# SUSY Search and Study Scenario at Linear $e^+e^-$ Colliders

## Experimental Aspects

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Introduction

SUSY Study Scenario

- A Sample Case
  - Sleptons
  - Charginos
  - Squarks
- 3-rd Generation
  - Stau
  - Stop
- If Only LSP is Available

Summary

# Introduction

## SUSY Physics

### Discovery Phase

$\exists$ Light $H^0$ ⇢ Probably SUSY

⋮

Not necessarily

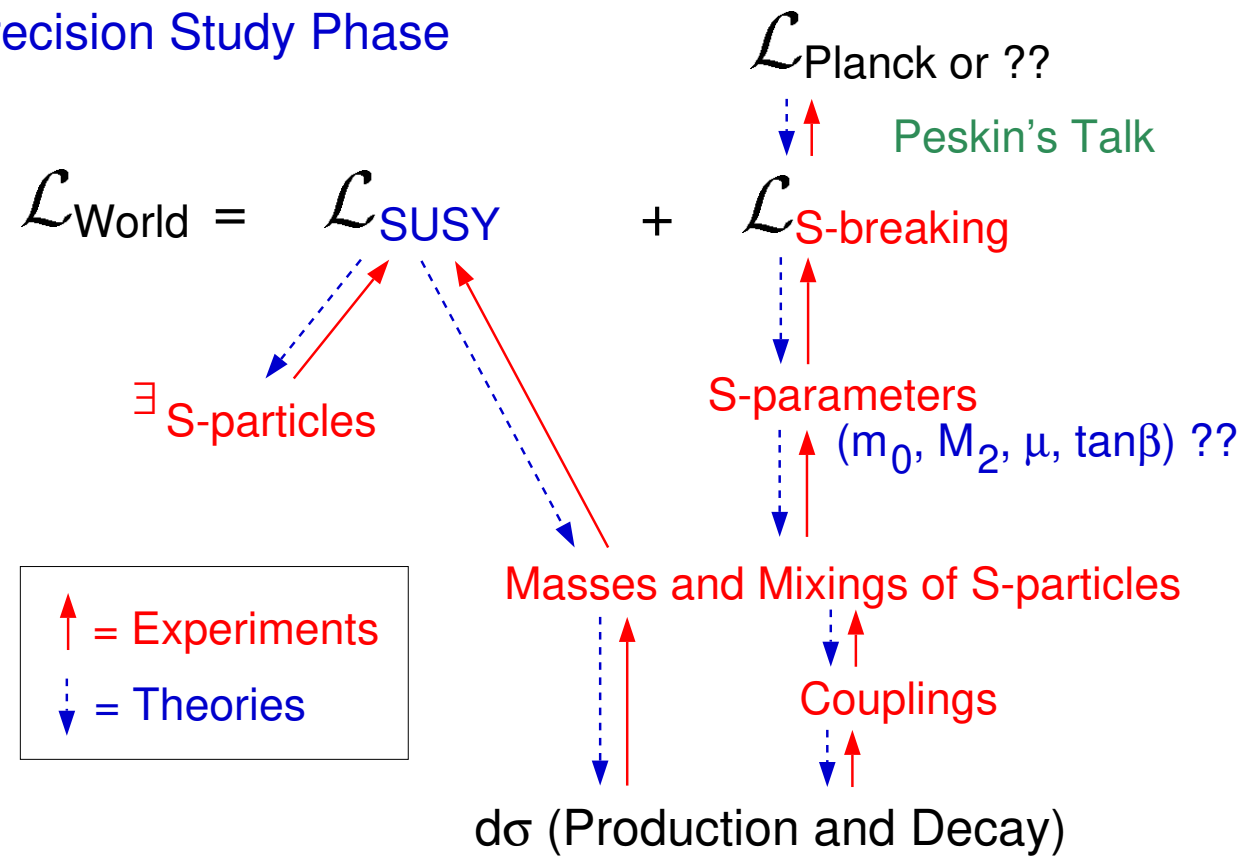
⇢ Need to find at least one SUSY particle!

Once a SUSY particle is found.....

It is only the beginning!

True excitements come after this!

### Precision Study Phase




Quantitative tests of $\mathcal{L}_{SUSY}$

Studies of $\mathcal{L}_{S\text{-}breaking}$ ⇢ $\mathcal{L}_{Planck}$ ?

# Mass Relations

## Gaugino Masses

GUT Relation

$$M_i = M \cdot \frac{\alpha_i(m_Z)}{\alpha_{GUT}} \quad : i = \begin{cases} 1\ U(1)_Y \\ 2\ SU(2)_L \\ 3\ SU(3)_C \end{cases}$$

$$M_1 : M_2 : M_3 \cong 1 : 2 : 7$$

$\tilde{B}$ $\tilde{W}$ $\tilde{g}$

Sizes of groups

$1 < 2 < 3$

$$m_{\tilde{\chi}_1^\pm} \lesssim \frac{1}{3} m_{\tilde{g}}$$

Lighter Chargino

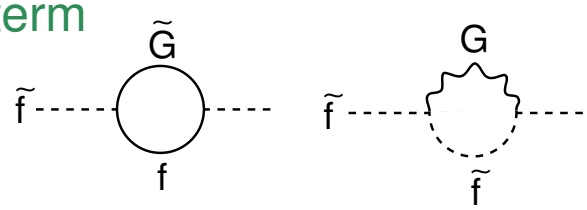
## Sfermion Masses

D-term $\lesssim O(1)$

$$m_{\tilde{f}}^2 = m_0^2 + G_{\tilde{f}} \cdot M^2 + D_{\tilde{f}} \cdot m_Z^2$$

Gauge term

Common scalar mass




$$\begin{cases} G_{\tilde{q}} > G_{\tilde{l}} & (3 > 2, 1) \\ G_{\tilde{f}_L} > G_{\tilde{f}_R} & (2 > 1) \end{cases}$$

$m_{colorless} < m_{colored}$

$m_{\tilde{f}_R} < m_{\tilde{f}_L}$

First SUSY Particle (FSP)

$$FSP = \begin{cases} \tilde{\chi}_1^\pm \\ \tilde{l}_R^\pm \end{cases}$$

Take a sample parameter set

Ref. PRD51(1995)3153

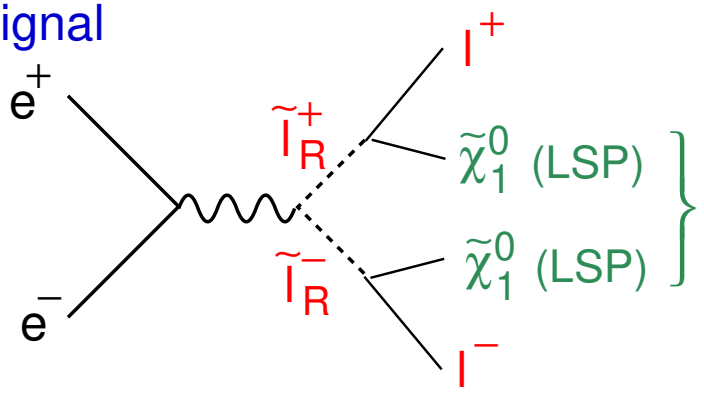
$(m_0, M_2, \mu, \tan\beta) = (70, 250, 400, 2)$ GeV $\leftrightarrow$ FSP = $\tilde{l}_R^\pm$

and see how a typical SUSY study scenario goes.

# SUSY Study Scenario; A Sample Case

## $\tilde{l}^{\pm}_R$ Study ($l \neq \tau$)

### Signal




### BG

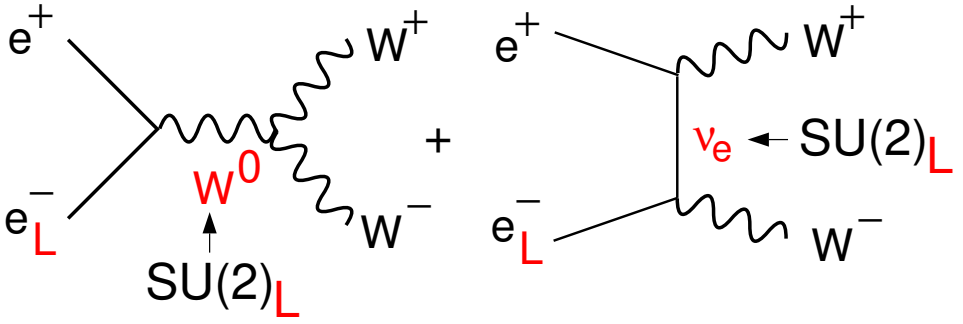
SM processes with $\nu$'s which mimic LSP.

### Key Point

Polarized $e^-$ beam is VERY POWERFUL!

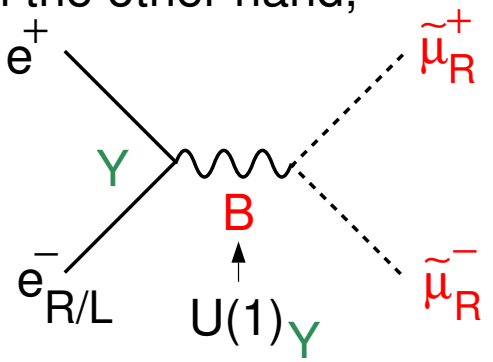
(e.g.) $e^+e^- \rightarrow W^+W^-(W \rightarrow l\nu)$

The LARGEST SM BG




In the symmetry limit, $\sigma_{WW} \rightarrow 0$ for $e^-_R$ !

On the other hand,




$$\begin{cases} Y_L = -1/2 : e^-_L \\ Y_R = -1 \quad : e^-_R \end{cases}$$

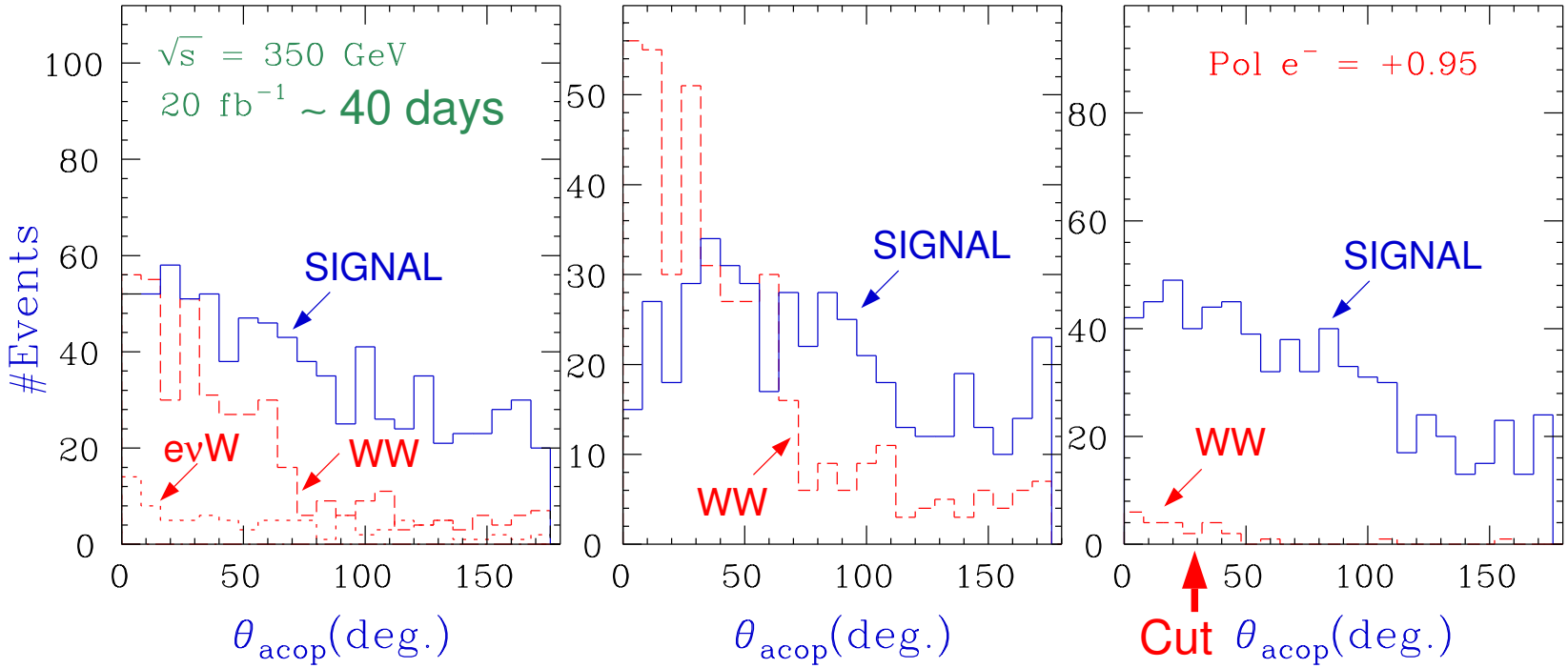
In the symmetry limit, $\sigma_R = 4\,\sigma_L$ !

