Introduction

Testing Supersymmetric Theories at Linear e^+e^- Colliders

Keisuke FUJII : KEK, Japan

∃ Light H^0 ⇢ Probably SUSY

Not necessarily

⇢ Need to find at least one SUSY particle!

Once a SUSY particle is found.....

It is only the beginning!

True excitements come after this!

Introduction

Precision Study Phase

SUSY Study Scenario

A Sample Case

$\mathcal{L}_{World} = \mathcal{L}_{SUSY} + \mathcal{L}_{S\text{-breaking}}$

$\mathcal{L}_{Planck}$ or ??

Peskin's Talk

∃ S-particles

Sleptons

Charginos

Squarks

S-parameters

$(m_0, M_2, \mu, \tan\beta)$??

Ref. Feng-Finnell

Masses and Mixings of S-particles

PRD49(1994)2369

Couplings

↑ = Experiments

↓ = Theories

Summary

dσ (Production and Decay)

Quantitative tests of $\mathcal{L}_{SUSY}$

Studies of $\mathcal{L}_{S\text{-breaking}}$

⇢ $\mathcal{L}_{Planck}$?

Mass Relations

Gaugino Masses

GUT Relation

$$M_i = M \cdot \frac{\alpha_i(m_Z)}{\alpha_{GUT}} \quad : i = \begin{cases} 1\ U(1)_Y \\ 2\ SU(2)_L \\ 3\ SU(3)_C \end{cases}$$

$$M_1 : M_2 : M_3 \cong 1 : 2 : 7$$

$\tilde{B}$ $\tilde{W}$ $\tilde{g}$

Sizes of groups

$1 < 2 < 3$

$$m_{\tilde{\chi}_1^\pm} \lesssim \frac{1}{3} m_{\tilde{g}}$$

Lighter Chargino

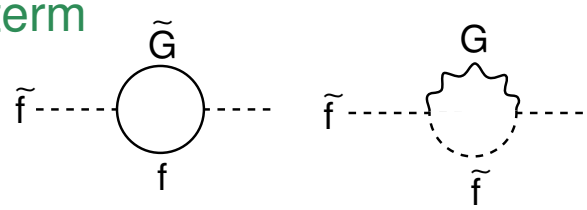
Sfermion Masses

D-term $\lesssim O(1)$

$$m_{\tilde{f}}^2 = m_0^2 + G_{\tilde{f}} \cdot M^2 + D_{\tilde{f}} \cdot m_Z^2$$

Gauge term

Common scalar mass



$$\begin{cases} G_{\tilde{q}} > G_{\tilde{l}} & (3 > 2, 1) \\ G_{\tilde{f}_L} > G_{\tilde{f}_R} & (2 > 1) \end{cases}$$

$m_{colorless} < m_{colored}$

$m_{\tilde{f}_R} < m_{\tilde{f}_L}$

First SUSY Particle (FSP)

$$FSP = \begin{cases} \tilde{\chi}_1^\pm \\ \tilde{l}_R^\pm \end{cases}$$

Take a sample parameter set

Ref. PRD51(1995)3153

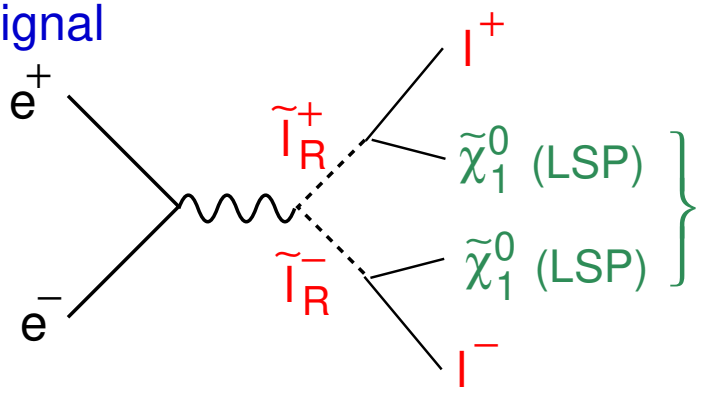
$(m_0, M_2, \mu, \tan\beta) = (70, 250, 400, 2)$ GeV $\leftrightarrow$ FSP = $\tilde{l}_R^\pm$

and see how a typical SUSY study scenario goes.

SUSY Study Scenario; A Sample Case

$\tilde{l}^{\pm}_R$ Study ($l \neq \tau$)

Signal



BG

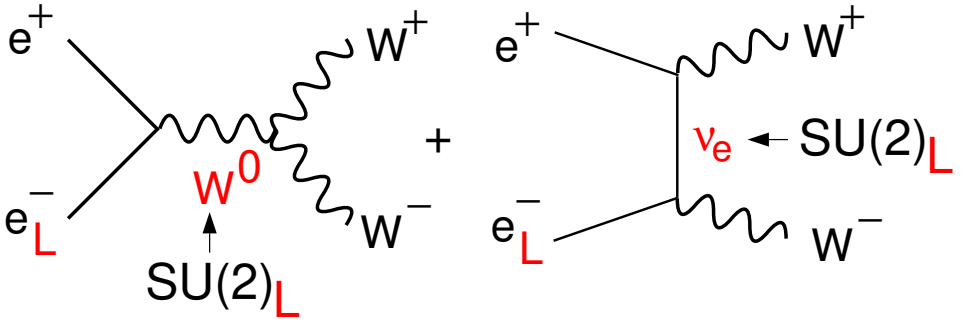
SM processes with ν's which mimic LSP.

Key Point

Polarized e^- beam is VERY POWERFUL!

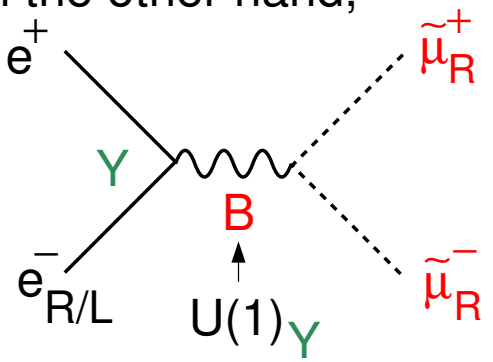
(e.g.) $e^+e^- \rightarrow W^+W^-(W \rightarrow l\nu)$

The LARGEST SM BG



In the symmetry limit, $\sigma_{WW} \rightarrow 0$ for e^-_R !

