GATE Simulation study

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Ryo Hamanishi

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- ROOT reference
  - the algorithm of peak search

ROOT reference

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- There are three references for TSpectrum2.
  - For peak finding
    - "Identification of peaks in multidimensional coincidence γ-ray spectra"
  - For deconvolution
    - "EFFICIENT ONE AND TWO DIMENTIONAL GOLD DECONVOLUTION AND ITS APPLICATIO TO γ-RAY SPECTRA DECOMPOSITION"
  - For eliminating background
    - "BACKGROUND ELIMINATION METHODS FOR MULTIDIMENSIONAL COINCIDENCE γ-RAY SPECTRA"

- These are published by Miroslav Morhac (Institute of Physics, Slovak Academy of Sciences)

Identification of peaks in multidimensional coincidence γ-ray spectra (for peak search)

- Study peak search algorithm for photon distribution

- This paper introduces the algorithm for multidimensional spectra measurement.

Introduction

- The progress in the nuclear structure understanding depends on the ability to analyze correctly multidimensional spectra.
- The positions of identified peaks can be fed as initial estimate into peak fitting.
- There are many well-known method of peak search for 1 dimensional spectra, but almost of them cannot be directly extended to search for peaks in multidimensional space.
- The algorithm introduced in this paper is used for especially 2 dimensional spectra. (2-fold coincidence γ-ray peaks)

Peak searching algorithm in 2-dimensional spectra

- Suppose that the number of counts in a 2-dimensional γ-ray spectrum in the channel x, y can be approximated by

$$\boxed{N(x,y) = G_1(x,y) + G_2(x) + G_3(y) + B + Cx + Dy}$$

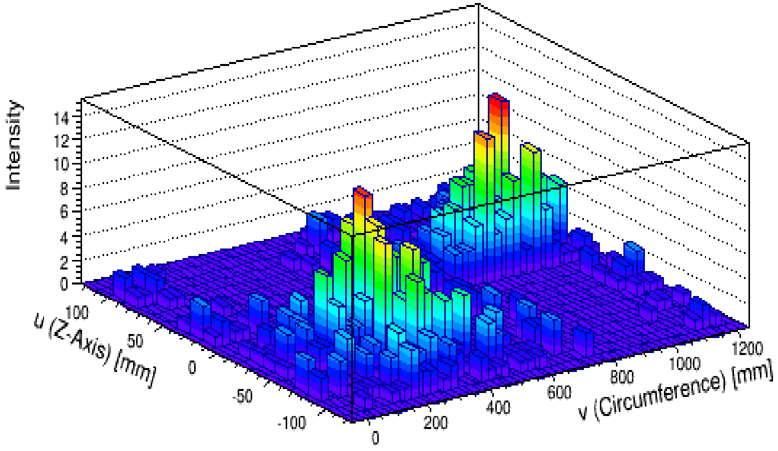
  - G_1 is 2-dimensional gauss.

$$\boxed{G_1(x,y) = A_1 \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-x_1)^2}{\sigma_{x_1}^2} - \frac{2\rho(x-x_1)(y-y_1)}{\sigma_{x_1}\sigma_{y_1}} + \frac{(y-y_1)^2}{\sigma_{y_1}^2}\right]\right\}}$$

  - G_2 and G_3 are 1-dimensional gauss.

$$\boxed{G_{2,3}(\alpha) = A\exp\left\{-\frac{(\alpha-\alpha_0)^2}{2\sigma_{\alpha_0}^2}\right\}}$$

  - $B+Cx+Dy$ is the assumed background



Peak searching algorithm in 2-dimensional spectra

- They shall replace the mixed partial second derivatives by the mixed partial second differences in both dimensions.

$$\boxed{\begin{aligned} S(i,j) = {} & N(i+1,j+1) - 2N(i,j+1) + N(i-1,j+1) \\ & - 2N(i+1,j) + 4N(i,j) - 2N(i-1,j) \\ & + N(i+1,j-1) - 2N(i,j-1) + N(i-1,j-1) \end{aligned}}$$

- In order to eliminate statistical fluctuations, they shall employ the smoothed mixed particle second differences in both dimensions. The smoothing is achieved by summing S(i, j) in a given window

$$\boxed{S_w(i,j) = \sum_{i_1=i-r}^{i+r} \sum_{j_1=j-r}^{j+r} S(i_1, j_1)} \qquad w = 2r + 1$$

- This is repeated z times to optimize the S_w. ($S_{z,w}$)