# Acoplanarity Distributions

$e^+e^- \to \tilde{e}_R^+ \tilde{e}_R^-$ Pol $e^-$ = 0

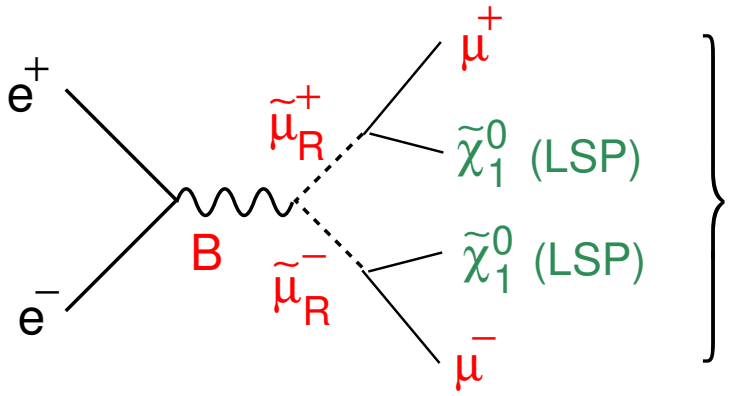
$e^+e^- \to \tilde{\mu}_R^+ \tilde{\mu}_R^-$ Pol $e^-$ = 0

Pol $e^-$ = +0.95 $\cong e_R^-$




Very Clean Sample for Precision Studies!

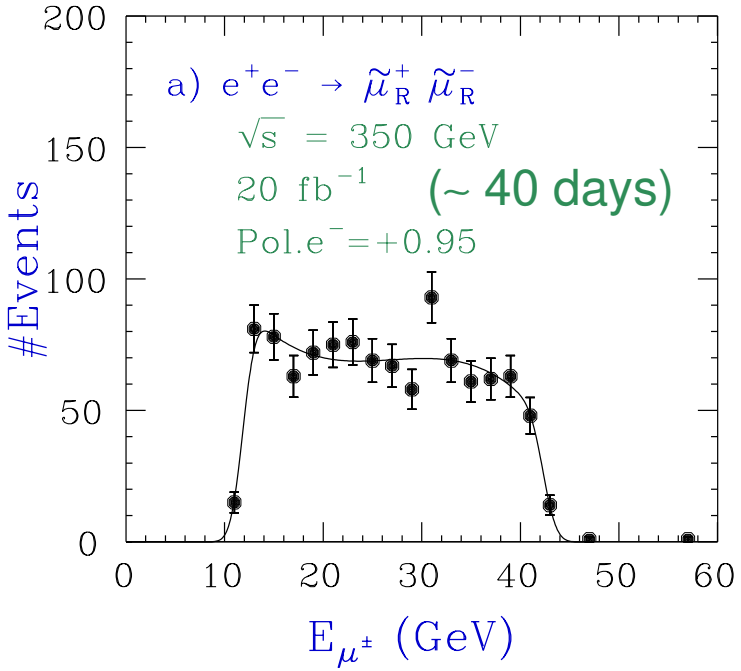
# Mass Determination

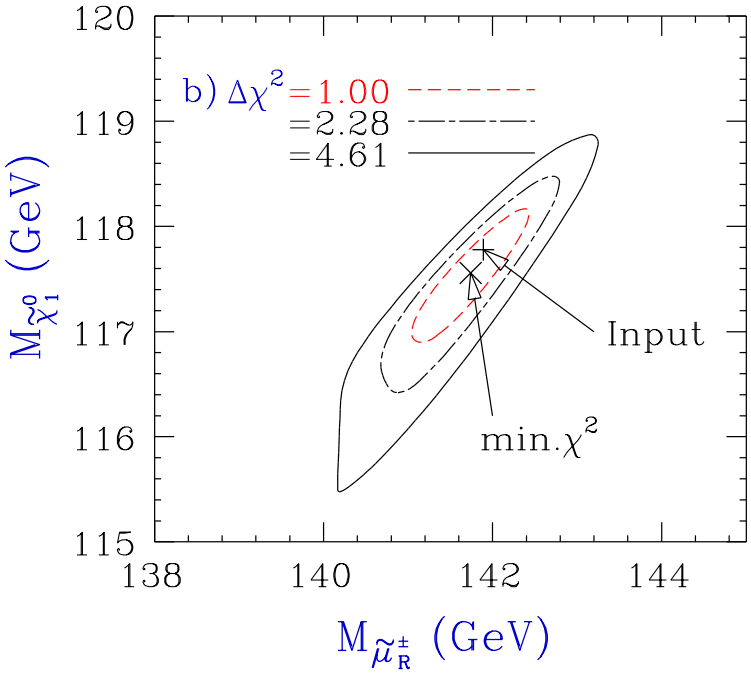



Measure $E_{\mu^{\pm}}$

End points

$\longrightarrow (m_{\tilde{\mu}^{\pm}_R}, m_{\tilde{\chi}^0_1})$



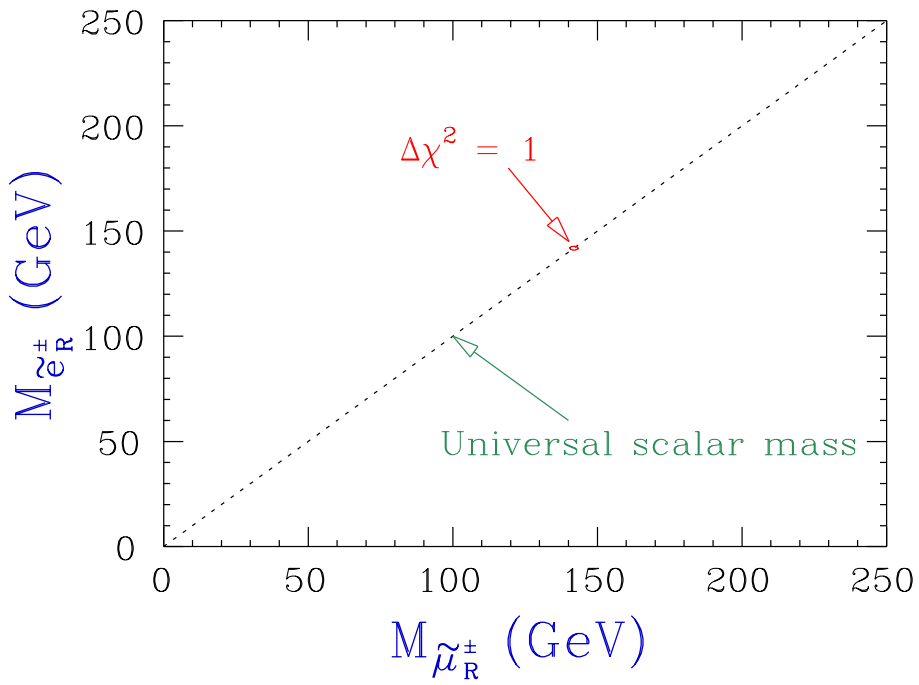

$\longrightarrow \Delta m/m \lesssim 1\%$ !

# Test of Generation Independence

## Hidden Sector SUSY Breaking

----------► Universal Scalar Mass

----------► $m_{\tilde{\mu}_R^\pm} = m_{\tilde{e}_R^\pm}$

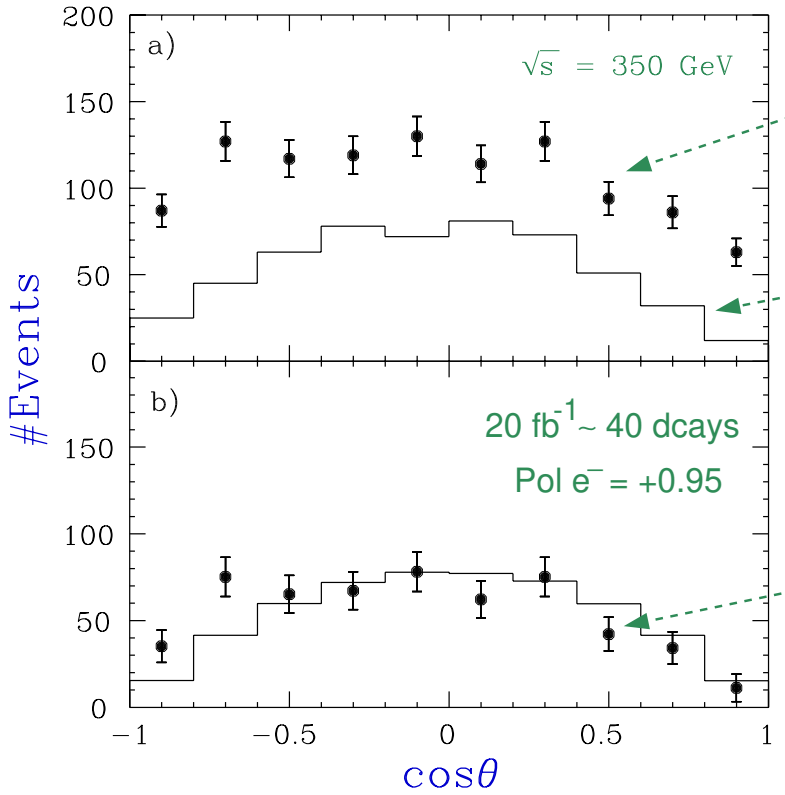



----------► Precision test ($\lesssim 1\%$) of universal scalar mass hypothesis is possible!

# Differential Cross Section Measurements

$(m_{\tilde{l}^{\pm}_R}, m_{\tilde{\chi}^0_1})$ : known ----► Can solve KINEMATICS up to 2-fold ambiguities!

