Ground Motion Models for Future Linear Colliders *

Andrei Seryi
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309 USA

Abstract
Optimization of the parameters of a future linear collider requires comprehensive models of ground motion. Both general models of ground motion and specific models of the particular site and local conditions are essential. Existing models are not completely adequate, either because they are too general, or because they omit important peculiarities of ground motion. The model considered in this paper is based on recent ground motion measurements performed at SLAC and at other accelerator laboratories, as well as on historical data. The issues to be studied for the models to become more predictive are also discussed.

1 INTRODUCTION
Ground motion is one of the limiting factors which would influence the future linear colliders by continuously misaligning its focusing and accelerating elements. Understanding the ground motion, finding driving mechanisms of the motion, studying the ground motion dependence with geology, conditions, etc., is essential for optimization of the configuration of the linear collider.

In order to accurately characterize the ground motion influence on the linear collider, an adequate mathematical model of ground motion has to be created. We consider in this paper one particular model based on measurements performed at SLAC site. We use this model to illustrate existing methods of modeling, as well as potential problems and oversimplifications of the models.

Since a completely new site may be chosen for a linear collider location, one of the goals is to be able to build a ground motion model for this location, taking into account only general information about geology, tunnel depth and construction technique, urbanization of the area, cultural noise sources to be installed in the tunnel or on the surface. Current understanding is not yet at this level. We discuss below some steps, that may help to achieve the necessary predictive capabilities.

2 GROUND MOTION FEATURES
Ground motion can be divided into fast motion and slow motion. Fast motion ($f \gtrsim$ a few Hz) cannot be adequately corrected by a feedback based on a pulse-to-pulse repetition rate of the collider and therefore results primarily in the beam offset at the IP. On the other hand, slow motion ($f \lesssim 0.1–1$ Hz) result only in beam emittance growth.

There is also another reason to divide ground motion into fast and slow. The nature of how the ground motion results in relative displacements of neighboring (separated by 10–100m) quadrupoles is different for low and high frequencies, and the boundary also seem to lie around 0.1 Hz.

The fast motion is usually represented by spectra of absolute motion $p(\omega)$ (see Fig.1) and by the correlation $c(\omega , L)$ which show how the motion of two points separated by distance $L$ differs. The fast motion consists primarily from elastic waves propagating with quite large velocity $v$ (of the order of km/s). The correlation is then defined by this velocity (or by the wavelength) and distribution of the sources, and the spectrum of relative motion can be found using this correlation and the power spectrum of absolute motion $p(\omega)$, as $p(\omega, L) = p(\omega)2(1 - c(\omega, L))$. For example, assuming that waves propagate on surface and are distributed uniformly over azimuthal angle, the correlation will be given by $c(\omega , L) = \langle \cos(\omega L/v \cos(\theta)) \rangle = J_0(\omega L/v)$ and the corresponding 2-D spectrum of ground motion $P(\omega, k) = p(\omega)2/\sqrt{\langle \omega^2 \rangle f} - k^2$, $|k| \leq \omega/\ell(f)$ [5].

As we see, at frequencies below 0.01Hz the wavelength quickly become larger than the earth crust and eventually the earth size so that, first, no waves would be possible anymore, and second, their contribution to relative displacement over reasonably short distance is negligible.

Figure 1: Power spectra measured in several places in different conditions [3, 4, 7]

Another causes, not the wave mechanism, are responsible for producing relative misalignments at low frequencies, which justify special treatment of slow motion. Among them are the variation of temperature in the tunnel, underground water flow, spatial variation of ground properties combined with some driving force, etc. These causes can produce rather short wavelength misalignments in spite they are very slow. Diffusive, or ATL model of ground motion [2] is a method to describe all these effects by a simple rule which states that the variance of misalignment $\Delta X^2$ is proportional to a coefficient $A$, time $T$ and separation $L$: $\Delta X^2 = ATL$. The spectrum of this model is given by

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\[ P(\omega, k) = A/(\omega^2 k^2) \]. Review of various measurements have shown [9] that in many cases this rule corresponds to measurements, however, typically either spatial or temporal behavior can only be studied from a particular measurement. Measurements where good statistics was collected both in time and in space in relevant region of parameters are sparse. For example, measurements at SLAC have shown that \( A \approx 10^{-7} - 10^{-6} \mu \text{m}^2/(\text{m}\cdot\text{s}) \) in wide frequency band of 0.01–10^{-6}Hz, however only two data points in \( L \) were studied: 30m and 1500m [6, 13]. Detailed investigations of slow motion is one of the urgent issues.

The driving causes of the slow motion should be identified. In the above mentioned studies a major driving term of the \( 1/\omega^2 \) motion was identified to be the temporal variation of atmospheric pressure coupled to spatial variations of ground properties [13]. The \( 1/k^2 \) behavior, while not confirmed or rejected directly, appears not to contradict to both the considered driving mechanism, and to another measurements [6] performed over shorter baselength.