On the other hand,



$$\begin{cases} Y_L = -1/2 : e^-_L \\ Y_R = -1 \quad : e^-_R \end{cases}$$

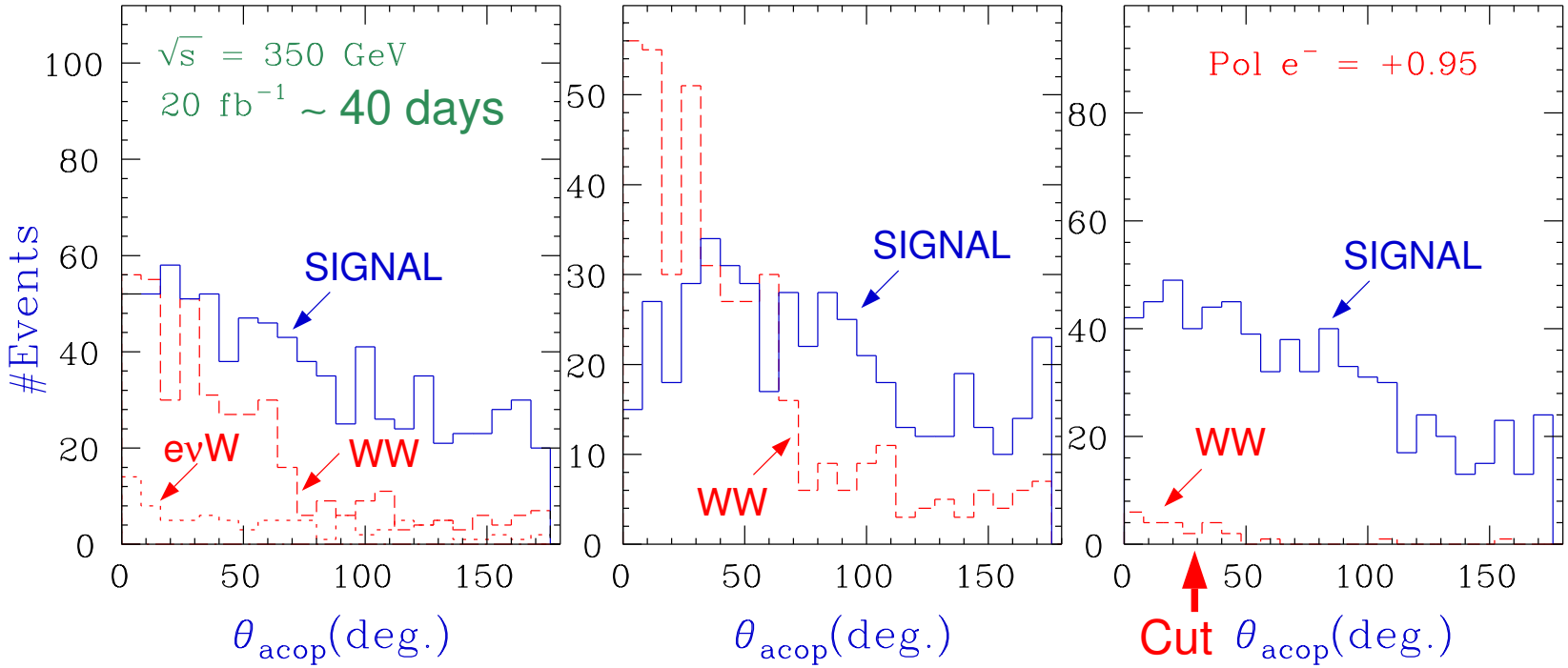
In the symmetry limit, $\sigma_R = 4\,\sigma_L$!

Acoplanarity Distributions

$e^+e^- \to \tilde{e}_R^+ \tilde{e}_R^-$ Pol e^- = 0

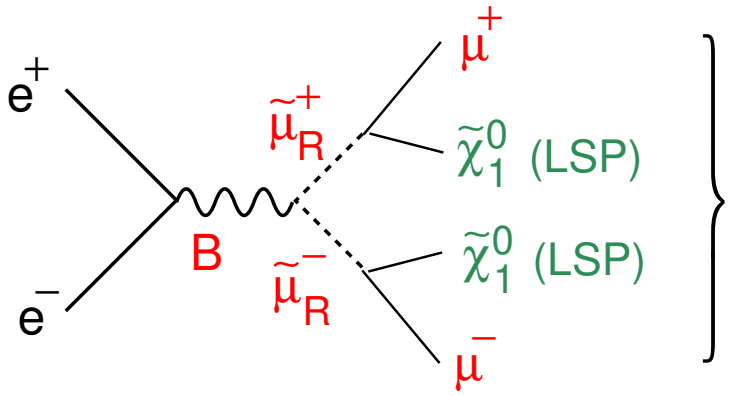
$e^+e^- \to \tilde{\mu}_R^+ \tilde{\mu}_R^-$ Pol e^- = 0

Pol e^- = +0.95 $\cong e_R^-$



Very Clean Sample for Precision Studies!