$e^+e^- \rightarrow \tilde{\mu}^+_R \tilde{\mu}^-_R$




Reconstructed 2 solutions

$\theta_{generated}$ for selected sample

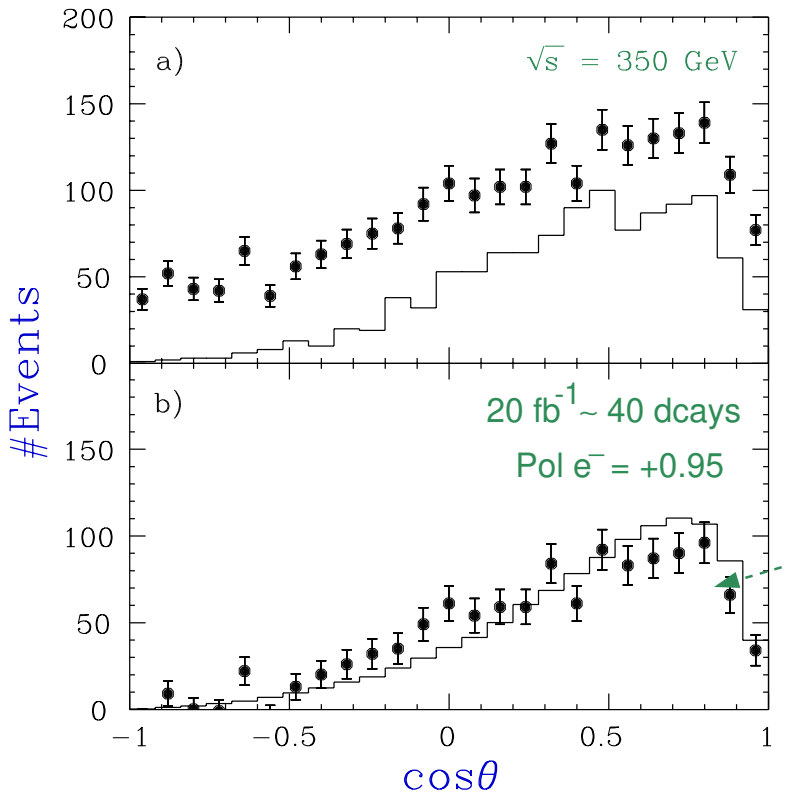
Wrong Solution = Flat BG

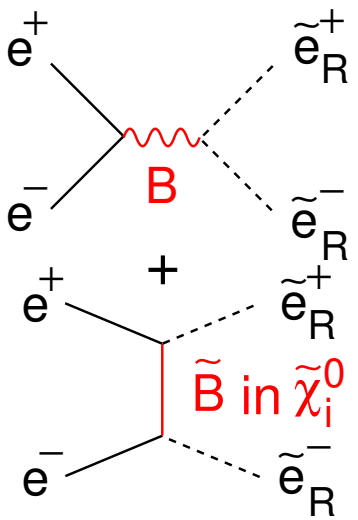
BG subtracted

$\sin^2\theta$

$\therefore J = 0$

$e^+e^- \rightarrow \tilde{e}^+_R \tilde{e}^-_R$



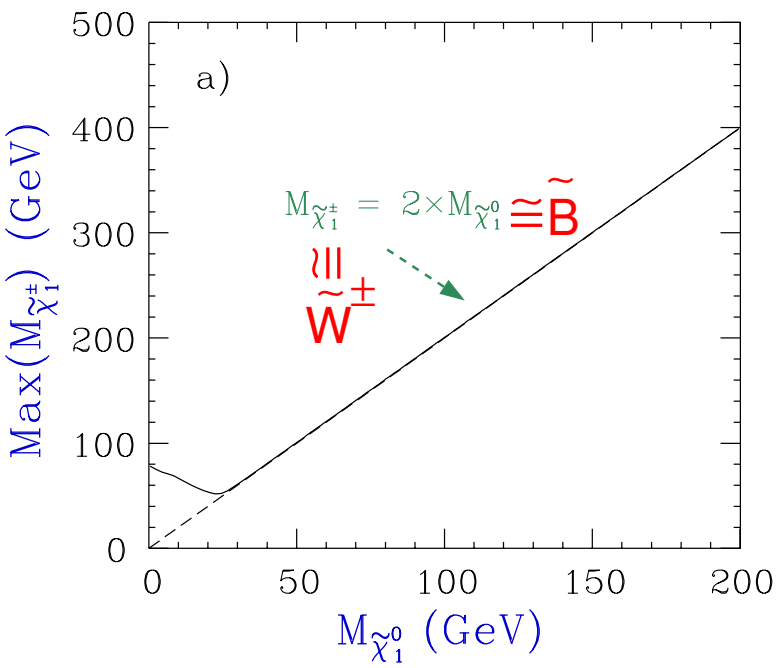

Forward Peak

$\therefore \exists$ t-channel

$\tilde{\chi}^0_1 \cong \tilde{B}$

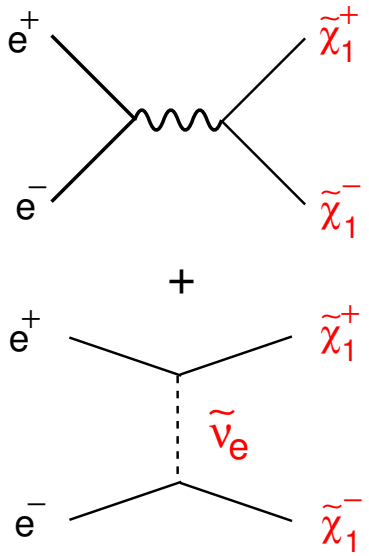
# $\tilde{\chi}_1^{\pm}$ Study

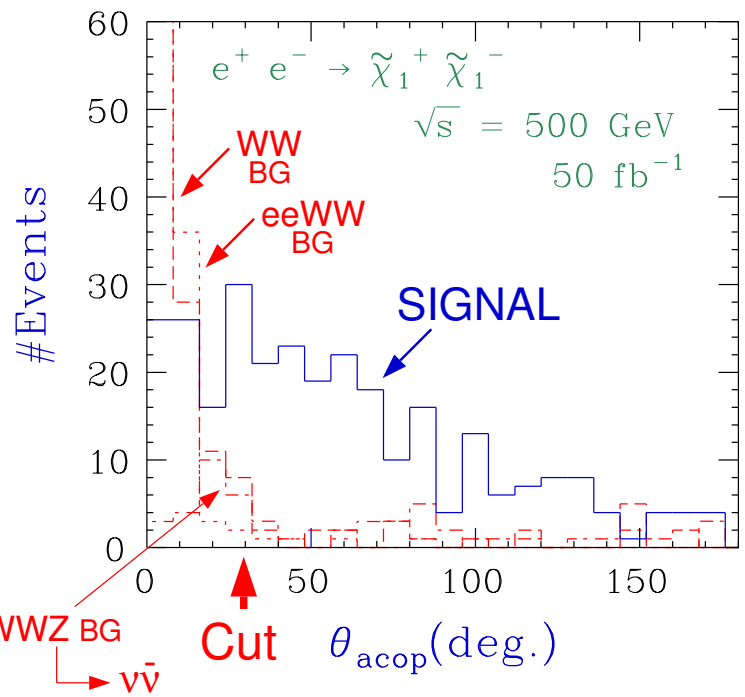
$m_{LSP} = m_{\tilde{\chi}_1^0}$ ($\tilde{l}_R^{\pm}$ Studies) ⟶ Upper limits on $m_{\tilde{\chi}_1^{\pm}}$




⟶ Set $\sqrt{s}$ just above $4\times m_{LSP}$, look for $\tilde{\chi}_1^{\pm}$

$\tilde{\chi}_1^{\pm} \to W^{\pm} \tilde{\chi}_1^0$ ⟶ Signal = Acoplanar $W^+W^-$ → $q\bar{q}'$ / $q\bar{q}'$ 4 Jets

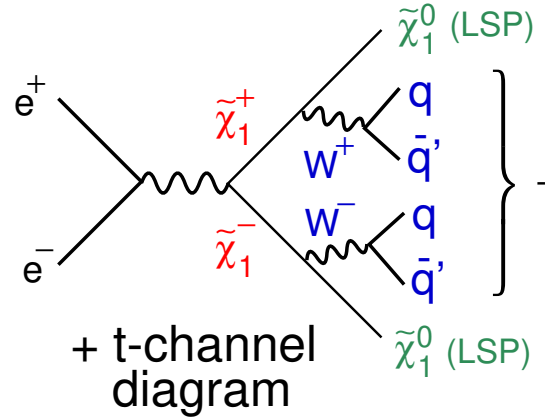



$\nu\nu WW$, $e\nu WZ$, $WWZ$ BG → $\nu\bar{\nu}$

⟶ Fairly Clean Sample for Precision Studies !