Peak searching algorithm in 2-dimensional spectra

- The contents of the channel *i, j* of this difference spectrum can be expressed :

$$S_{z,w}(i,j)=\sum_{i_1=i-r}^{i+r}\sum_{j_1=j-r}^{j+r}C_{z,w}(i_1-i,j_1-j)N(i_1,j_1)$$

$r=z\cdot m+1$

$m=(w-1)/2$

- Elements of the filter matrix $C_{z,w}(i, j)$ can be facterized to produces of vector elements

$$C_{z,w}(i,j)=C_{z,w}(i)\cdot C_{z,w}(j)$$

$$C_{z,w}(i)=\sum_{i_1=i-r}^{i+r}C_{z-1,w}(i_1)$$

- Standard deviation of $S_{z,w}$ is defined as :

$$F_{z,w}(i,j)=\sqrt{\sum_{i_1=i-r}^{i+r}\sum_{j_1=j-r}^{j+r}C_{z,w}^{2}(i_1-i)C_{z,w}^{2}(j_1-j)N(i_1,j_1)}$$

Peak searching algorithm in 2-dimensional spectra

- Another way to compute the coefficient of the filter vector C

$$\boxed{C_\sigma = \frac{d^2}{dx^2}\exp\left(-\frac{x^2}{2\sigma^2}\right)}$$

$x = i$

σ : standard deviation of searched peak

- The choice of these calculations is optional and depends on application.
- In general independently of the way of calculation of the $S_{z,w}$ filter can be expressed :

$$\boxed{S(i,j) = \sum_{i_1=i-r}^{i+r}\sum_{j_1=j-r}^{j+r} C(i_1-i)C(j_1-j)N(i_1,j_1)}$$

$$\boxed{F(i,j) = \sqrt{\sum_{i_1=i-r}^{i+r}\sum_{j_1=j-r}^{j+r} C^2(i_1-i)C^2(j_1-j)N(i_1,j_1)}}$$

Peak searching algorithm in 2-dimensional spectra

- Example1
  - $G_{1A}(A_{1A}=100, x_{1A}=60, y_{1A}=8)$, $G_{1B}(A_{1B}=50, x_{1B}=180, y_{1B}=200)$
  - $G_{2A}(A_{2A}=200, x_{2A}=60)$, $G_{2B}(A_{2B}=180, x_{2B}=180)$
  - $G_{3A}(A_{3A}=200, x_{3A}=80)$, $G_{3B}(A_{3B}=500, x_{3B}=200)$
  - Background
    - $800 - 1.176x - 1.96y$
  - σ was set to 6 and $\rho=0$

$$\boxed{N(x,y) = G_1(x,y) + G_2(x) + G_3(y) + B + Cx + Dy}$$

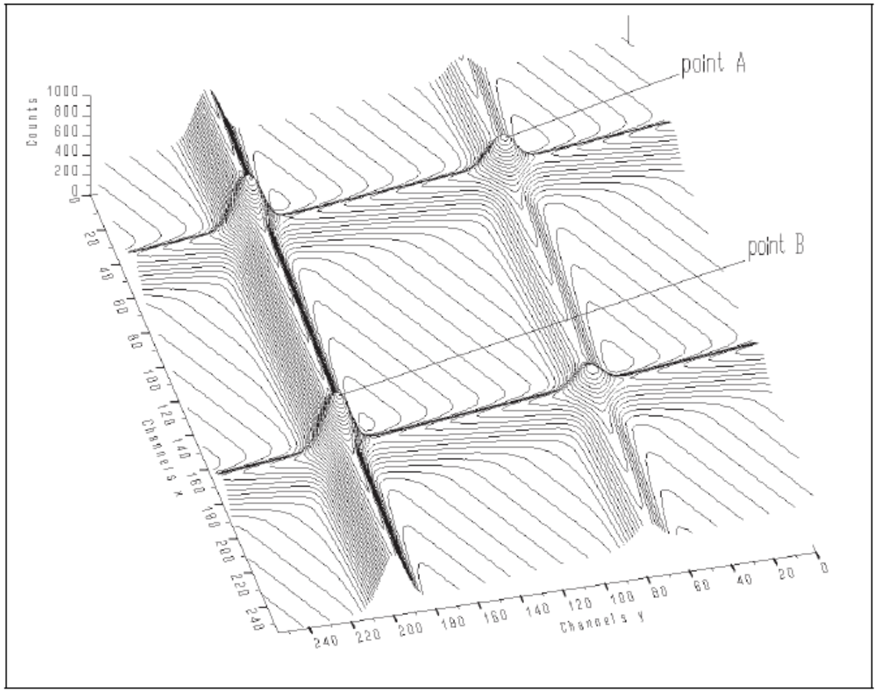


Figure 1: Synthetic spectrum with two two-dimensional Gaussians located at the points A, B