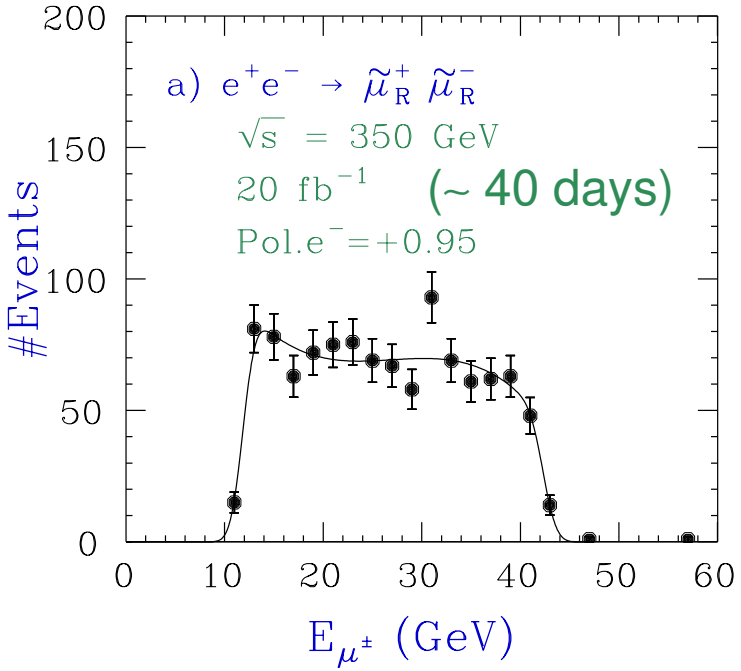
Mass Determination

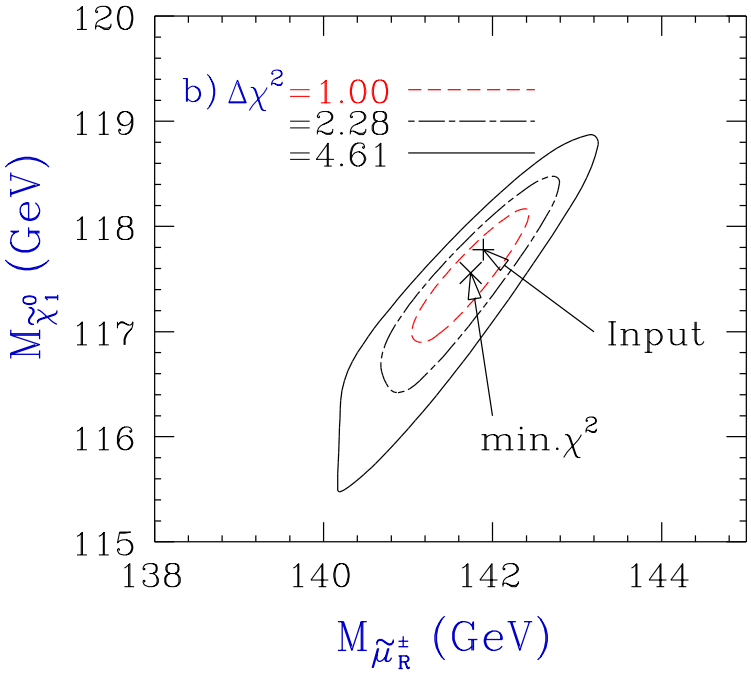


Measure $E_{\mu^{\pm}}$

End points

$\longrightarrow (m_{\tilde{\mu}^{\pm}_R}, m_{\tilde{\chi}^0_1})$





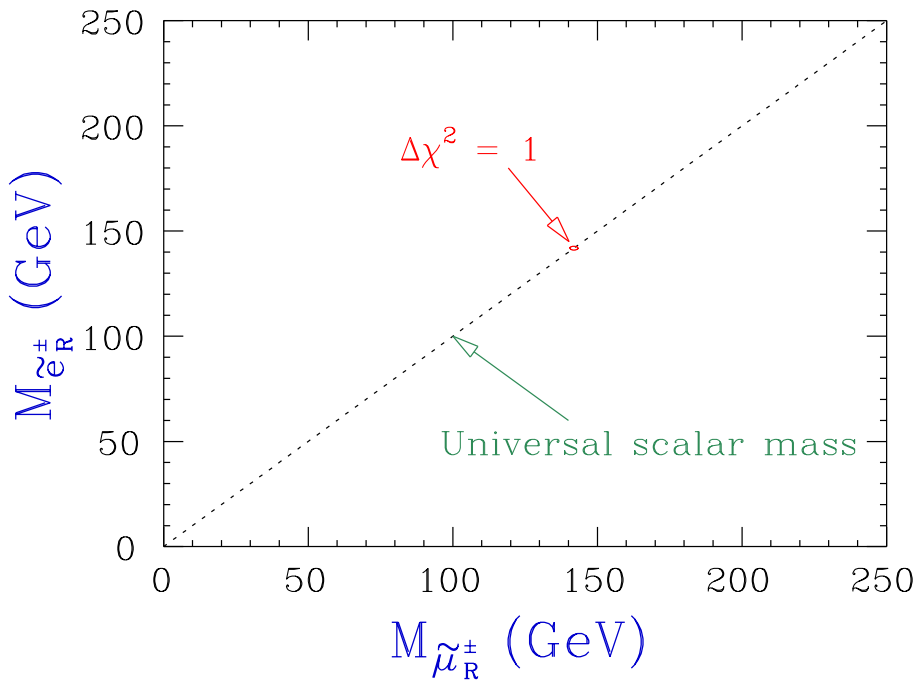
$\longrightarrow \Delta m/m \lesssim 1\%$!

Test of Generation Independence

Hidden Sector SUSY Breaking

----------► Universal Scalar Mass

----------► $m_{\tilde{\mu}_R^\pm} = m_{\tilde{e}_R^\pm}$

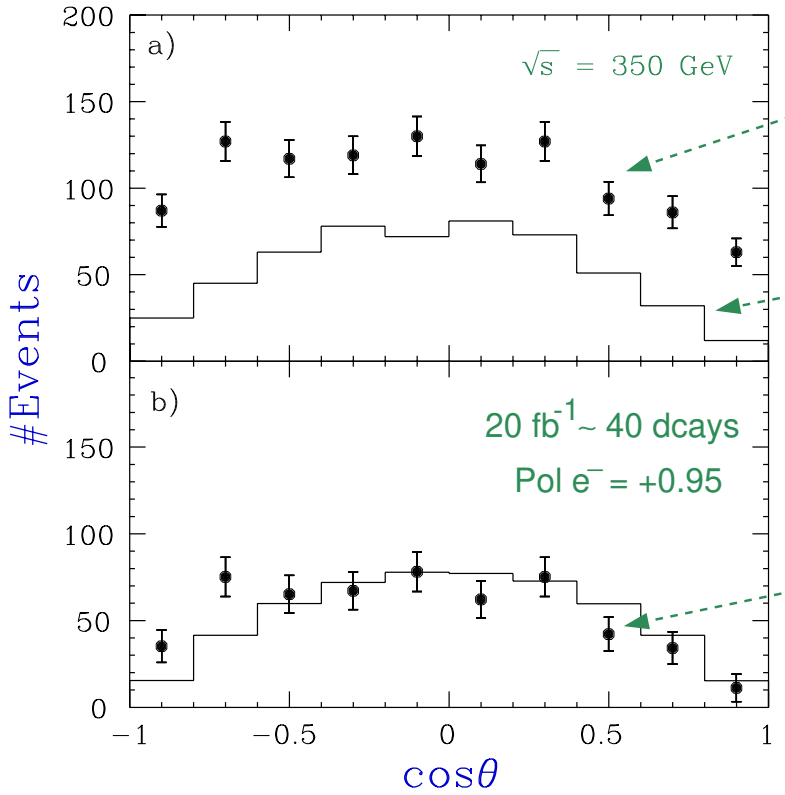


----------► Precision test ($\lesssim 1\%$) of universal scalar mass hypothesis is possible!

Differential Cross Section Measurements

$(m_{\tilde{l}^{\pm}_R}, m_{\tilde{\chi}^0_1})$: known ----► Can solve KINEMATICS up to 2-fold ambiguities!