Transition region from fast to slow motion is of particular interest. For a fast motion whose spectrum drops as about \( 1/\omega^4 \) with frequency, for the short time scale the variance of the absolute motion grows as \( \Delta X^2 \propto T^2 \) (or the rms grows linearly in time in systematic manner). If the slow motion is described by the ATL-rule, it has to fail at short time or at high frequencies, since the ATL given relative motion, \( \Delta X^2 \propto T \), would otherwise exceed the absolute motion (maximal value of relative rms amplitude is \( \sqrt{2} \) of the absolute rms motion of a single point). In spectral sense it means that the diffusive spectrum \( AL/\omega^2 \) should make transition to \( 1/\omega^4 \) dependence. There are not enough measured data which can clarify details of the transition. In [5] it was assumed that this transition occurs as \( p(\omega,L) \approx \min(AL/\omega^2,B/\omega^4) \). As we see, in this case the transition frequency becomes a function of separation, and the short time misalignment for the ”corrected ATL” is given by \( \Delta X^2 \propto T^2 \sqrt{T} \) which has quite odd spatial behavior. If, however, we suppose that the driving causes of the diffusive motion appear below certain frequency \( \omega_0 \), then this modified spectrum may have the form \( p(\omega,L) \approx \min(AL/\omega^2,AL\omega_0^2/\omega^4) \) in which case the short term misalignment would behave as \( \Delta X^2 \propto T^2 L \). In reality, a combination of those, or more complex transition may occur. Though details of this transition do not seem to be very critical for a linear collider, at least for NLC and for SLAC ground motion, this could be not the case in other conditions and other parameters of the collider. One should note that the above given consideration of the transition, which occurs at \( \sim 0.1\text{Hz} \), may have interesting analogy for very slow motion.

Very slow motion, observed in a year-to-year time scale at SLAC, LEP or other places, appears to be systematic in time, i.e. \( \Delta X^2 \propto T^2 \) [8]. For example the motion of SLAC linac tunnel measured for 17 years [1] showed almost linear in time motion with rate up to 1mm/year in many locations along the linac. In the case of SLAC this motion may be caused primarily by settlement effects, while in the case of LEP the causes may be different, underground water being one of the first candidates [8].

(Not) surprisingly, the spatial characteristics of systematic motion seem to follow the \( 1/k^2 \) (or \( \Delta X^2 \propto L \)) behavior (see Fig.3). The spectrum of displacements observed at SLAC for 17 years follows \( 1/k^2 \) in the range of \( \lambda \) about 20–500m. Comparing the \( \Delta X^2 \propto T^2 L \) behavior of the systematic motion with transition region of the diffusive motion described above, one can come to a hypothesis that this systematic motion may again be a transition, since at lower frequencies some new driving causes of motion may appear. At much larger time scale (years?), above the transition, this motion may lose its systematic character.

### 3 A MODEL

To illustrate these considerations, we give below some results obtained with a SLAC site ground motion model. The absolute power spectrum of the fast motion, assumed for the model, corresponds to measurements performed at 2 a.m. at the sector 10 of the linac, the spatial properties of the fast motion are defined by the fit of phase velocity to measured correlation \( v(f) = 450 + 1900 \exp(-f/2) \) (with \( v \) in m/s, \( f \) in Hz) [7]. The slow motion is represented by ATL motion with \( A = 5 \cdot 10^{-7} \mu \text{m}^2/(\text{m}\cdot\text{s}) \) and by system-
atic motion corresponded to 17-year observation of SLAC linac tunnel displacements. The transition from fast to slow motion is done in similar manner as in [5]. The absolute spectrum \( p(\omega) \) and the spectrum of relative motion \( p(\omega, L) \) are shown in Fig.3. The systematic motion is not seen in this figure as it corresponds to much smaller frequencies, but is seen in Fig.4 where the rms \( \Delta X \) for \( L = 30 \, \text{m} \) is shown as a function of time. One can see that this curve can be distinctly divided into three regions: wave dominated \( (T \lesssim 10^3 \, \text{s}) \), ATL-dominated \( (10 \lesssim T \lesssim 10^4 \, \text{s}) \) and systematic motion dominated \( (T \gtrsim 10^7 \, \text{s}) \).

This model, which basically give a phenomenological approximation of the \( P(\omega, k) \) spectrum, can then be used to evaluate analytically the collider performance in terms of the IP beam offset or emittance growth, for example, as it is done in [5] or [7]. Such analytical evaluation of ground motion, with use of the \( P(\omega, k) \) spectrum and spectral response functions of the transport lines is included in the PWK module of the final focus design and analysis code PFADA [10]. Analytical treatment is not always useful or reliable. Direct simulations of ground motion can be done, for example, by summing of harmonics whose amplitudes are given by the 2-D spectrum. In this case, if done, for example, by summation of harmonics whose amplitudes are given by the 2-D spectrum.

Figure 4: Rms relative motion versus time for \( L = 30 \, \text{m} \) for the 2 a.m. SLAC site ground motion model.

Ground motion modeling, specifically to future accelerators, is a fast developing area. A lot of progress have been made in classification of ground motion, understanding its specific features and driving causes. Many issues have to be addressed in further studies, some of them were listed in this paper, for example understanding and careful modeling of cultural noises, investigation of slow motion, studies of the influence of tunnel location, geometry, etc.

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4 CONCLUSION

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5 REFERENCES