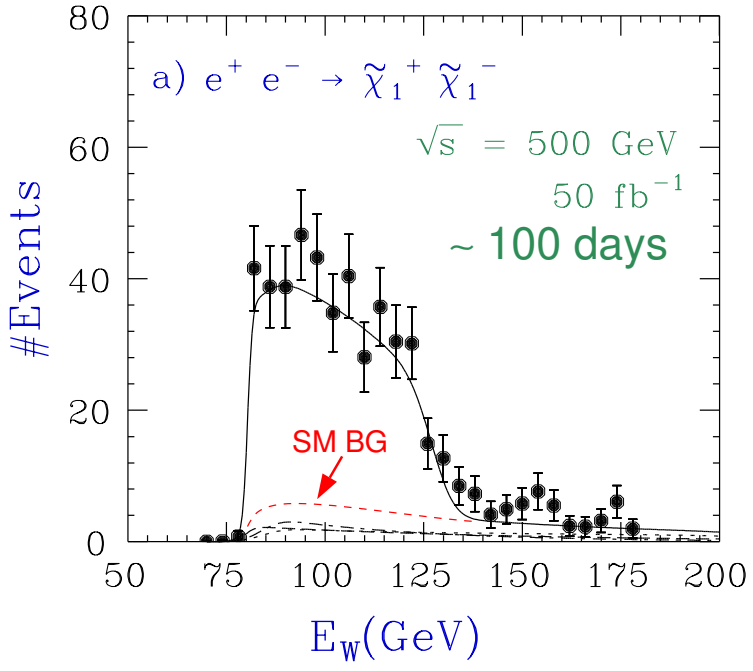
# Mass Determination

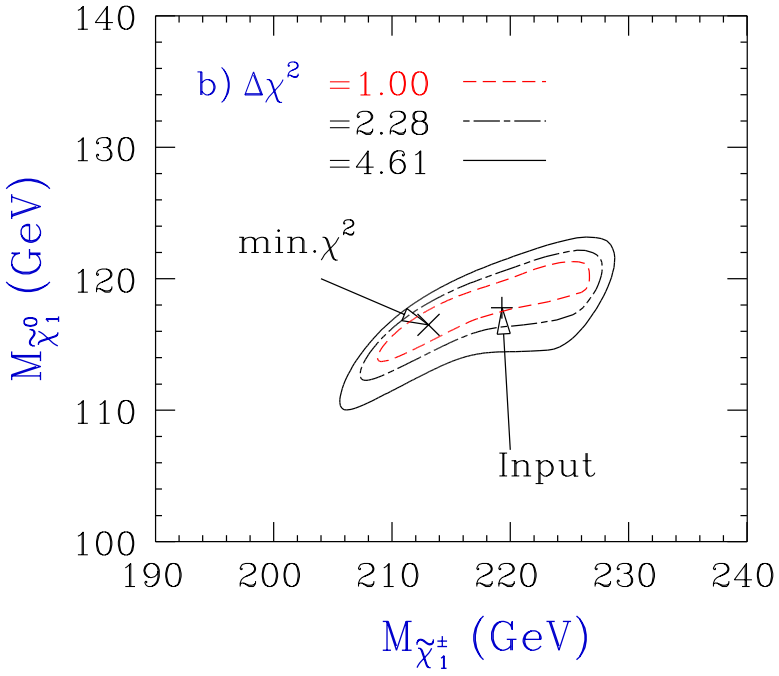



+ t-channel diagram

Measure $E_{W^\pm}$

End points ⟶ $(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_1^0})$

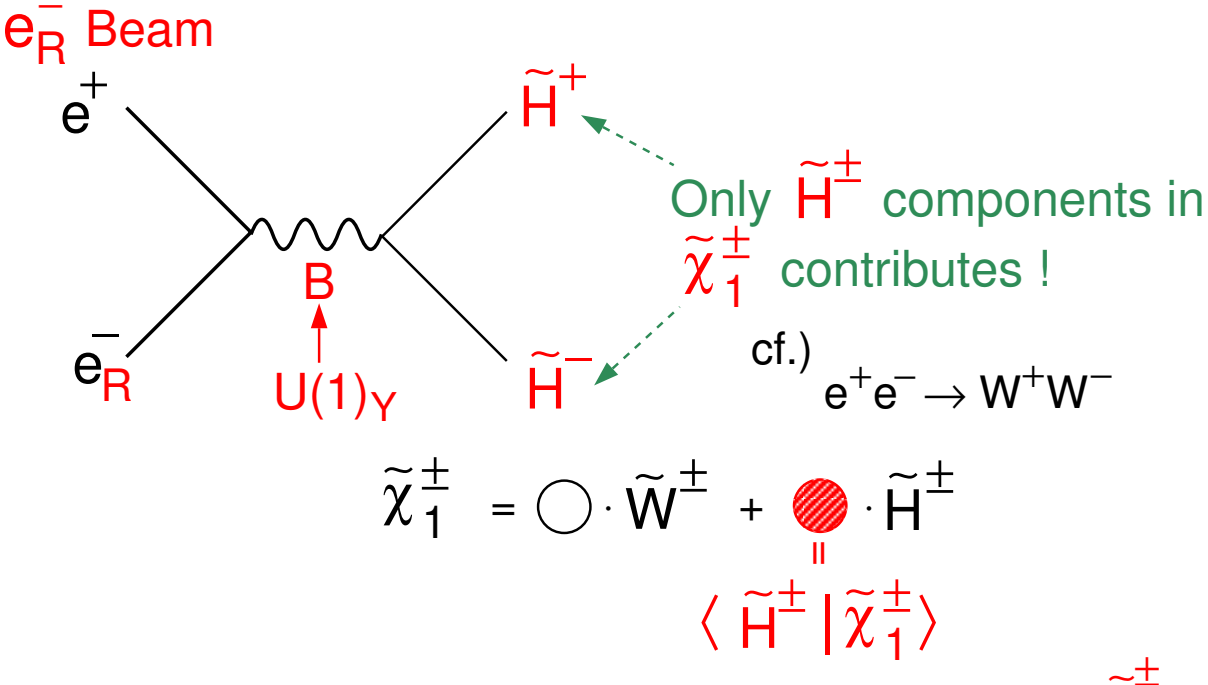



⟶ $\Delta m_{\tilde{\chi}_1^\pm} \cong 8$ GeV ⟶ 5 GeV

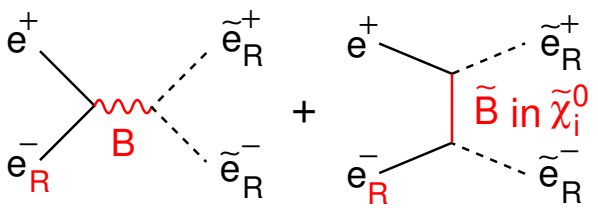
$m_{\tilde{\chi}_1^0}$ from $\tilde{l}_R^\pm$ studies

# $\sigma_{\tilde{\chi}_1^+\tilde{\chi}_1^-}$ Measurement




⇢ We can determine the composition of $\tilde{\chi}_1^\pm$ !

## Global Fit




We know

$m_{\tilde{\chi}_1^0}$ = LSP $\quad m_{\tilde{\chi}_1^\pm}$ $\quad \sigma(e^+e_R^- \to \tilde{e}_R^+\tilde{e}_R^-)$ $\quad \sigma(e^+e_R^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$

Mass Matrices $\quad \langle \tilde{B} | \tilde{\chi}_1^0 \rangle \quad \langle \tilde{H}^\pm | \tilde{\chi}_1^\pm \rangle$

$(M_1, M_2, \mu, \tan\beta)$

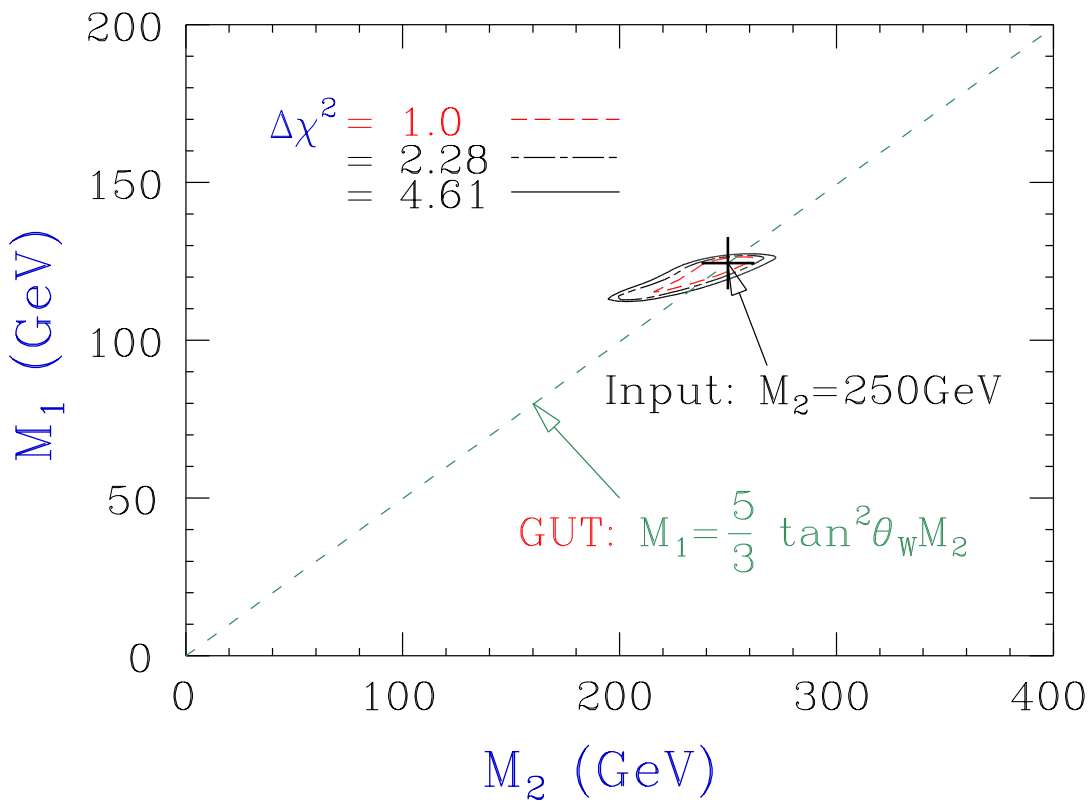
Forget the GUT relation and test if the GUT relation between $M_1$ and $M_2$ actually holds !

# Test of GUT Relation

Common Gaugino Mass at the GUT scale

GUT $\dashrightarrow$ $\dfrac{M_i}{\alpha_i(m_Z)} = \dfrac{M}{\alpha_{GUT}}$

$i = \begin{cases} 1\ U(1)_Y \\ 2\ SU(2)_L \\ 3\ SU(3)_C \end{cases}$



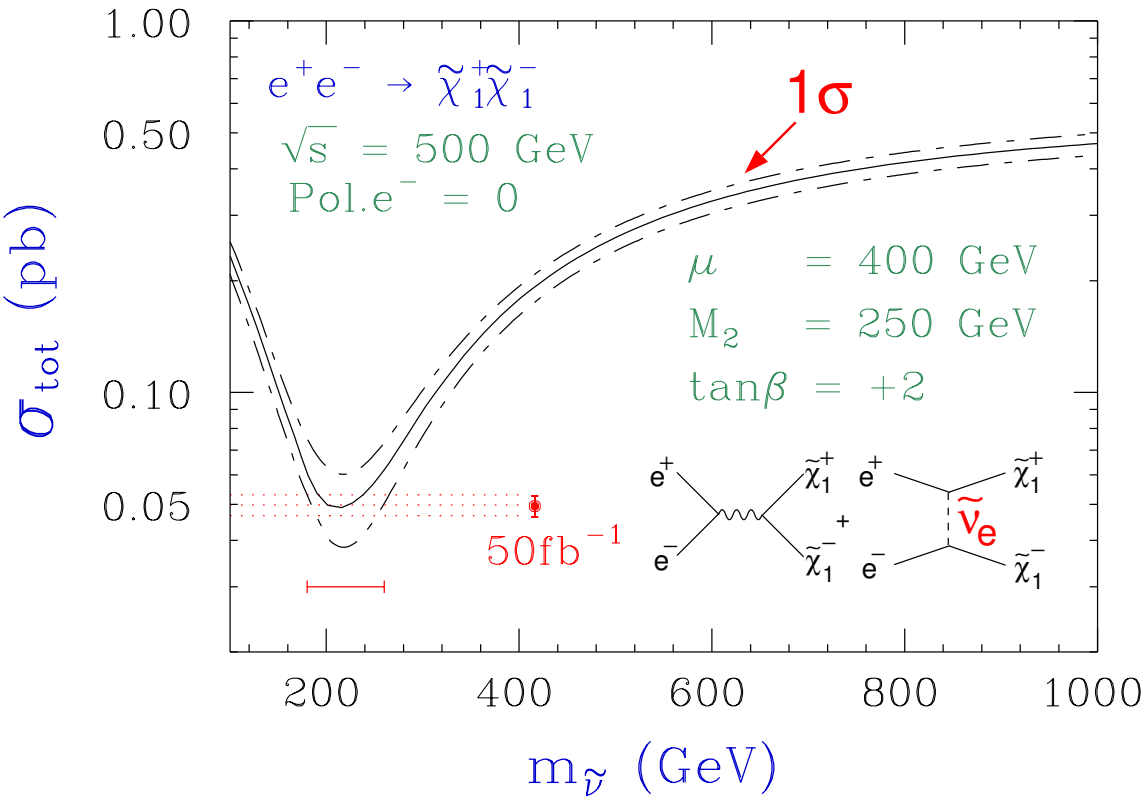

If things turn out this way, we will be forced to believe GUT !

# Upper Limit on $m_{\tilde{l}_L^{\pm}}$

Since we know $m_{\tilde{\chi}_1^{\pm}}$ and

$$\tilde{\chi}_1^{\pm} = \bigcirc \cdot \tilde{W}^{\pm} + \bullet \cdot \tilde{H}^{\pm}$$

only UNKNOWN in $\sigma_{\tilde{\chi}_1^+\tilde{\chi}_1^-}$ is $m_{\tilde{\nu}_e}$ !




Measure $\sigma_{\tilde{\chi}_1^+\tilde{\chi}_1^-}$ (Pol $e^-$ = 0) to get $m_{\tilde{\nu}_e}$

$$m_{\tilde{e}_L^{\pm}}^2 \leq m_{\tilde{\nu}_e}^2 + 0.77\ m_Z^2$$

We now have an upper limit on $m_{\tilde{e}_L^{\pm}}$, look for $\tilde{e}_L^+$ in $e^+e_R^- \to \tilde{e}_L^+\tilde{e}_R^-$ !

# Quantitative Test of SUSY

If there is substantial mixing

Ref. Feng-Peskin-Murayama-Tata

$\tilde{e}^{\pm}_{L,R}$ Beam Study

Consider the case in which $\tilde{\chi}^{\pm}_1 \approx \tilde{W}^{\pm}$

$\tilde{\chi}^{\pm}_1 = \bigcirc \cdot \tilde{W}^{\pm} + \bullet \cdot \tilde{H}^{\pm}$

Question

Signal

Only $\tilde{H}^{\pm}$ components in $\tilde{\chi}^{\pm}_1$ contributes !

Observables to Use : $d\sigma \cong d\sigma(m_{\tilde{\chi}^{\pm}_1}, m_{\tilde{\nu}_e}, g_{\chi})$

Question

To kill SM BG

$M_{\tilde{\chi}^{\pm}_1} = \left( \frac{M_2}{\sqrt{2}\, m_W \cos\beta^{\chi}} \right)$ known $\rightarrow m_W^{\chi} = m_W$ ?

$\sigma_L = \sigma(e^+ e^-_L \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1)$

$A_L^{\chi} = \dfrac{\sigma_L(0<\cos\theta<0.707) - \sigma_L(-1<\cos\theta<0)}{\sigma_L(-1<\cos\theta<0.707)}$

If SUSY, YES !

20 fb

Observables to Use:

$\sigma_R$ and $A_R^{\chi} \leftrightarrow A_R^{had=W}$

$A_L^{had=W}$

Test of SUSY

Test of SUSY

$(m_{\tilde{l}}, m_{\tilde{q}}, M_2, \mu, \tan\beta) = (400, 700, 210, -195, 4)$

$(m_{\tilde{l}}, m_{\tilde{q}}, M_2, \mu, \tan\beta) = (400, 700, 170, -500, 4)$

BG

SIGNAL

10 fb$^{-1}$

100 fb$^{-1}$

SUSY

SUSY

$m_{\tilde{\chi}^{\pm}_1} = 172$ GeV

$m_{\tilde{\chi}^0_1} = 105$ GeV

$m_{\tilde{\chi}^{\pm}_1} = 172$ GeV

$m_{\tilde{\chi}^0_1} = 86$ GeV

$e^+ \sigma_L$ carries information on $\tilde{e}_L$ !