Peak searching algorithm in 2-dimensional spectra

- two free parameters were set to w=7 and z=7.
- The different spectrum calculated with these are shown following :

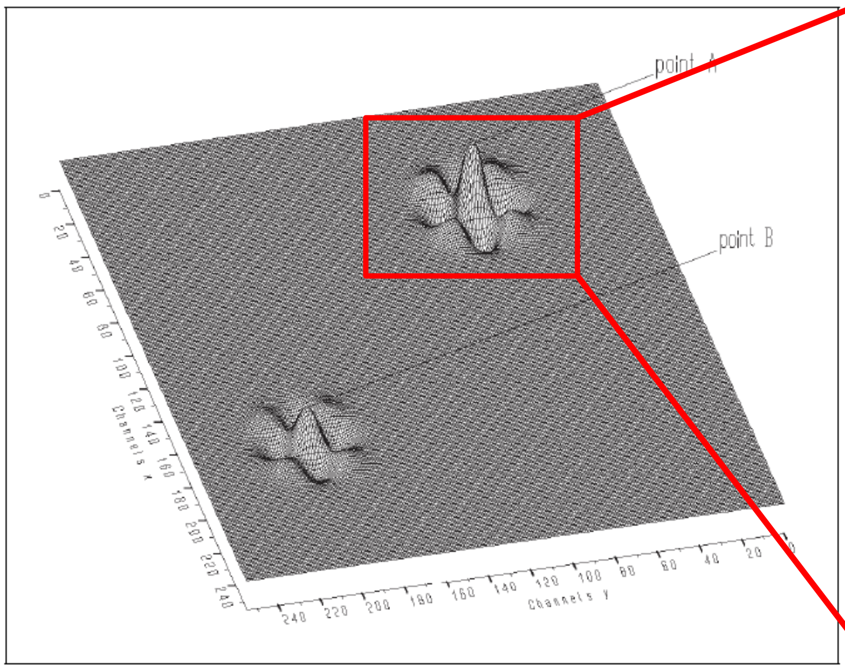


Figure 2: The difference spectrum for the spectrum from Fig. 1

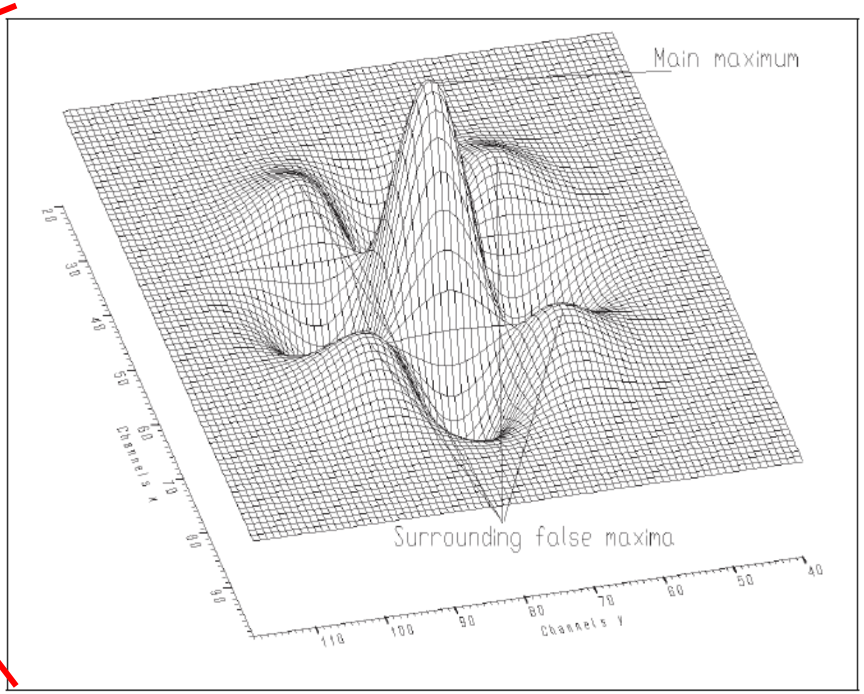


Figure 5: True and false local maxima for the two-dimensional peak

Peak searching algorithm in 2-dimensional spectra

- The position of one of the local maxima located in the middle of the surrounding ones corresponds to the position of two-dimensional peak.
  - true peak (middle peak)
  - false peak (other 4 peaks)

