Lectures on Brane World Scenarios V

Today's topics:

``Diluting the cosmological constant in infinite volume extra dimensions''

by G. Dvali, G. Gobadadze and M. Shifman

PRD 67, 044020 (2003) , and references therein & thereof

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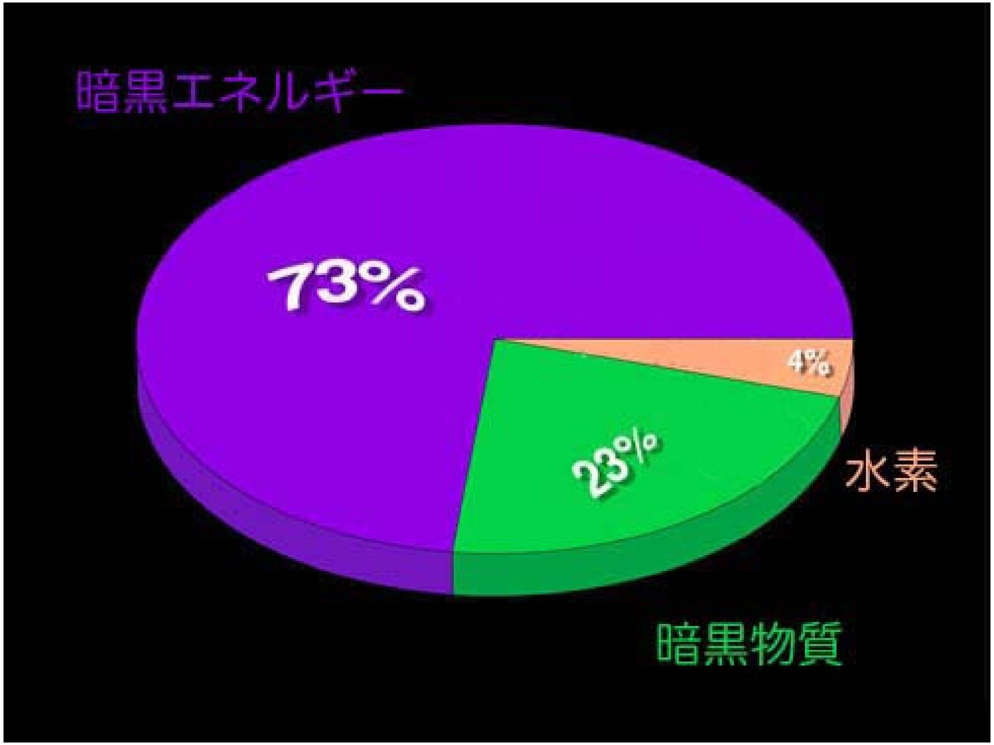
The cosmological constant problem

Observations of the Universe: after WMAP results

The flat universe dominated by unknown energy densities



Dark energy: 73%

Dark Matter: 23%

Baryon: 4%

Standard 4D cosmology:

$$H^2 = \frac{1}{3\bar{M}_P^2}(\mathcal{E}_4 + \rho_D + \rho_B) \sim H_0^2 \sim (10^{-33}\text{eV})^2$$

Reduced Planck mass: $\bar{M}_P \sim 10^{18}$ GeV

Tiny, non-zero dark energy exist! $\mathcal{E}_4 \sim \left(10^{-3}\text{eV}\right)^4$

If this is the cosmological constant → vacuum energy

In field theories, $\Delta V_0 \sim \Lambda_{cut}^4$

→ fine-tuning is necessary

$$\mathcal{E}_4 = V_{tree} + \Delta V_0 = \mathcal{O}(\Lambda_{cut}^4) - \mathcal{O}(\Lambda_{cut}^4) \sim \left(10^{-3}\text{eV}\right)^4$$

The cosmological constant problem

- **Why is it non-zero?**
- **Why is it so tiny** $\mathcal{E}_4 \ll \Lambda_{cut}^4$ **?**

Naturally, $\Lambda_{cut} \sim M_P \sim 10^{19}\,\text{GeV}$

Introduction of SUSY may be a remarkable way,

but it cannot solve the problem

If SUSY is manifest $V_0 = \Delta V_0 = 0$

SUSY should be broken → $\mathcal{E}_4 \sim M_{SUSY}^4 \geq (1\,\text{TeV})^4$

It seems to be very hard to obtain a tiny vacuum energy

Other possibilities?