Very Clean Sample for Precision Studies !

$g_{\chi}\phi_{\chi}$ (degrees)

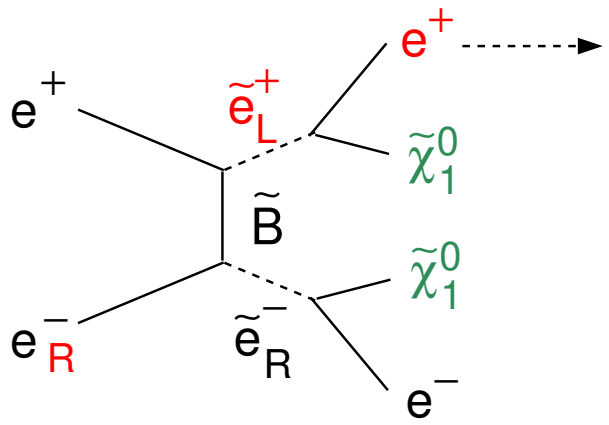
$E_{e^+}$ (GeV)

$\Phi_{\pm}$ (degrees)

$m_{\tilde{\nu}_e}$ (GeV)

Situation is much better than the gaugino-rich case !

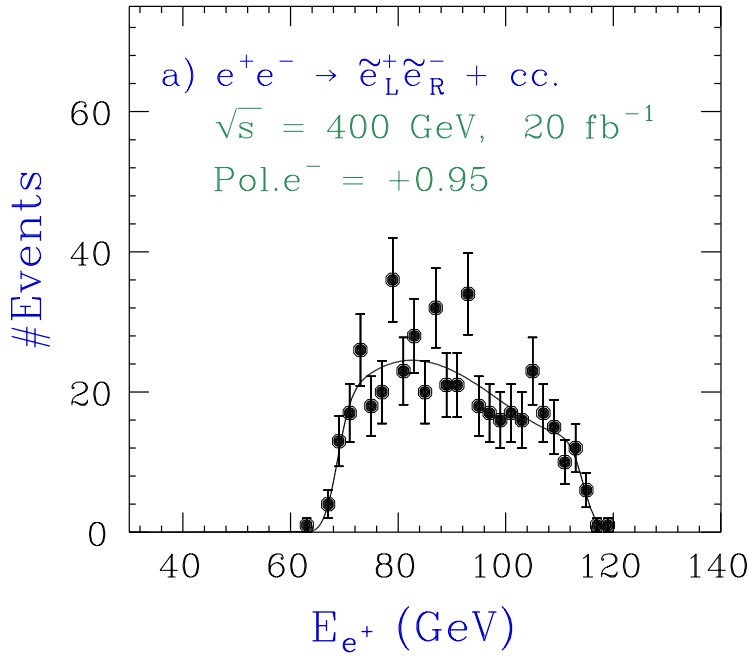
# Mass Determination

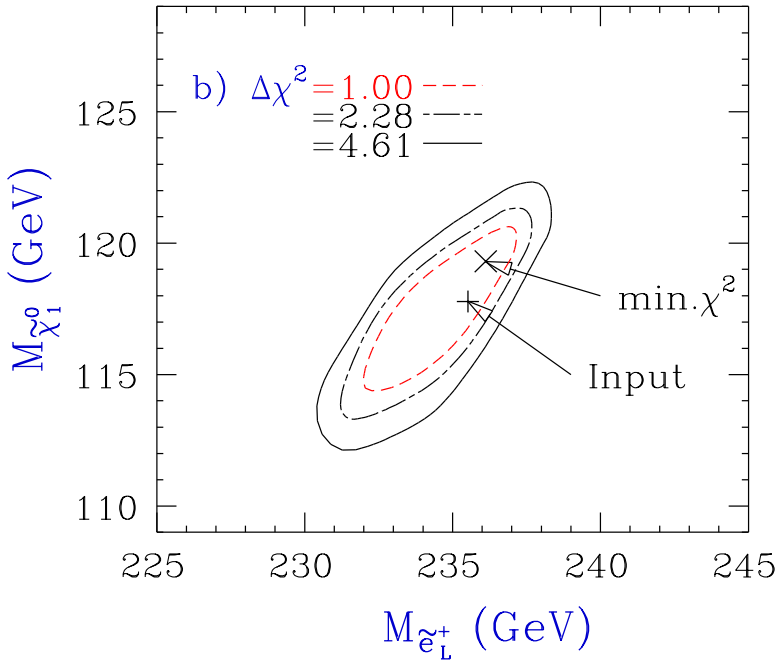



Measure $E_{e^+}$

End points

$\longrightarrow (m_{\tilde{e}_L^\pm}, m_{\tilde{\chi}_1^0})$




$\longrightarrow \Delta m/m \lesssim 1\%$ !

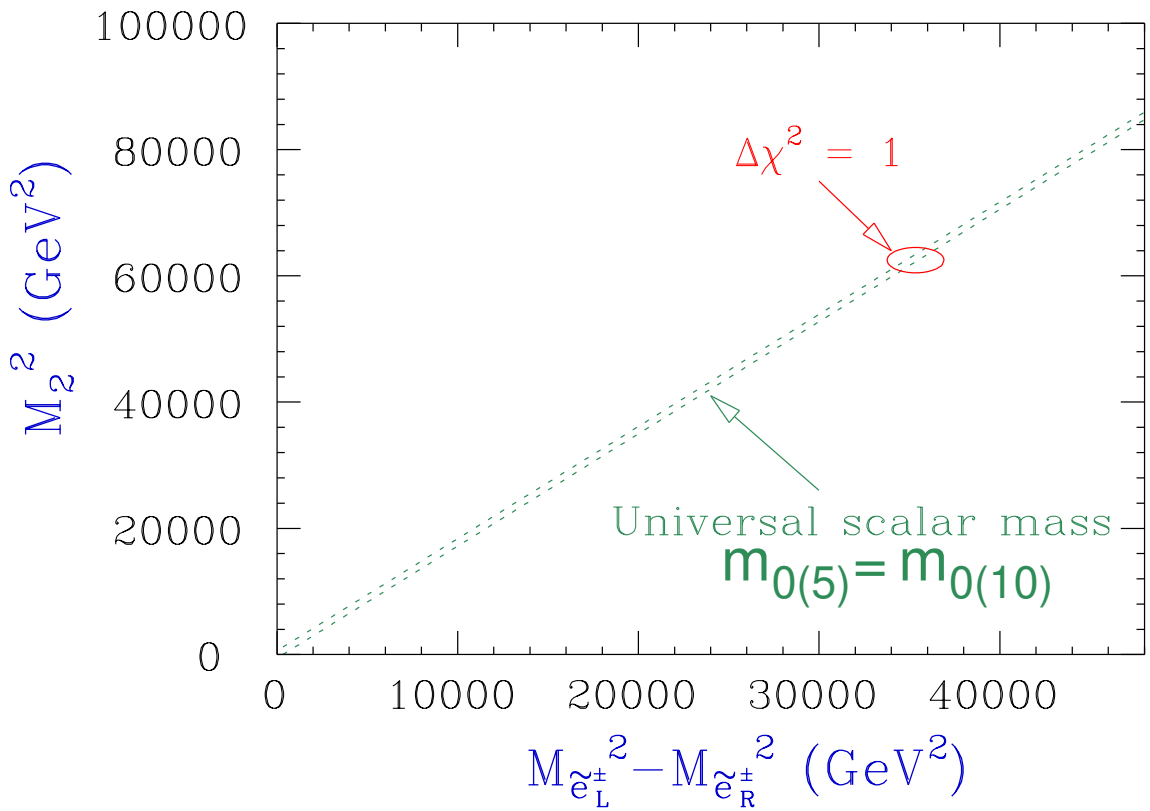
# Test of Universal Scalar Mass

## Sfermion Mass Formula

$$m^2_{\tilde{e}_L} - m^2_{\tilde{e}_R} = m^2_{0(5)} - m^2_{0(10)} + 0.53\, M_2^2 - \underbrace{0.44\, D}_{\text{Small}} \approx m_Z^2 \cos 2\beta < 0$$

$m^2_{\tilde{e}_L}$: 5 $\quad$ $m^2_{\tilde{e}_R}$: 10

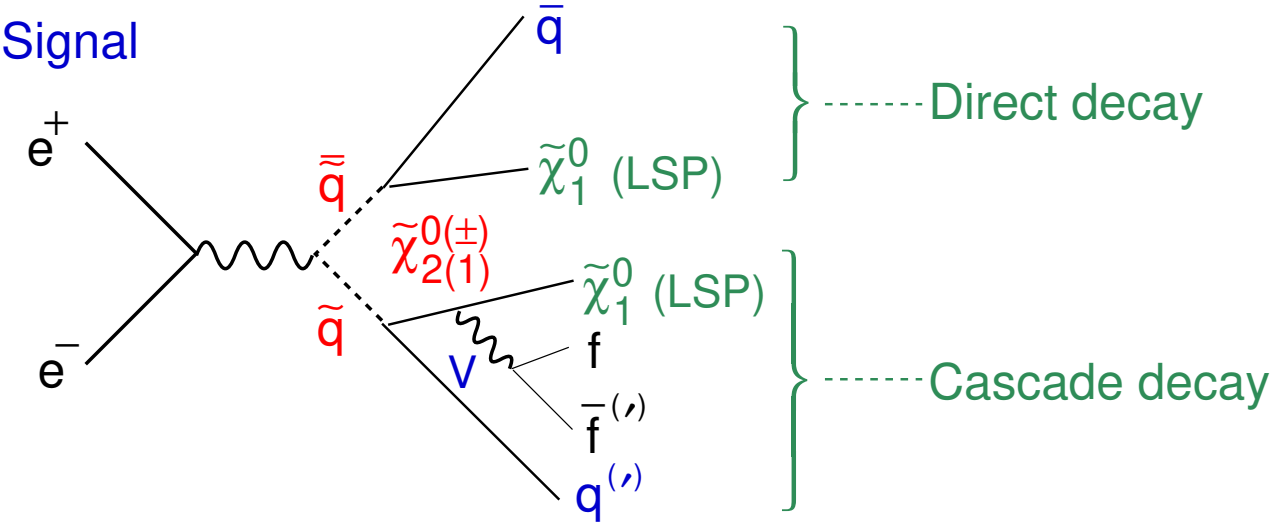
## Test of Representation Independence




Once the GUT relation is confirmed and the universal scalar mass hypothesis is verified in the slepton sector, we can set, with confidence, the next target energy for $\tilde{q}$ pair production !

## $\tilde{q}_G$ Study (G=1,2)

### Signal




⇢ Acoplanar 2 jets (+ leptons / V=W/Z)

### BG

WW
eνW
ννZ
$\tilde{\chi}^+_1\tilde{\chi}^-_1$
$\tilde{\chi}^0_i\tilde{\chi}^0_j$

⇢ Negligible for $m_{q\bar{q}} > 100$ GeV, $\not{P}_T > 35$ GeV, $\theta_A > 30°$

$m_{q\bar{q}} = m_{W^{(*)}/Z^{(*)}} < 100$ GeV !

⇢ We can use both $e^-_L$ and $e^-_R$ beams !

### Chirality Selection

Much better than the slepton case !