- False peak should be ignored by the searching algorithm

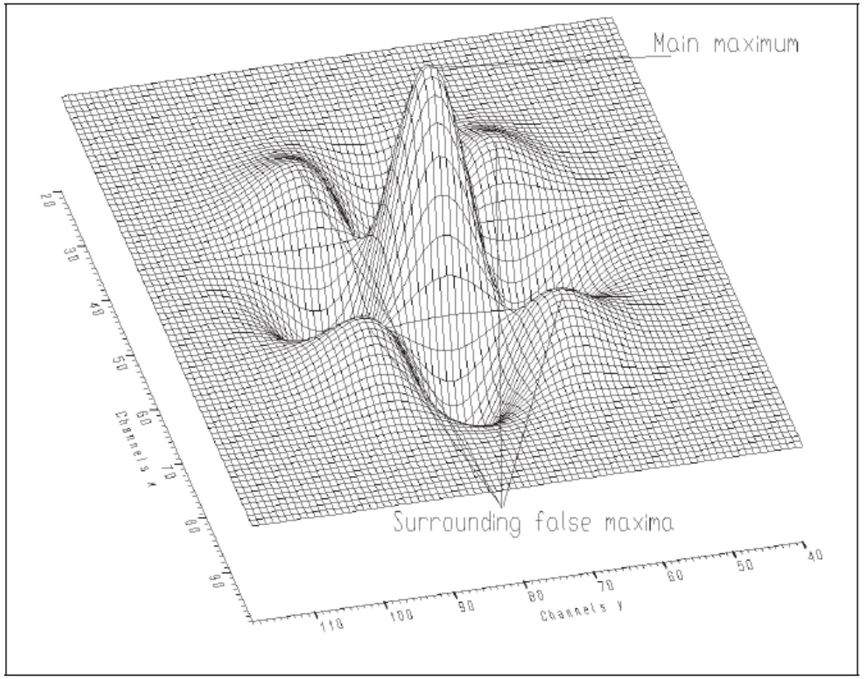


Figure 5: True and false local maxima for the two-dimensional peak

Peak searching algorithm in 2-dimensional spectra

- Example2
  - one peak from experiment of positron annihilation

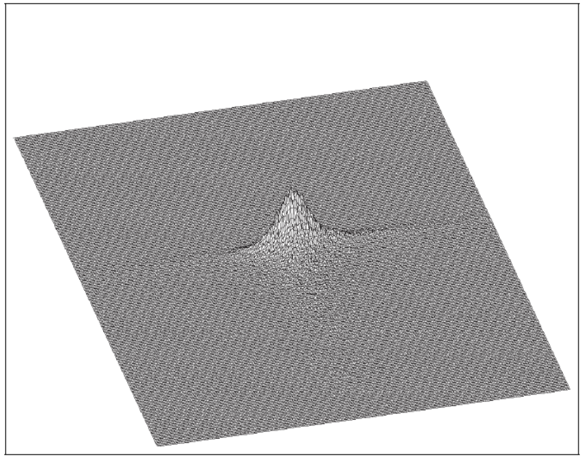


Figure 6: Spectrum with one peak from experiment of positron annihilation

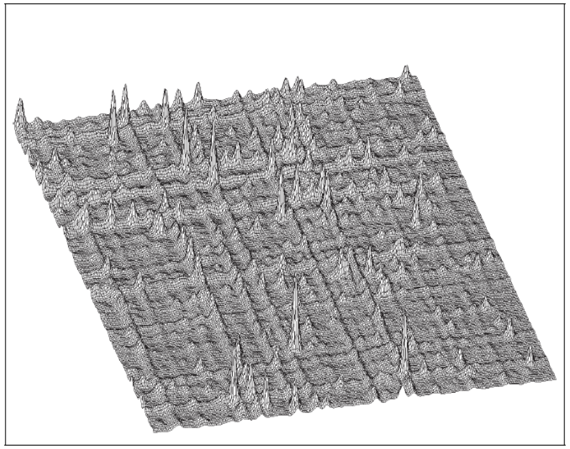


Figure 8: Two-dimensional γ-ray spectrum

- It is impossible to find five maxima in γ-ray spectrum.
- It is needed to suppress in a way the false maxima in the $S_{z,w}$.