Give up to reduce the vacuum energy,

but modify the 4D cosmological relation: $H^2 \sim \frac{\mathcal{E}_4}{\bar{M}_P^2}$

4+N extra-dimensional theory:

the relation could be different

since $\mathcal{E}_4$ **could curve extra-dimensions**

In this paper, the authors try to solve the cosmological constant problem based on the brane induced gravity model with infinite extra-dimensions

Modification of the cosmological relations

The author claim the modification

In 4D	→	**In 4+N D**
$\bar{M}_P^2 H^2 \sim \mathcal{E}_4$		$M_*^{2+N} H^{2-N} \sim \mathcal{E}_4$

M_* **is the 4+N dim. Planck scale**

Example: $H \sim 10^{-33}\text{eV}\ (H^{-1} \sim 10^{28}\text{cm})$

→ $N = 4,\ M_* \sim 10^{-3}\text{eV} \to \mathcal{E}_4 \sim (1\text{TeV})^4$

$N = 6,\ M_* \sim 10^{-3}\text{eV} \to \mathcal{E}_4 \sim (\bar{M}_P)^4$

The model is <u>very different</u> from the usual extra-dim. models

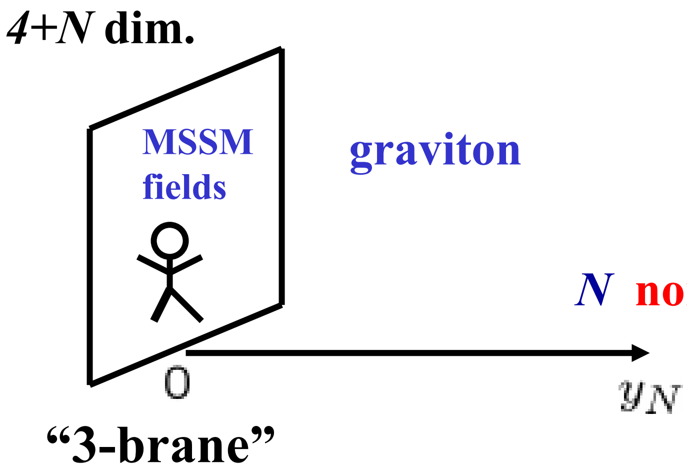
It is crucial that M_* is the model parameter independent of $\bar{M}_P$

***Remember the usual relation** $M_P^2 = M_*^{2+N} V_N$

The model:

so-called ``brane induced gravity model''

Dvali, Gabadadze and Porrati, PLB485, 208 (2000)



N non-compact extra-dimensions

$$S = M_*^{2+N} \int d^4x d^N y \sqrt{G} R + \int d^4 x \sqrt{\bar{g}} \left[\Lambda_{cut}^2 \bar{R} + \mathcal{E}_4 + \mathcal{L}_{MSSM} \right]$$

I II III

I: Bulk gravity with the 4+N dim. Planck scale independent of $\bar{M}_P$

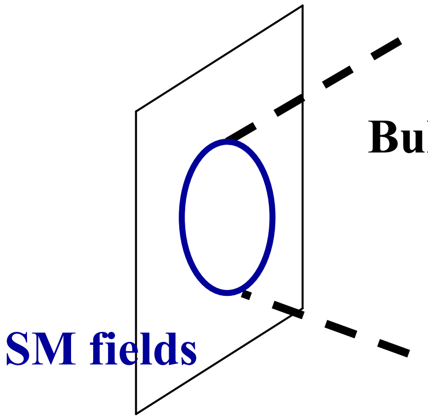
Bulk is supersymmetric → $\mathcal{E}_{bulk} = 0$

SUSY is broken on the brane → but still $\mathcal{E}_{bulk} = 0$

$$\Delta m_{SUSY} \propto V_N^{-1} \to 0$$

(infinite) volume suppression

II: induced gravity on the brane



Bulk graviton $\sim \Lambda_{cut}^2 \bar{R}$

is induced on the brane

We take $\Lambda_{cut} \sim \bar{M}_P$

III: cosmological constant on the brane

SUSY breaking on the brane → $\mathcal{E} \sim M_{SUSY}^4$

Can the model be consistent with experiments?