$e^+e^- \rightarrow \tilde{\mu}^+_R \tilde{\mu}^-_R$



Reconstructed 2 solutions

$\theta_{generated}$ for selected sample

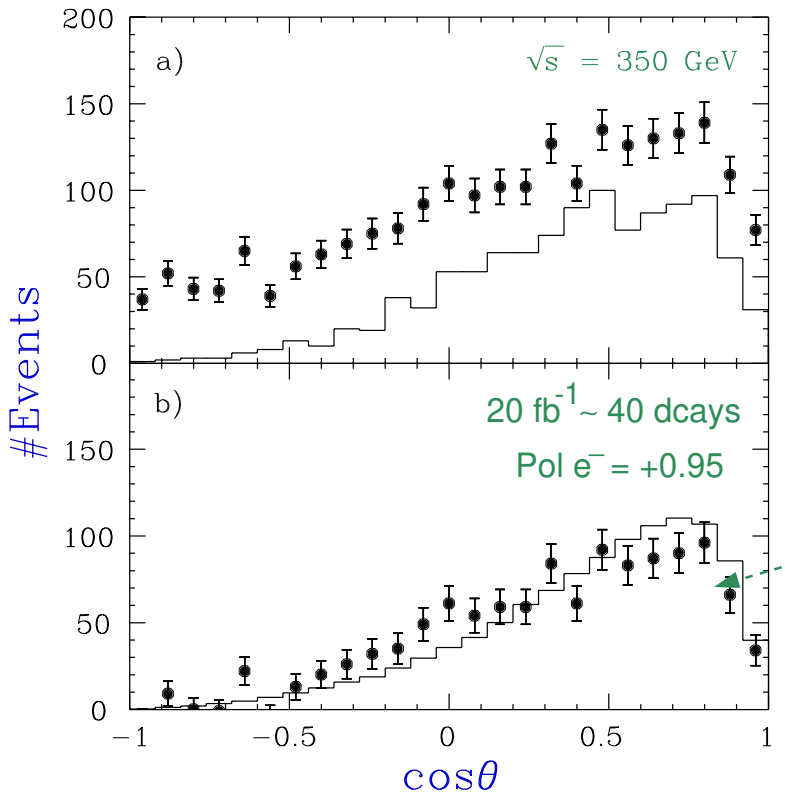
Wrong Solution = Flat BG

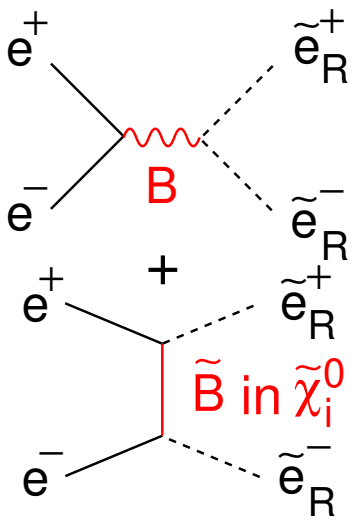
BG subtracted

$\sin^2\theta$

$\therefore J = 0$

$e^+e^- \rightarrow \tilde{e}^+_R \tilde{e}^-_R$





Forward Peak

$\therefore \exists$ t-channel

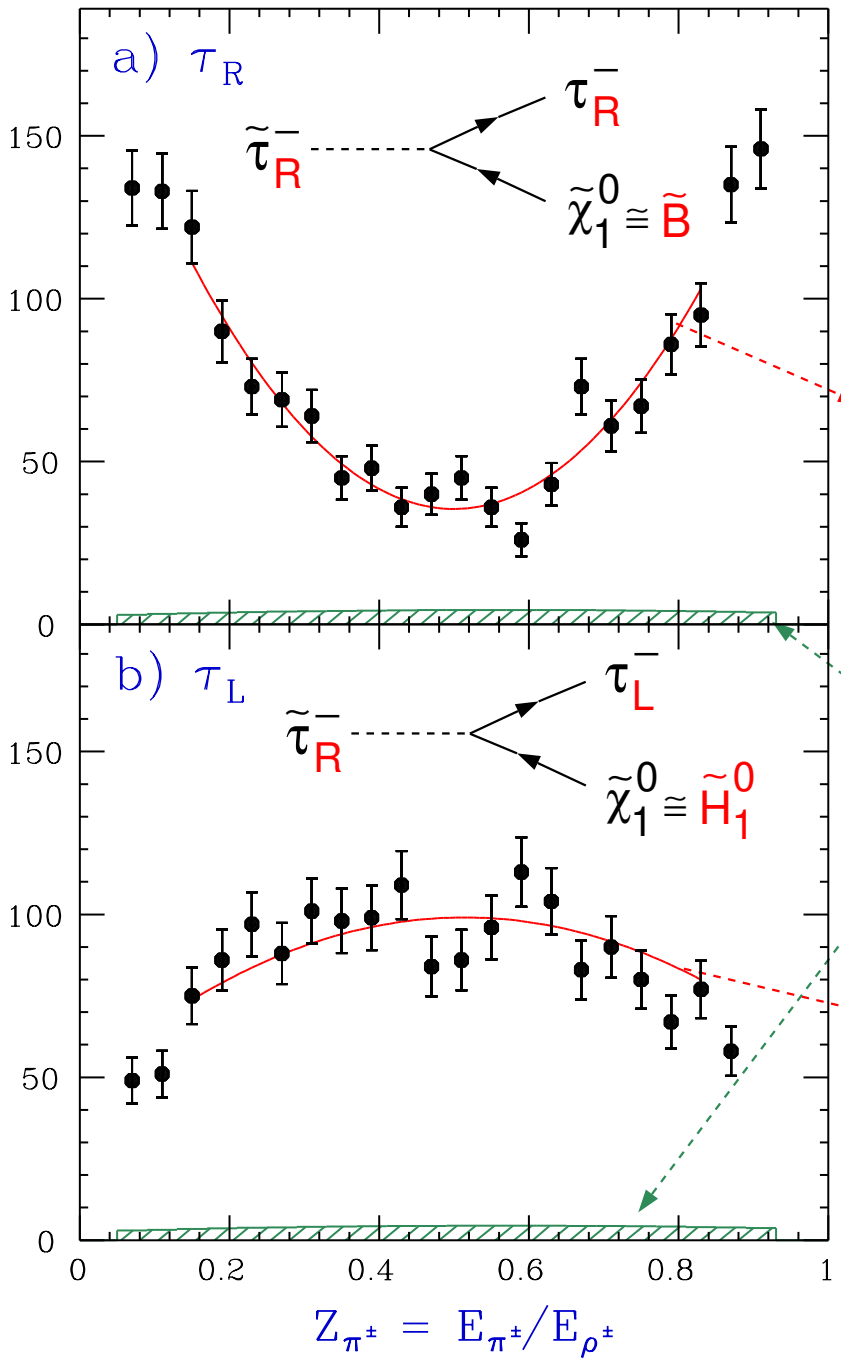
$\tilde{\chi}^0_1 \cong \tilde{B}$

$\tilde{\tau}^{\pm}$ Study

Measurement of P_τ

Nojiri-KF-Tsukamoto
Preliminary

$\sqrt{s}$ = 500 GeV
Pol e^- = +0.95
100 fb^{-1}



$\begin{cases} m_{\tilde{\tau}} = 150 \text{ GeV} \\ m_{\tilde{\chi}_1^0} = 100 \text{ GeV} \\ \sigma_{\tilde{\tau}\tilde{\tau}} = 100 \text{ fb} \end{cases}$