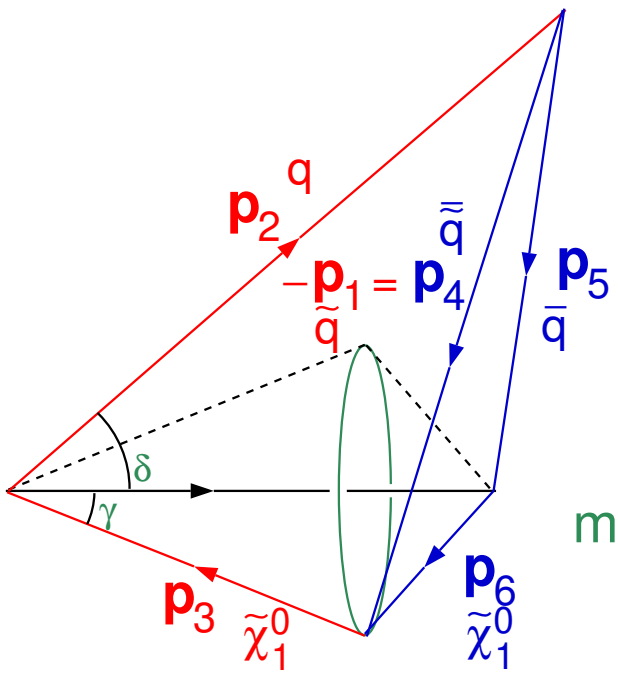
$$\sigma_R(\tilde{q}_R\bar{\tilde{q}}_R) : \sigma_R(\tilde{q}_L\bar{\tilde{q}}_L) \cong \sigma_L(\tilde{q}_L\bar{\tilde{q}}_L) : \sigma_L(\tilde{q}_R\bar{\tilde{q}}_R) \cong 10 : 1$$

if $m_{\tilde{q}_R} \cong m_{\tilde{q}_L}$

# Mass Determination

We can use the endpoint method as with $\tilde{l}^{\pm}$ and $\tilde{\chi}_1^{\pm}$

Better quantity proposed by Feng-Finnell




$m_{\tilde{\chi}_1^0}$ : known

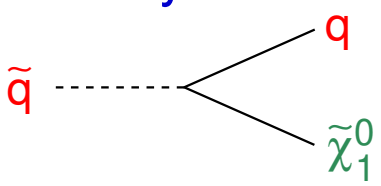
## Minimal Kinematically Allowed Squark Mass

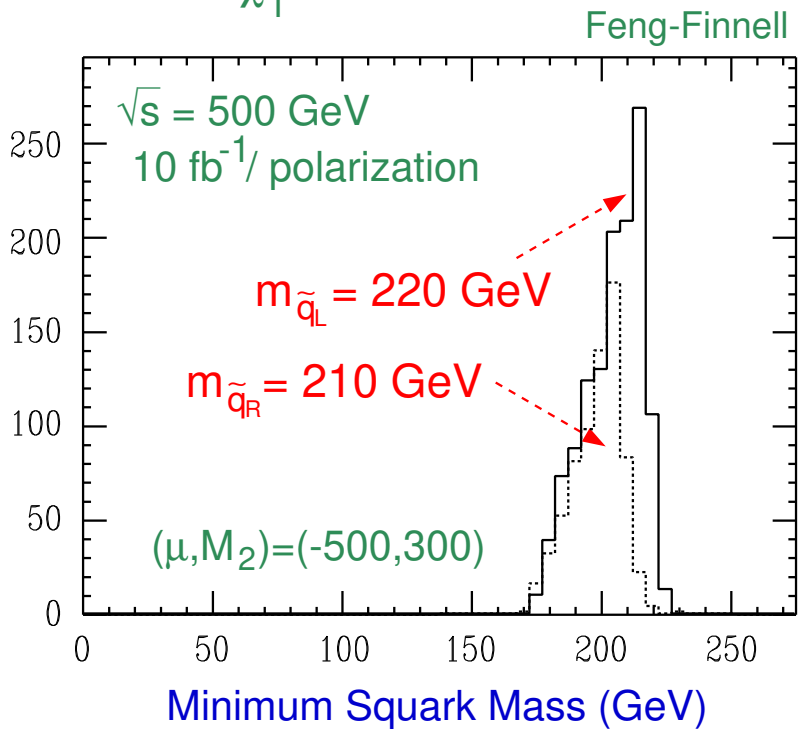
$$(m_{\tilde{q}}^{min})^2 = E_{BM}^2 - |\mathbf{p}_1|_{max}^2$$
$$= E_{BM}^2 - |\mathbf{p}_2|^2 - |\mathbf{p}_3|^2 + 2\,|\mathbf{p}_2| \cdot |\mathbf{p}_3| \cos(\delta + \gamma)$$

Use the information contained in

the relative configuration of q and $\bar{q}$ !

## Direct Decays

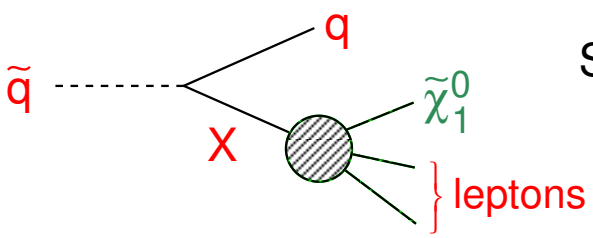



$\Delta m_{\tilde{q}} \cong$ 1 GeV at 95% CL

if $m_{\tilde{\chi}_1^0}$ : known

## Cascade Decays




Select

2 q-jets + leptons

$m_X \cong m_{\tilde{\chi}_1^\pm} \cong m_{\tilde{\chi}_2^0}$ ⟶ Replace $\tilde{\chi}_1^0$ by X = $\tilde{\chi}_1^\pm$/ $\tilde{\chi}_2^0$

$m_{\tilde{q}}^{min}$ ⟶ $\Delta m_{\tilde{q}} \cong$ 2.4 GeV at 95% CL

$(\mu, M_2)$=(-500,200)

if $m_{\tilde{\chi}_1^\pm} \cong m_{\tilde{\chi}_2^0}$ : known

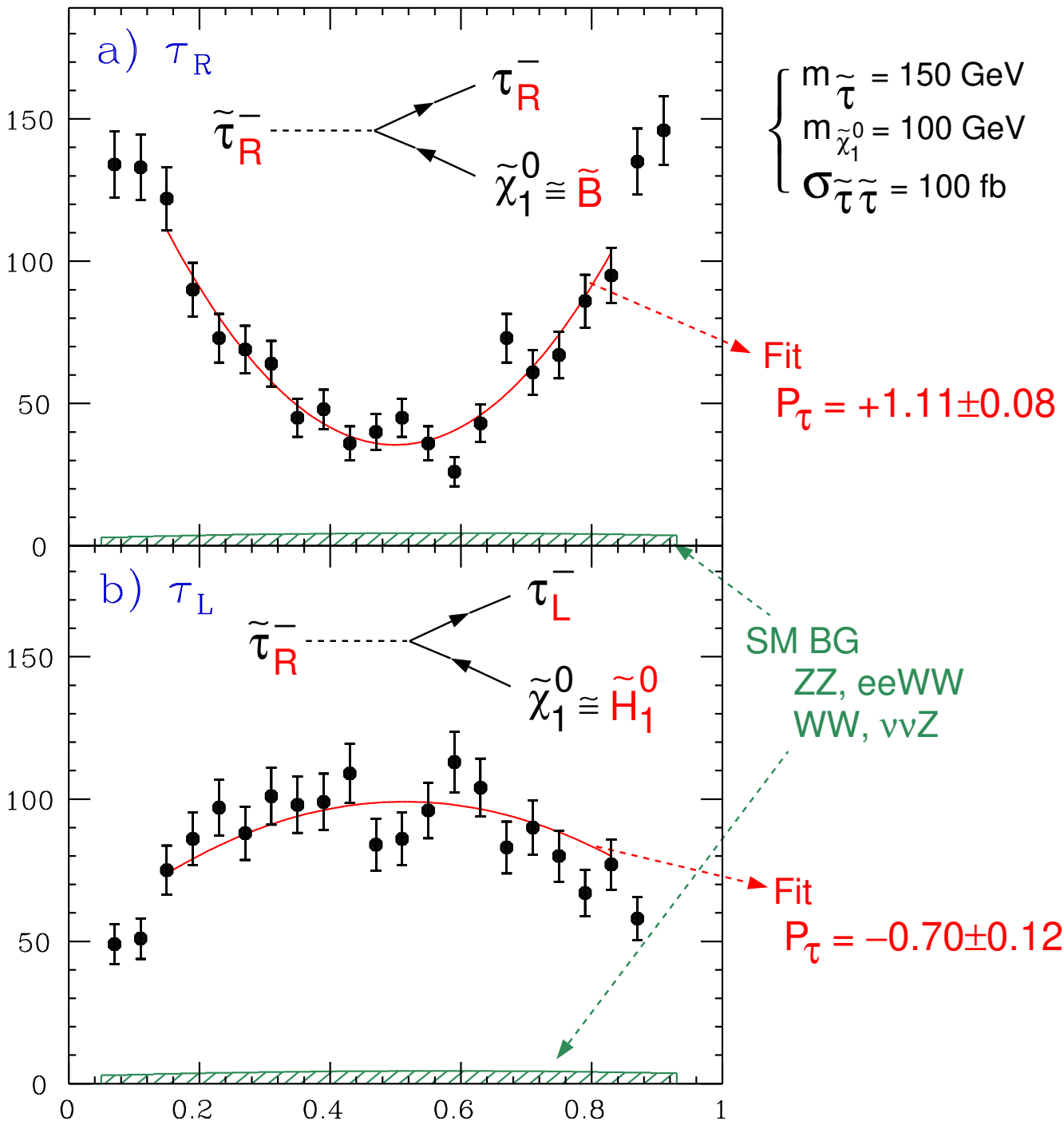
# SUSY Study Scenario; 3rd Generation

## $\tilde{\tau}^{\pm}$ Study

Ref. Nojiri's Talk

### Measurement of $P_\tau$

$\sqrt{s}$ = 500 GeV  Pol $e^-$ = +0.95  100 fb$^{-1}$



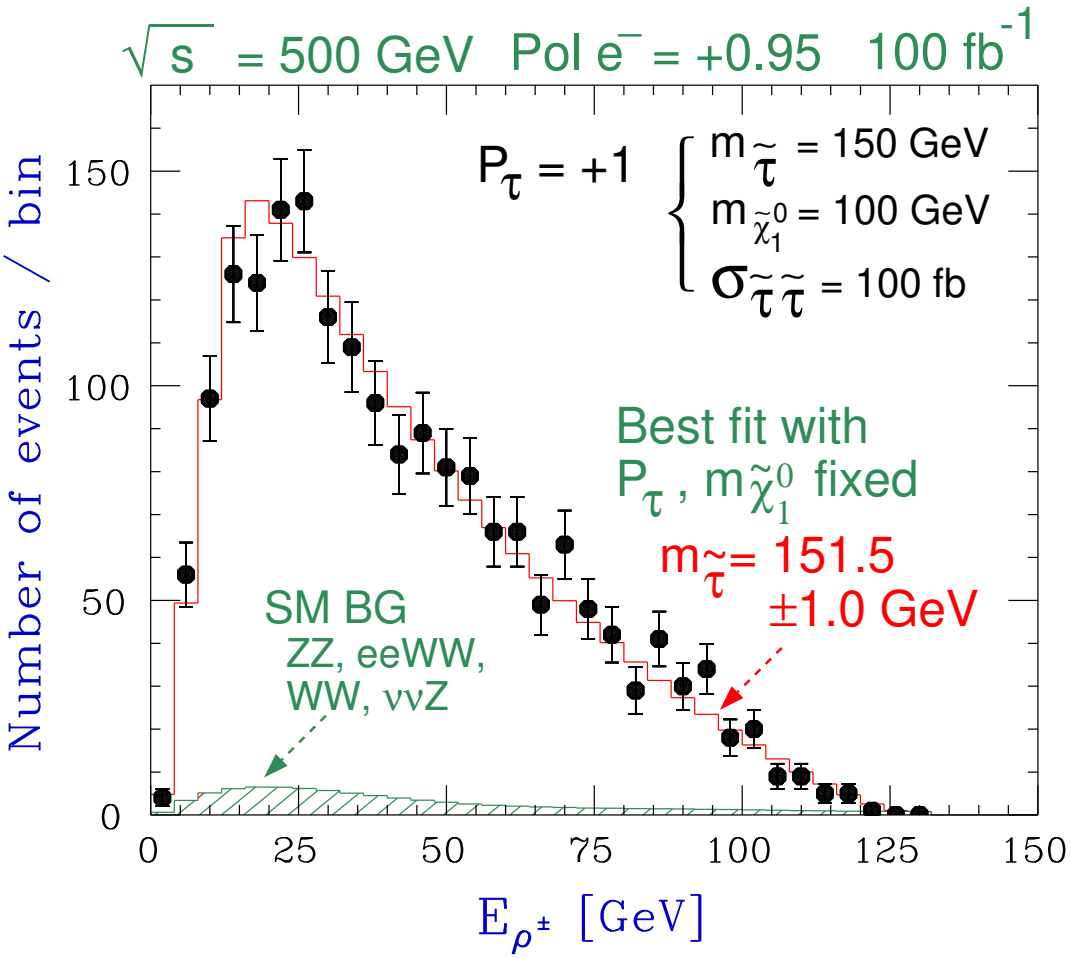

$m_{\tilde{\tau}}$ = 150 GeV
$m_{\tilde{\chi}^0_1}$ = 100 GeV
$\sigma_{\tilde{\tau}\tilde{\tau}}$ = 100 fb