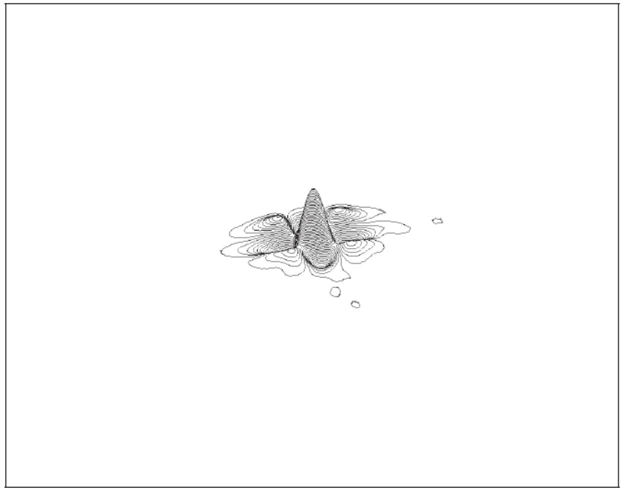


Figure 7: Two-parameter SSD spectrum from Fig. 6

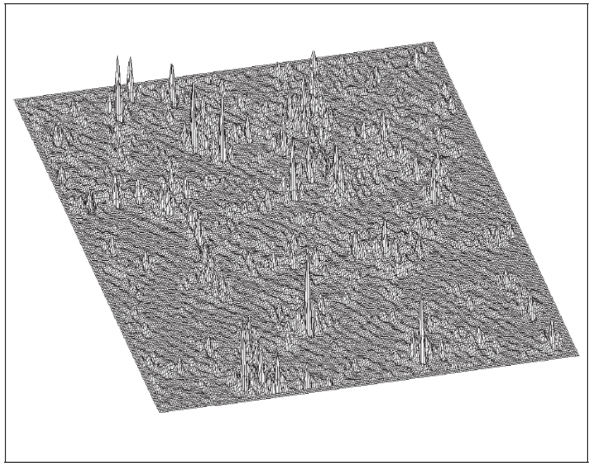


Figure 9: Two-parameter SSD spectrum from Fig. 8

Peak searching algorithm in 2-dimensional spectra

- Define spectra of the one-parameter smoothed $S_{z,w}$ in both x and y independent variables

$$\boxed{X(i,j) = \sum_{i_1=i-r}^{i+r} C(i_1 - i) N(i_1, j)} \qquad \boxed{Y(i,j) = \sum_{j_1=j-r}^{j+r} C(j_1 - j) N(i, j_1)}$$

- Example
  - $z=w=0$
- From the filters defined in this way we can observe that in the expected position of two-dimensional peak

	$C_s(i,j)$			$C_x(i,j)$			$C_y(i,j)$		
j_1 \ i_1	$i-1$	i	$i+1$	$i-1$	i	$i+1$	$i-1$	i	$i+1$
$j-1$	1	-2	1	1	-2	1	1	1	1
j	-2	4	-2	1	-2	1	-2	-2	-2
$j+1$	1	-2	1	1	-2	1	1	1	1

Table 1: Examples of the filters $C_s(i,j)$ for $z = w = 0$ according to (10) for two-parameter SSD and $C_x(i), C_y(j)$ for one-parameter SSDs in two-dimensional space.

Peak searching algorithm in 2-dimensional spectra

- In the position of true local maximum of $S(i_t, j_t)$ the following conditions satisfied (S is different from $S_{z,w}$)

$$\underline{S(i_t, j_t)>0 \text{ and } X(i_t, j_t)<0 \text{ and } Y(i_t, j_t)<0}$$

- Along with this condition also the following conditions, saying that in the point X, Y in both dimensions have local minima, must be satisfied

$$\underline{X(i_{t-1}, j_t) \geq X(i_t, j_t) \leq X(i_{t+1}, j_t) <0}$$

$$\underline{Y(i_t, j_{t-1}) \geq Y(i_t, j_t) \leq Y(i_t, j_{t+1}) <0}$$

- The above condition is not satisfied in the point of false local maxima

$$\underline{S(i_f, j_f)>0 \text{ and } (X(i_f, j_f) \geq 0 \text{ or } Y(i_f, j_f) \geq 0)}$$