Fit
$P_\tau = +1.11 \pm 0.08$

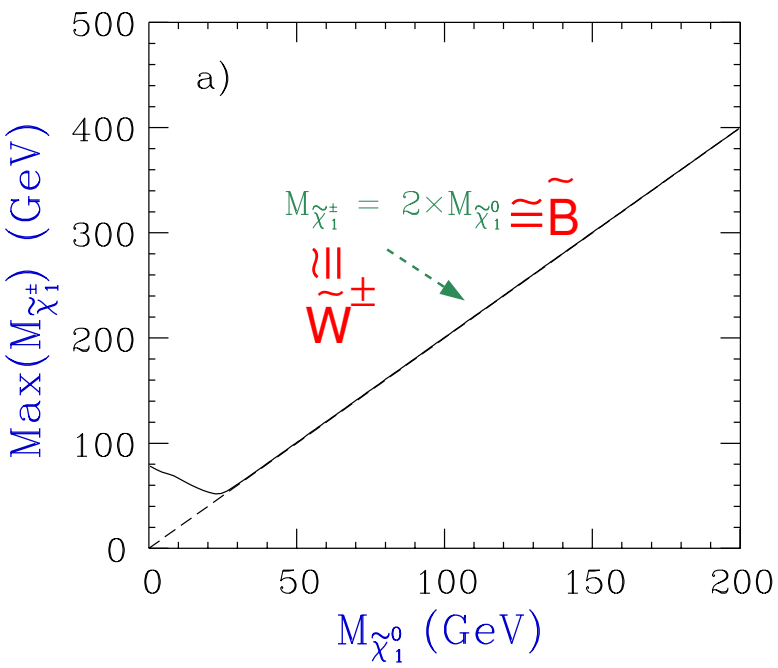
SM BG
ZZ, eeWW
WW, ννZ

Fit
$P_\tau = -0.70 \pm 0.12$

$\Delta P_\tau \cong 0.1$ possible with 100 fb^{-1}

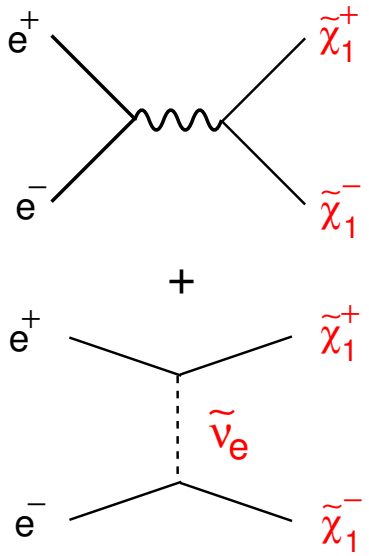
$\tilde{\chi}_1^{\pm}$ Study

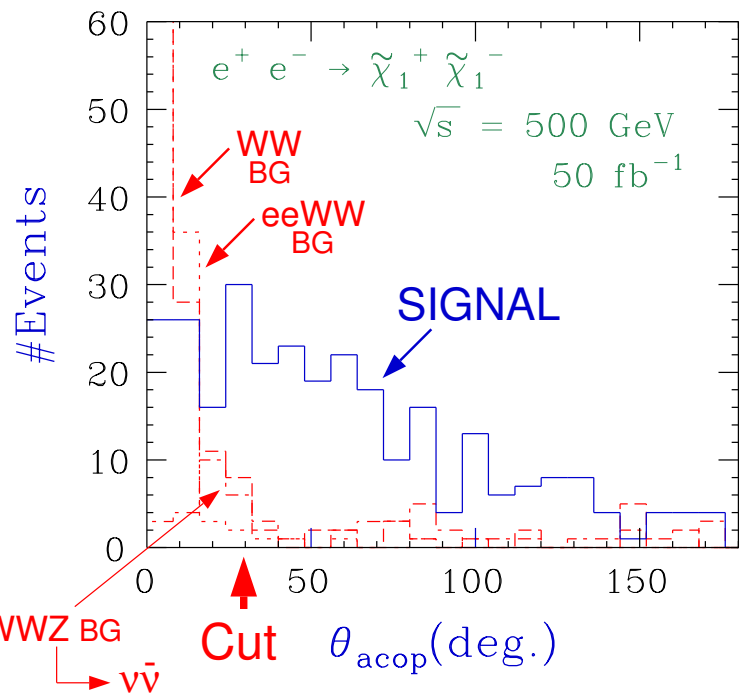
$m_{LSP} = m_{\tilde{\chi}_1^0}$ ($\tilde{l}_R^{\pm}$ Studies) ⟶ Upper limits on $m_{\tilde{\chi}_1^{\pm}}$



⟶ Set $\sqrt{s}$ just above $4\times m_{LSP}$, look for $\tilde{\chi}_1^{\pm}$

$\tilde{\chi}_1^{\pm} \to W^{\pm} \tilde{\chi}_1^0$ ⟶ Signal = Acoplanar W^+W^- → $q\bar{q}'$ / $q\bar{q}'$ 4 Jets

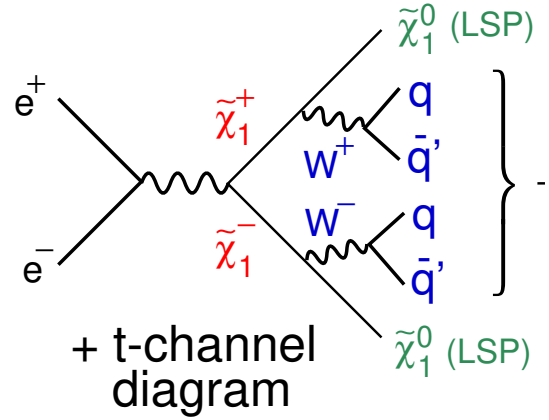




$\nu\nu WW$, $e\nu WZ$, WWZ BG → $\nu\bar{\nu}$

⟶ Fairly Clean Sample for Precision Studies !