$\Delta P_\tau \cong 0.1$ possible with 100 fb$^{-1}$

# Mass Determination

Can't measure $E_\tau$ because of a missing $\nu$, however,

$m_{\tilde{\chi}_1^0}$ from $\tilde{e}_R^\pm$ and $\tilde{\mu}_R^\pm$ studies
$E_{\rho^\pm}$ distribution
} ⇢ $m_{\tilde{\tau}}$




In practice, the errors on $P_\tau$ and $m_{\tilde{\chi}_1^0}$ introduces additional errors, but

$\Delta m_{\tilde{\tau}} / m_{\tilde{\tau}} \lesssim 1\%$ possible with 100 fb$^{-1}$

# Determination of Mixing Angle

Staus can mix:

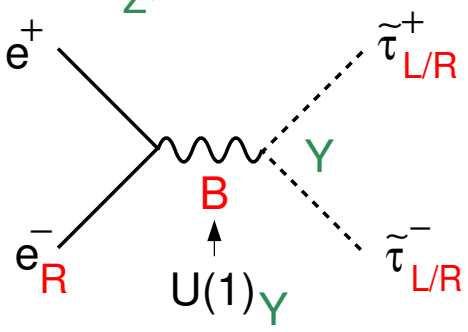
$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_\tau & \sin\theta_\tau \\ -\sin\theta_\tau & \cos\theta_\tau \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}$$

through the off-diagonal element of the stau mass marix:

$$-m_\tau(A_\tau + \mu \tan\beta)$$

$\longrightarrow$ $m_{\tilde{\tau}_1}$ and $\sigma_{\tilde{\tau}_1^+\tilde{\tau}_1^-}$ : functions of $\theta_\tau$

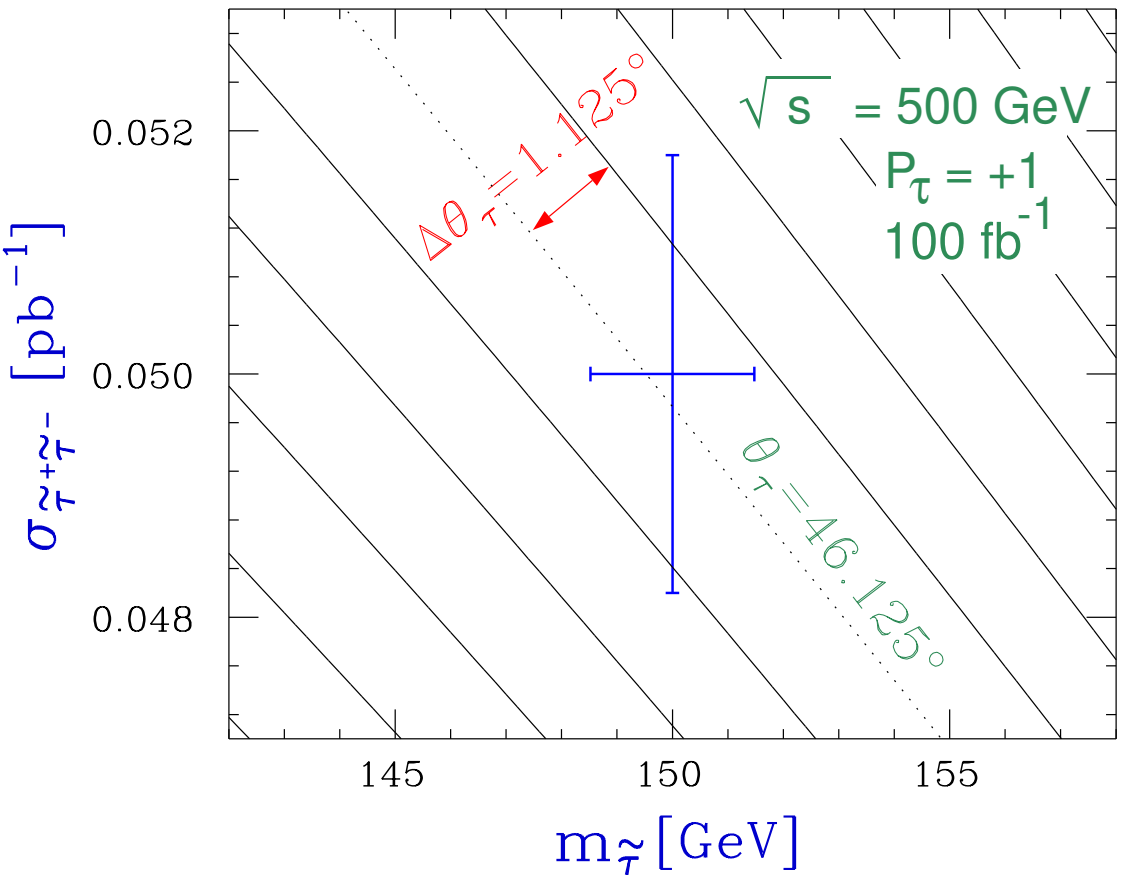
If $\sqrt{s} \gg m_Z$,




$$\begin{cases} Y_L = -1/2 \\ Y_R = -1 \end{cases} \qquad \sigma_R = 4\,\sigma_L$$

$$\sigma_{\tilde{\tau}_1^+\tilde{\tau}_1^-} = (\cos^2\theta_\tau + 2\sin^2\theta_\tau)^2 \sigma_L$$

## Expected Precision



# Stop Search and Study

Stops can mix:

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_t & \sin\theta_t \\ -\sin\theta_t & \cos\theta_t \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$

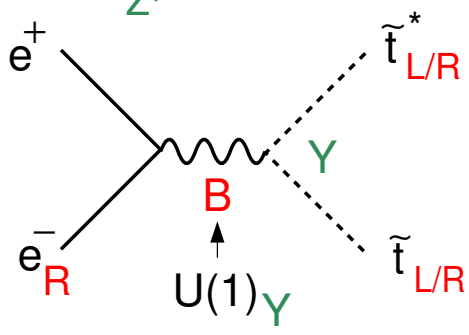
through the off-diagonal element of the stop mass marix:

$$-m_t (A_t + \mu \cot\beta)$$

$\longrightarrow$ $m_{\tilde{t}_1}$ and $\sigma_{\tilde{t}_1 \tilde{t}_1^*}$ : functions of $\theta_t$

## Production

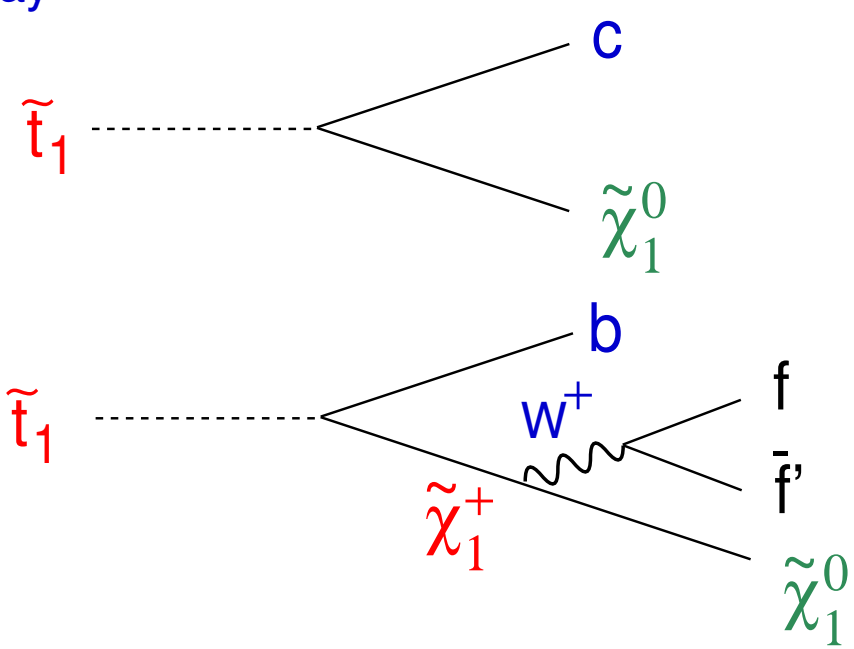
If $\sqrt{s} \gg m_Z$,




$$\begin{cases} Y_L = 1/6 \\ Y_R = 2/3 \end{cases} \qquad \sigma_R = 16\,\sigma_L$$

$$\sigma_{\tilde{t}_1 \tilde{t}_1^*} = (\cos^2\theta_t + 4\sin^2\theta_t)^2 \sigma_L$$

## Decay



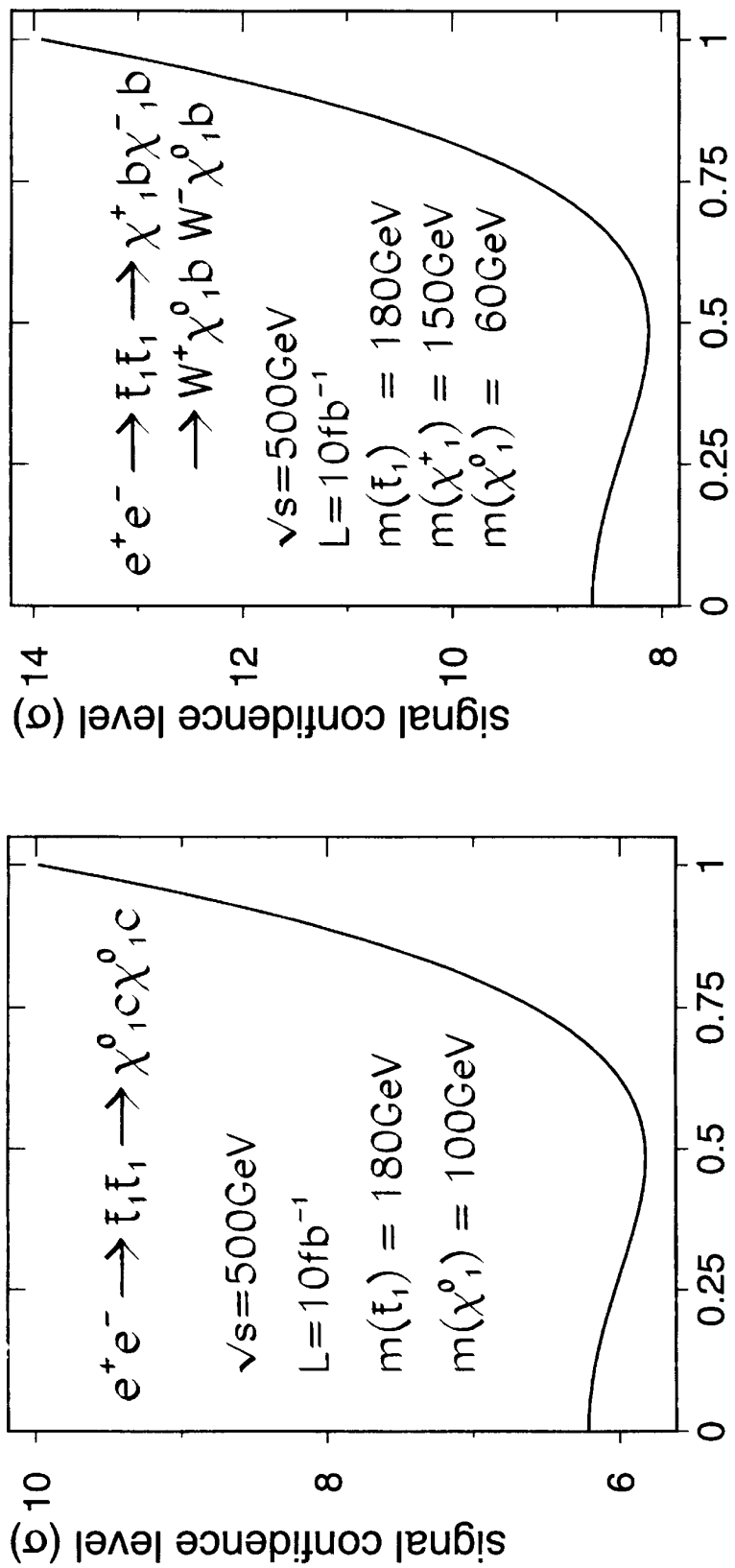
# Event Selection

By A.Spczak

By A.Sopczak

Complet

Expected Sensitivity




| | | |

# If Only LSP Is Available

Is single photon experiment possible ?