Normally $M_P^2 = M_*^{2+N} V_N \to \infty$ **for infinite extra-dim.**

→ inconsistent results

The brane induced gravity part plays a crucial role $S_{ind} = \int d^4x \sqrt{\bar{g}} \bar{M}_P^2 \bar{R}$

→ ``Shielding effect''

for the observer on the brane, the gravity looks like

- **4D gravity at short distance**
- **4+N dim. gravity at long distance**

Toy model: **bulk scalar with kinetic term on the brane**

$$S = M_*^{2+N} \int d^4x d^N y \, (\partial_M \Phi(x,y))^2 + \bar{M}_P^2 \int d^4x \, (\partial_\mu \Phi(x, y=0))^2$$

Green's function

$$\left(M_*^{2+N} \partial_M \partial^M + \bar{M}_P^2 \delta^{(N)}(y) \partial_\mu \partial^\mu \right) G_R(x,y,0,0) = -\delta^{(4)}(x) \delta^{(N)}(y)$$

$$\rightarrow \quad \left(M_*^{2+N} (p^2 - \Delta_N) + \bar{M}_P^2 \, p^2 \, \delta^{(N)}(y) \right) \tilde{G}_R(p,y) = \delta^{(N)}(y)$$

$$\tilde{G}_R(p,0) = \frac{1}{\bar{M}_P^2 p^2 + M_*^{2+N} D^{-1}(p,0)}$$

$$(p^2 - \Delta_N) D(p, y_N) = \delta^{(N)}(y)$$

$$D(p, y_N \to 0) \to 1/y^{N-2} \sim M_*^{N-2}$$

Singularity at y=0 would be softened by higher derivative terms for graviton (N>2)

$$\tilde{G}_R(p,0) \sim \frac{1}{\bar{M}_P^2\, p^2 + M_*^4}$$

Therefore $\tilde{G}_R(p,0) \sim \frac{1}{\bar{M}_P^2\, p^2}$ **for** $p_c = 1/r_c \geq \frac{M_*^2}{\bar{M}_P}$

→Green's function of 4D gravity is recovered

at a distance $\leq r_c \sim \frac{\bar{M}_P}{M_*^2} \sim 10^{-33}\text{eV} \sim H_0^{-1}$

(for $M_* \sim 10^{-3}$eV)

Compare to usual model with { **compactified extra-dim** / **no brane induced term** }

- $p > m_{KK} \rightarrow$ **N extra-dim. is going to be revealed**
- $p < m_{KK} \rightarrow$ **KK models are decoupled → 4D like**

Cosmological solution with $\mathcal{E}_4 \neq 0$

Start with a static solution:

$$ds^2 = A^2(y)\eta_{\mu\nu}dx^\mu dx^\nu - B^2(y)dy^2 - C^2(y)y^2 d\Omega_{N-1}^2$$

$$A(y), B(y), C(y) \sim \left(1 - \left(\frac{y_g}{y}\right)^{N-2}\right)^\alpha$$ **was found**

Naked singularity appears at $y = y_g \sim \frac{1}{M_*}\left(\frac{\mathcal{E}_4}{M_*^4}\right)^{\frac{1}{N-2}}$

because the solution is constrained ← static

→ **The authors expect that the singularity would be removed if an inflate solution is considered**

There are a number of examples, where a inflating solution removes the singularity with the relation $\frac{1}{H} \sim y_g$

If we apply the relation to our case

$$H \sim M_* \left(\frac{M_*^4}{\mathcal{E}_4}\right)^{\frac{1}{N-2}}$$

Example: $H \sim 10^{-33}\mathrm{eV}\ (H^{-1} \sim 10^{28}\mathrm{cm})$

→

$$N = 4,\ M_* \sim 10^{-3}\mathrm{eV} \rightarrow \mathcal{E}_4 \sim (1\mathrm{TeV})^4$$

$$N = 6,\ M_* \sim 10^{-3}\mathrm{eV}^4 \rightarrow \mathcal{E}_4 \sim (\bar{M}_P)^4$$

Even if $\mathcal{E}_4 \gg (10^{-3}\mathrm{eV})^4$ **we can obtain** $H \sim 10^{-33}\mathrm{eV}$ **consistent with the observations of the Universe**

Summary

Brane induced gravity model with infinite extra-dimensions

Three independent parameters: $M_*,\ \bar{M}_P,\ \mathcal{E}_4$

- **4D gravity is obtained at a distance shorter than** $r_c \sim \frac{\bar{M}_P}{M_*^2} \sim H_0^{-1}$
- **Hubble parameter is obtained through** $H \sim M_* \left(\frac{M_*^4}{\mathcal{E}_4}\right)^{\frac{1}{N-2}}$