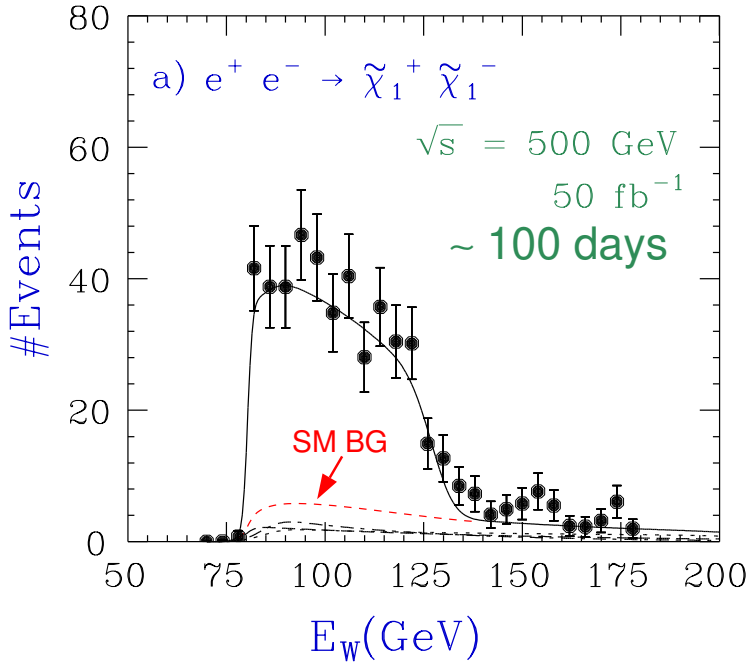
Mass Determination

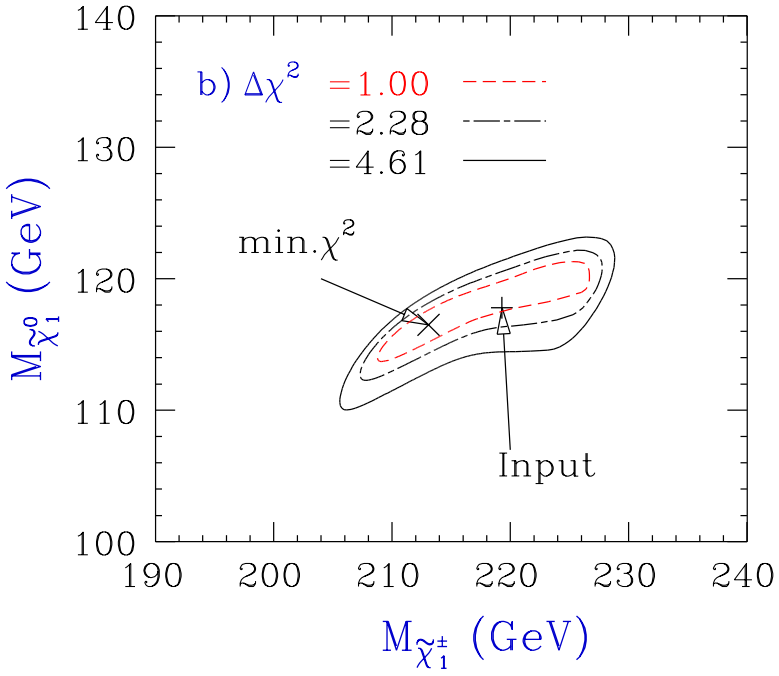


+ t-channel diagram

Measure $E_{W^\pm}$

End points ⟶ $(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_1^0})$

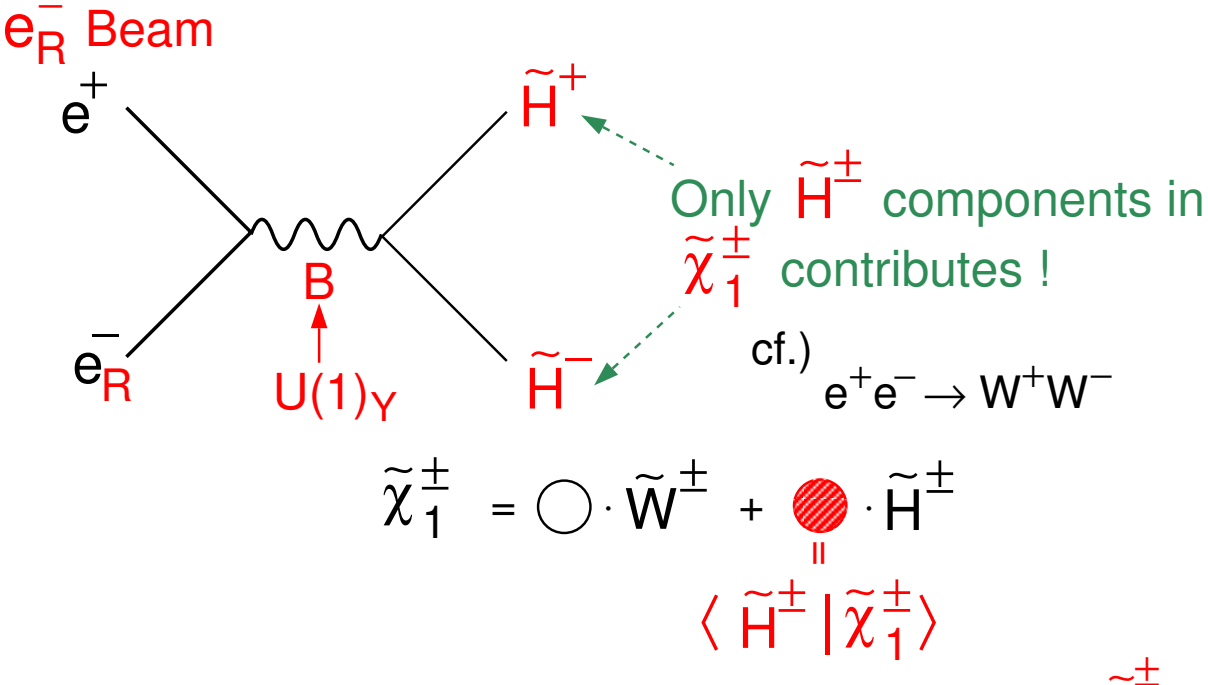




⟶ $\Delta m_{\tilde{\chi}_1^\pm} \cong 8$ GeV ⟶ 5 GeV

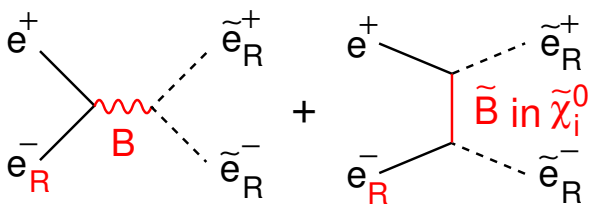
$m_{\tilde{\chi}_1^0}$ from $\tilde{l}_R^\pm$ studies

$\sigma_{\tilde{\chi}_1^+\tilde{\chi}_1^-}$ Measurement



⇢ We can determine the composition of $\tilde{\chi}_1^\pm$!

Global Fit



We know

$m_{\tilde{\chi}_1^0}$ = LSP $\quad m_{\tilde{\chi}_1^\pm}$ $\quad \sigma(e^+e_R^- \to \tilde{e}_R^+\tilde{e}_R^-)$ $\quad \sigma(e^+e_R^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$

Mass Matrices $\quad \langle \tilde{B} | \tilde{\chi}_1^0 \rangle \quad \langle \tilde{H}^\pm | \tilde{\chi}_1^\pm \rangle$

$(M_1, M_2, \mu, \tan\beta)$

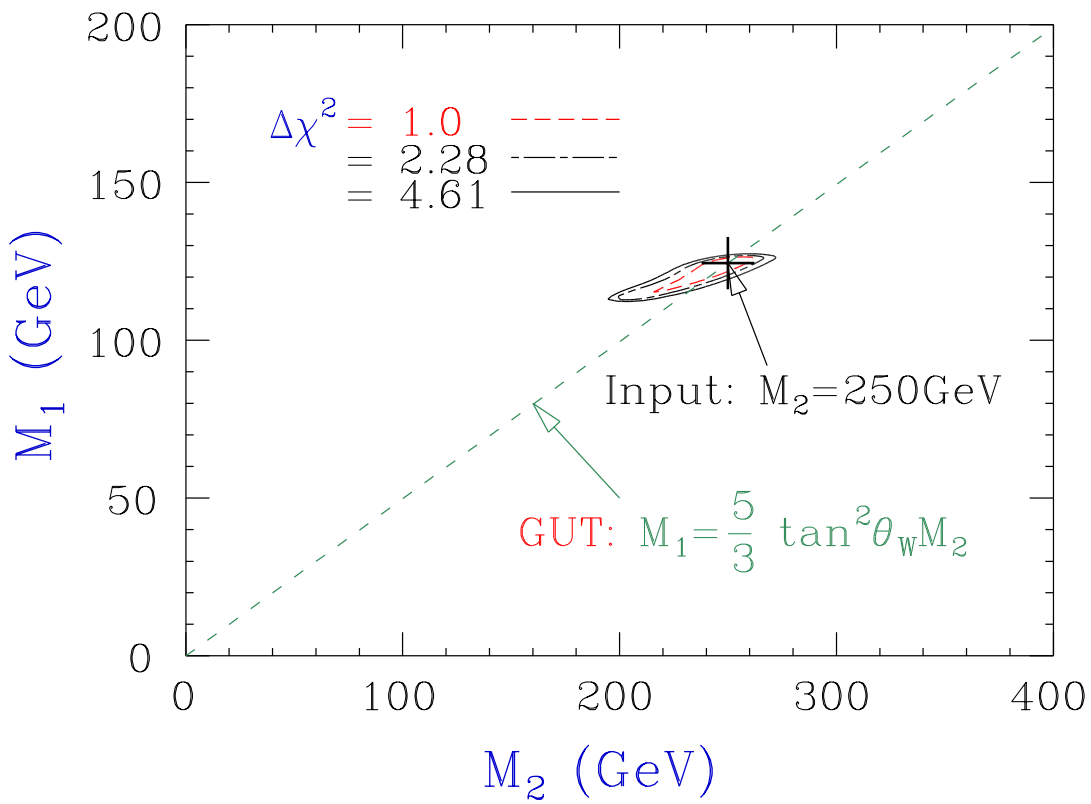
Forget the GUT relation and test if the GUT relation between M_1 and M_2 actually holds !