## General Belief

Abe-Kon-KF
with GRACE

BG Mask and Final Quads

Huge SM BG

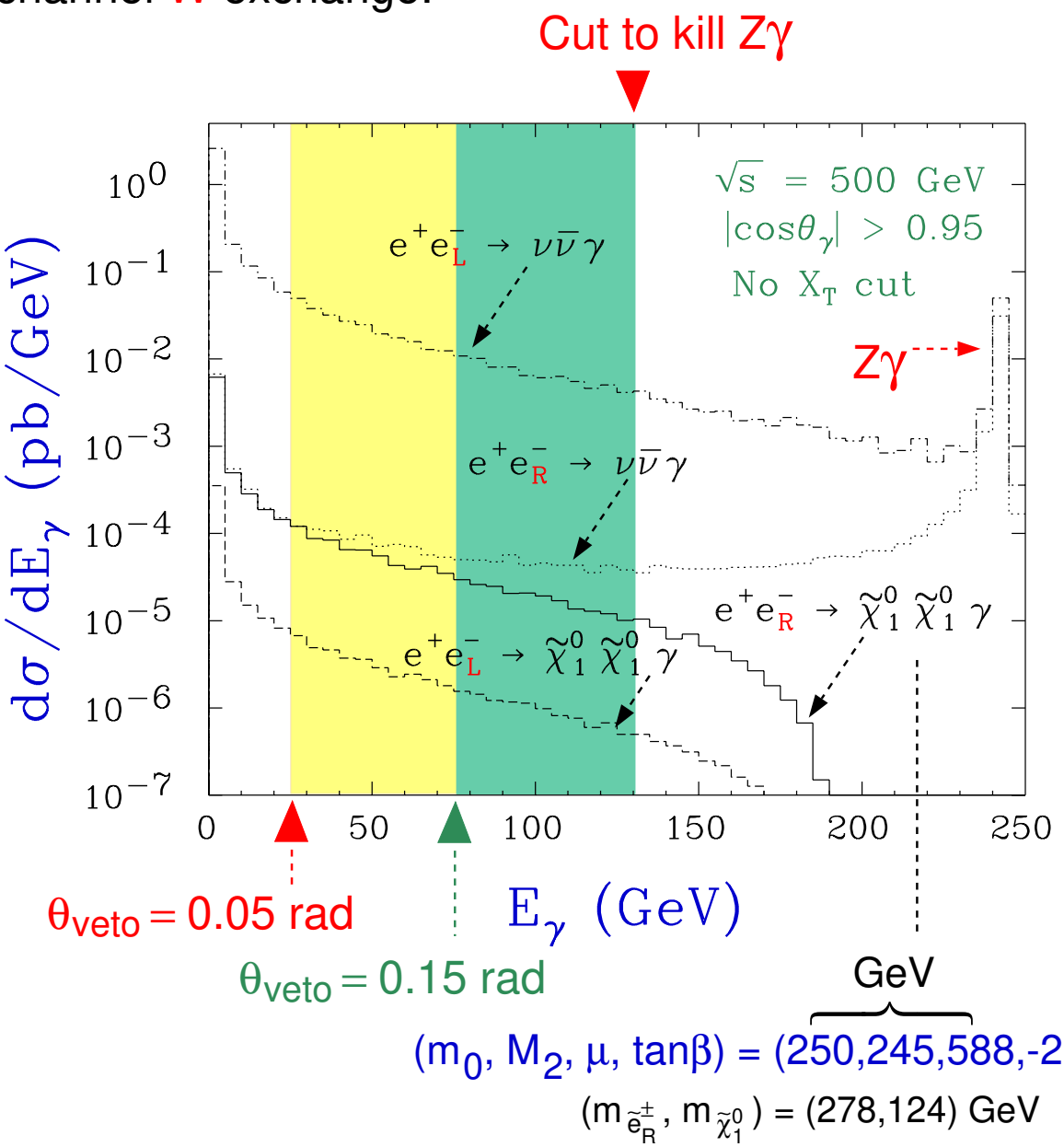
Hard to get $\theta_{veto}$ small !

$$e^+e^- \rightarrow \nu\bar{\nu}\gamma$$

Very difficult if not impossible !

## Possible Way Out

Polarized Beam to kill BG, which mainly comes from t-channel W exchange.




GeV

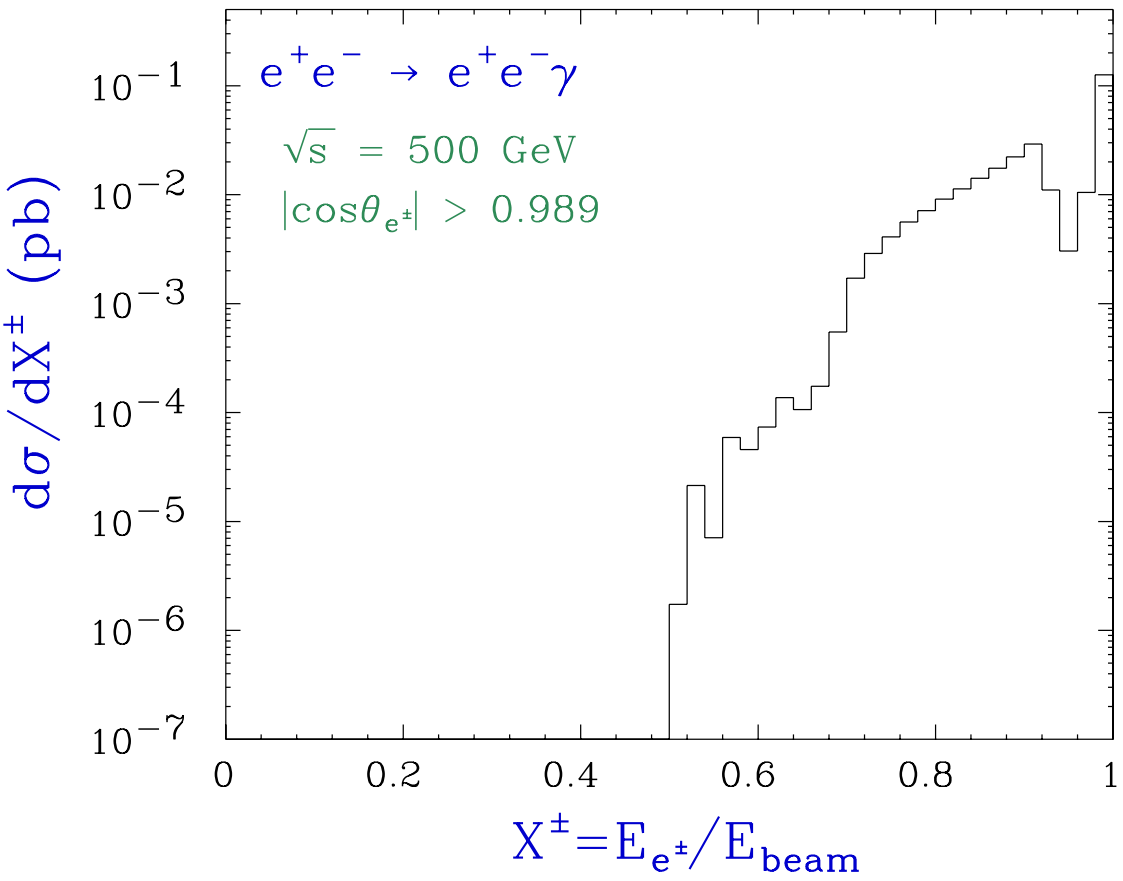
$(m_0, M_2, \mu, \tan\beta) = (250, 245, 588, -2)$

$(m_{\tilde{e}_R^\pm}, m_{\tilde{\chi}_1^0}) = (278, 124)$ GeV

# Energy of Electron/Positron to Veto

Need to put a veto counter in the beam mask

Is it possible ?




Energy threshold can be pretty high!

----------→ Seems feasible !

Let's assume that we can hopefully go down to

$\theta_{veto}$ = 50 mrad

and see how S/B ratio improves!

# Signal to BG Ratio

## Sample Parameter Set

$(m_0, M_2, \mu, \tan\beta) = (\overbrace{250,245,588}^{\text{GeV}},-2)$

$\dashrightarrow (m_{\tilde{e}_R^{\pm}}, m_{\tilde{\chi}_1^0}) = (278,124)$ GeV

$\sqrt{s} = 500$ GeV $\quad |\cos\theta_\gamma| < 0.95$

## Effective Cross Sections

$P_e = +0.95$

| | $\nu\bar{\nu}\gamma$ | $\tilde{\chi}_1^0\,\tilde{\chi}_1^0\,\gamma$ | S/N |
|---|---|---|---|
| $X_T > 0.3$, $\theta_{veto} = 150$ mrad | 180 fb | 0.57 fb | 0.3% |
| $X_T > 0.1$, $\theta_{veto} = 50$ mrad | 209 | 3.06 | 1.5% |
| $X_T > 0.1$, $E_\gamma < 130$ GeV | 31.7 | 2.8 | 8.8% |

If a good polarized positron beam available

$P_{e^-} = +0.95 \otimes P_{e^+} = -0.8 \dashrightarrow P_e = +0.995$

| | $\nu\bar{\nu}\gamma$ | $\tilde{\chi}_1^0\,\tilde{\chi}_1^0\,\gamma$ | S/N |
|---|---|---|---|
| $X_T > 0.1$, $E_\gamma < 130$ GeV | 7.9 | 2.9 | 37.% |

# Summary

## Pol $e^-$ Beam

Powerful tool to

i) suppress SM BG and enhance SIGNAL !

ii) sort out sparticle mixings !

## Precision Measurements

$$\dashrightarrow \begin{cases} \Delta m/m \lesssim O(1\%) \text{ !} \\ d\sigma/d\Omega \end{cases}$$

$\therefore$ We can learn a lot from the FSP alone !

$\dashrightarrow$ Systematic and Consistent Searches and Studies

FSP $\dashrightarrow$ NSP $\dashrightarrow$ ..... $\dashrightarrow$ $N^nSP$

S-parameters

$\dashrightarrow$ $\mathcal{L}_{\text{S-breaking}} \dashrightarrow \mathcal{L}_{\text{Planck?}}$ !

First realistic step toward GUT/Planck scale physics !