] + \left[\int d^2\theta e^{-3T\kappa\pi} W_\pi + h.c.\right]$$

SUSY vacuum condition $\partial W / \partial T = 0$ → $\langle T \rangle = r_c \rightarrow \infty$

i) Alternative compactification is realized through SUSY condition

ii) No gauge hierarchy problem ← SUSY

Other interesting topics and problems:

- **brane cosmology**
- **symmetry breaking by geometry**
- **other SUSY breaking transmission mechanism**
- **TeV scale string v.s. field theory**
- **origin of brane ← origin of setup**
- **………..**

Brane world scenario is worth investigating further!