Test of GUT Relation

Common Gaugino Mass at the GUT scale

GUT $\dashrightarrow$ $\dfrac{M_i}{\alpha_i(m_Z)} = \dfrac{M}{\alpha_{GUT}}$

$i = \begin{cases} 1\ U(1)_Y \\ 2\ SU(2)_L \\ 3\ SU(3)_C \end{cases}$



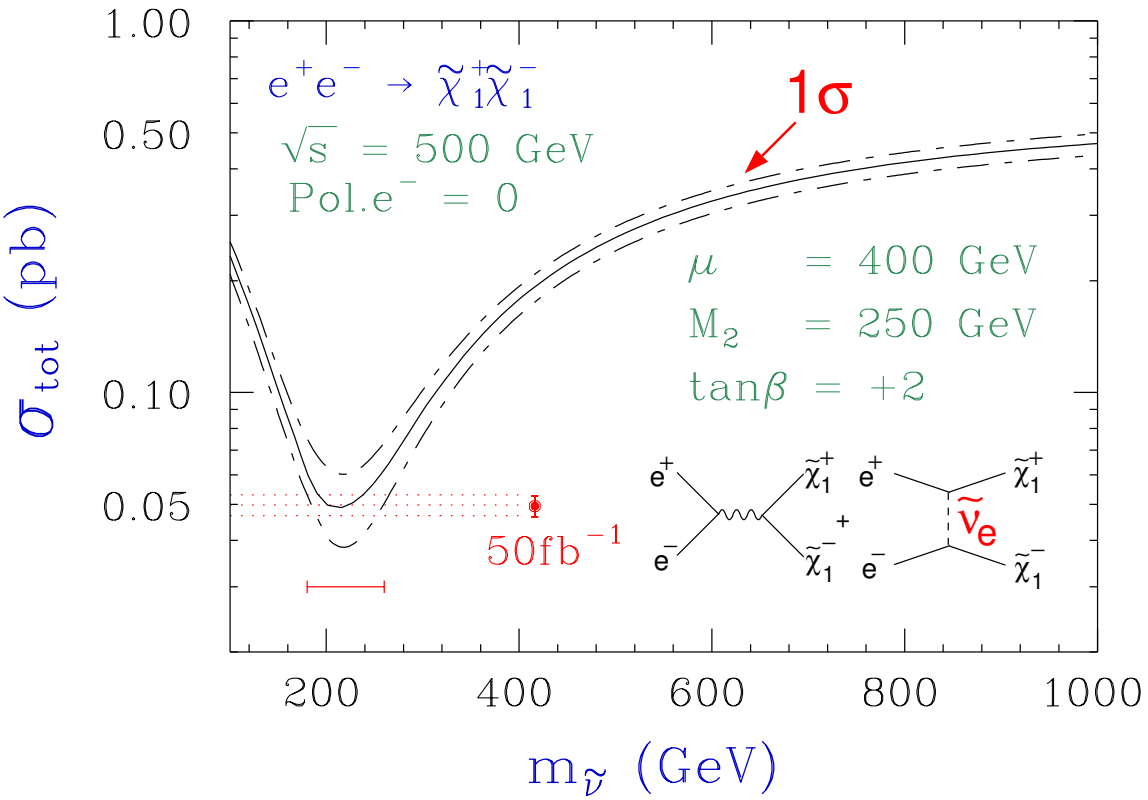
If things turn out this way, we will be forced to believe GUT !

Upper Limit on $m_{\tilde{l}_L^{\pm}}$

Since we know $m_{\tilde{\chi}_1^{\pm}}$ and

$$\tilde{\chi}_1^{\pm} = \bigcirc \cdot \tilde{W}^{\pm} + \bullet \cdot \tilde{H}^{\pm}$$

only UNKNOWN in $\sigma_{\tilde{\chi}_1^+\tilde{\chi}_1^-}$ is $m_{\tilde{\nu}_e}$!



Measure $\sigma_{\tilde{\chi}_1^+\tilde{\chi}_1^-}$ (Pol e^- = 0) to get $m_{\tilde{\nu}_e}$

$$m_{\tilde{e}_L^{\pm}}^2 \leq m_{\tilde{\nu}_e}^2 + 0.77\ m_Z^2$$

We now have an upper limit on $m_{\tilde{e}_L^{\pm}}$, look for $\tilde{e}_L^+$ in $e^+e_R^- \to \tilde{e}_L^+\tilde{e}_R^-$!

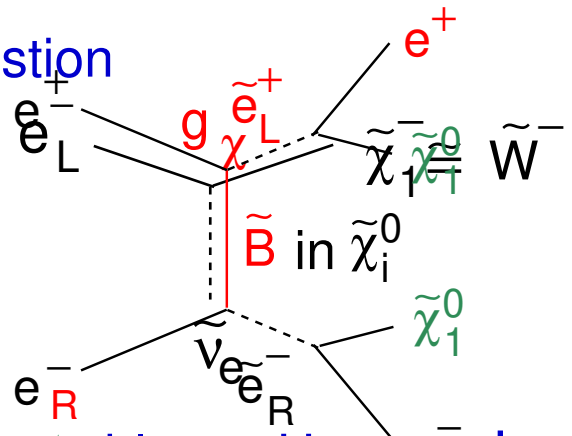
Quantitative Test of SUSY

Ref. Feng-Peskin-Murayama-Tata

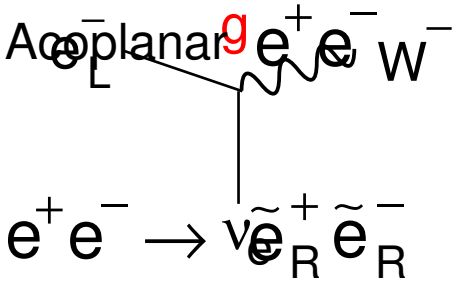
$\tilde{e}^{\pm}_L$ Study

Consider the case in which $\tilde{\chi}^{\pm}_1 \cong \tilde{W}^{\pm}$

Question



Signal



BG

Observables to Use : $d\sigma \cong d\sigma(m_{\tilde{\chi}^{\pm}_1}, m_{\tilde{\nu}_e}, g_{\chi})$