Peak searching algorithm in 2-dimensional spectra

- Applied the conditions to

figure7 and figure9

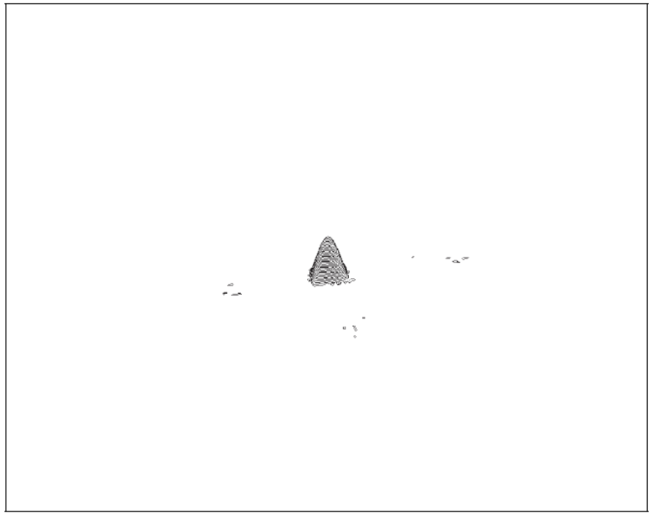


Figure 10: Two-parameter SSD spectrum from Fig. 7 after application of the condition (20)

the algorithm to search for peaks in two-dimensional spectrum is as follows:

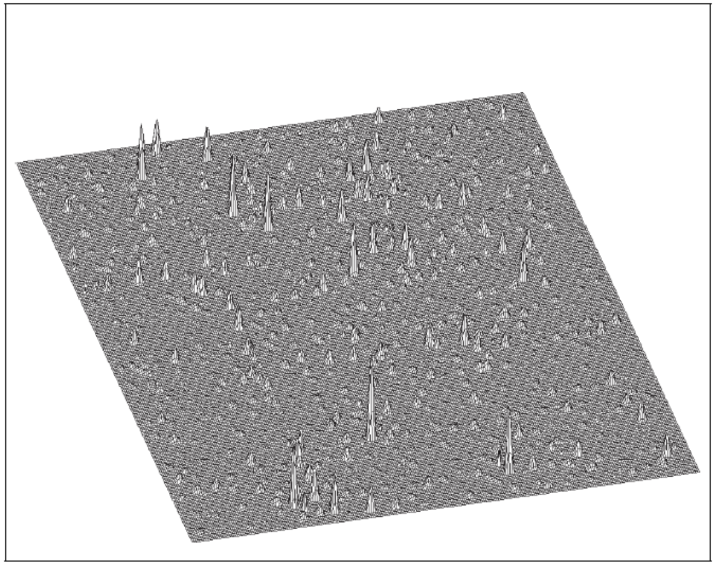


Figure 11: Two-parameter SSD spectrum from Fig. 9 after application of the condition (20)

Peak searching algorithm in 2-dimensional spectra

- Algorithm
  - 1. using $\boxed{S(i,j)=\sum_{i_1=i-r}^{i+r}\sum_{j_1=j-r}^{j+r}C(i_1-i)C(j_1-j)N(i_1,j_1)}$ it calculates spectrum of the smoothed second differences in both dimensions
  - 2. using $\boxed{F(i,j)=\sqrt{\sum_{i_1=i-r}^{i+r}\sum_{j_1=j-r}^{j+r}C^2(i_1-i)C^2(j_1-j)N(i_1,j_1)}}$ it calculates spectrum of standard deviation of the smoothed second differences
  - 3. for $i\in$ <0,n_1-1>, $j\in$ <0,n_2-1>, where n_1,n_2 are sizes of the spectrum, we search for local maxima in the spectrum of S
  - 4. once it has found a local maximum in the position i_l, j_l then using $\boxed{X(i,j)=\sum_{i_1=i-r}^{i+r}C(i_1-i)N(i_1,j)}$ $\boxed{Y(i,j)=\sum_{j_1=j-r}^{j+r}C(j_1-j)N(i,j_1)}$ it calculates spectra of X, Y respectively.Then applying condition <u>$S(i_l,j_l)>0$ and $X(i_l,j_l)<0$ and $Y(i_l,j_l)<0$</u> it decides whether the found local maximum is true of false.
  - 5. It tests $S(i,j_l)$, $F(i,j_l)$, $i\in$ <0,n_1-1> for the shape of the peak in x dimension. Likewise it tests y dimension.
  - 6. If the shape of the peak in both dimensions satisfied these criteria we have found two-dimensional peak in the position i_l, j_l.
  - 7. It repeats the whole procedure for another local maximum from the point 3 on.