known

To kill SM BG

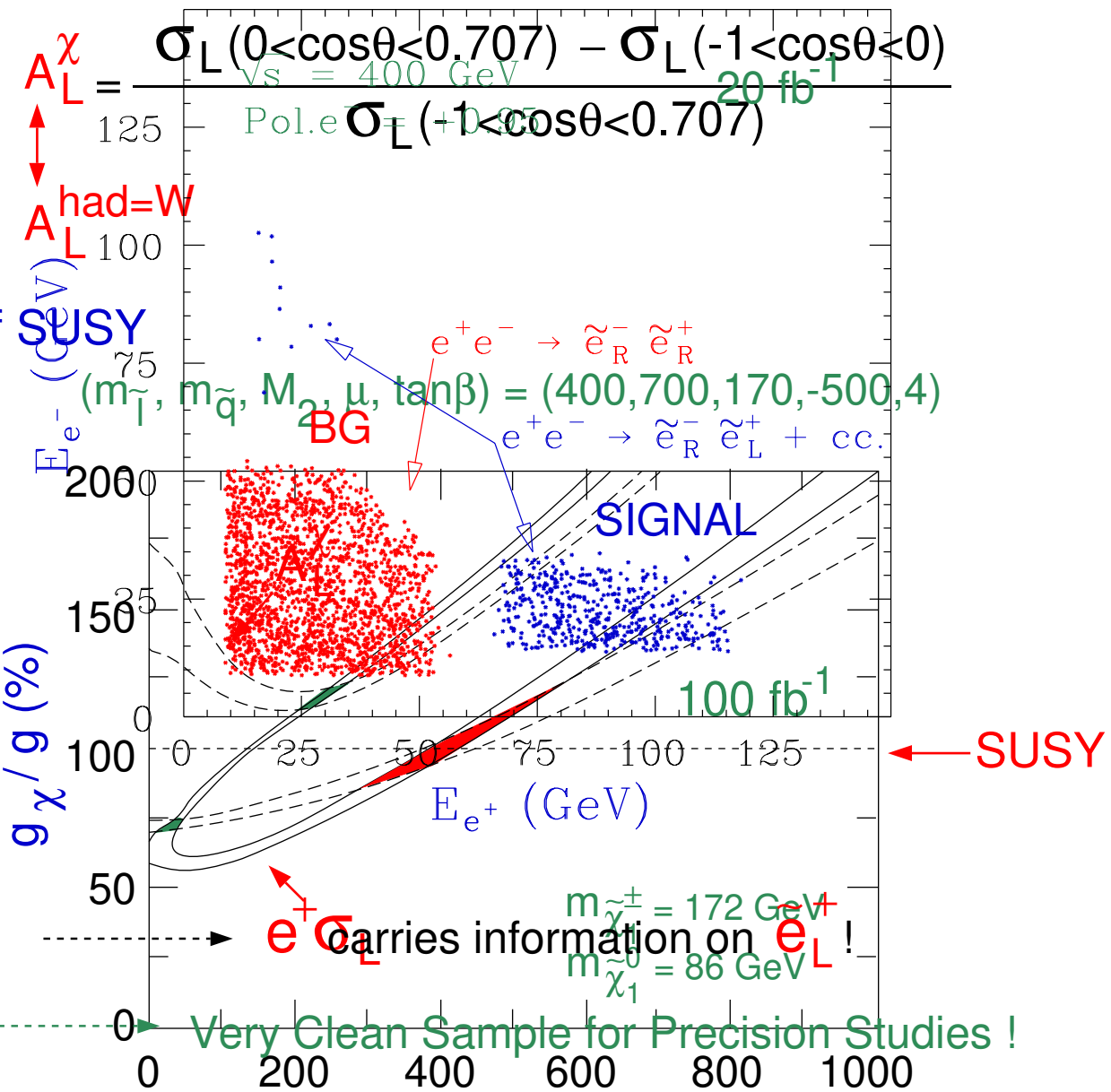
$$\sigma_L = \sigma(e^+e^-_L \to \tilde{\chi}^+_1\tilde{\chi}^-_1)$$

$$A^{\chi}_L = \frac{\sigma_L(0<\cos\theta<0.707) - \sigma_L(-1<\cos\theta<0)}{\sigma_L(-1<\cos\theta<0.707)}$$

$A^{had=W}_L$

Test of SUSY

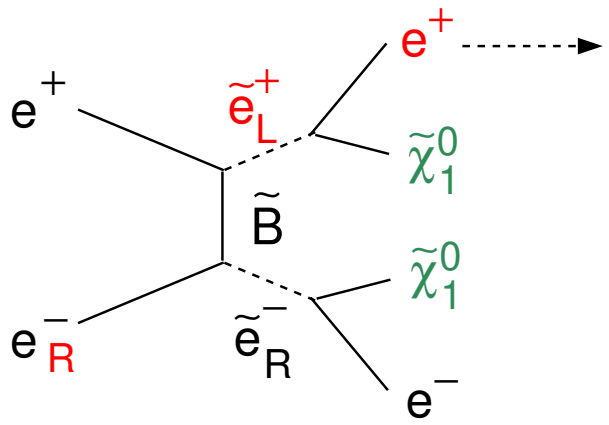
$(m_{\tilde{l}}, m_{\tilde{q}}, M_2, \mu, \tan\beta) = (400,700,170,-500,4)$



$e^+\sigma_L$ carries information on $\tilde{e}_L$!

Very Clean Sample for Precision Studies !

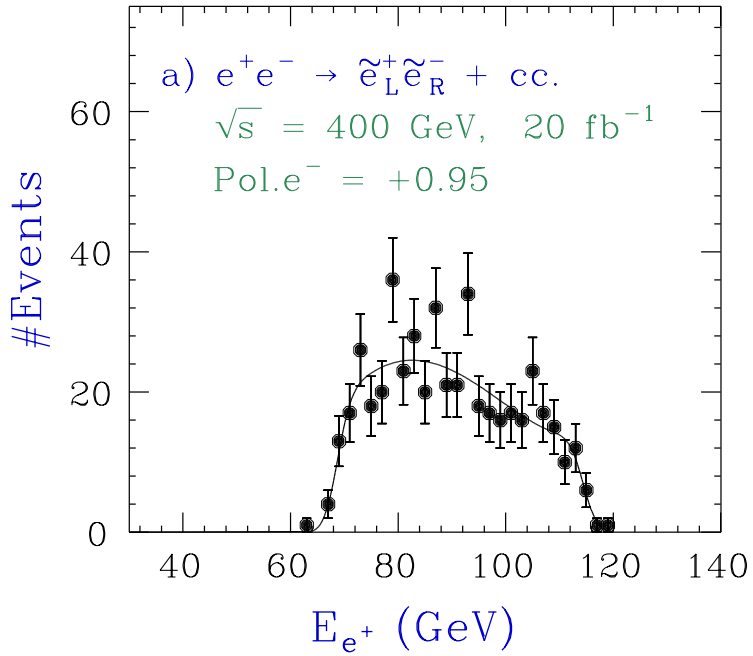
Mass Determination

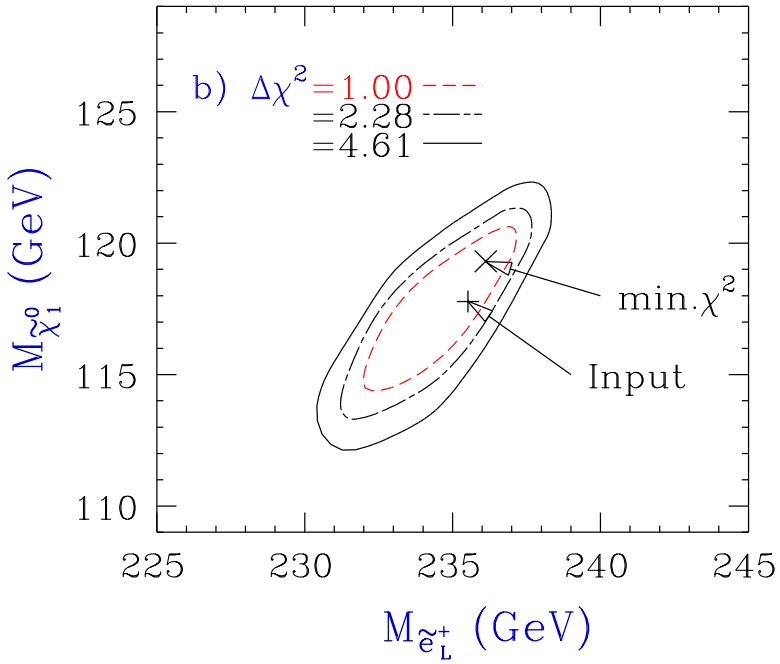


Measure E_{e^+}

End points

$\longrightarrow (m_{\tilde{e}_L^\pm}, m_{\tilde{\chi}_1^0})$





$\longrightarrow \Delta m/m \lesssim 1\%$!

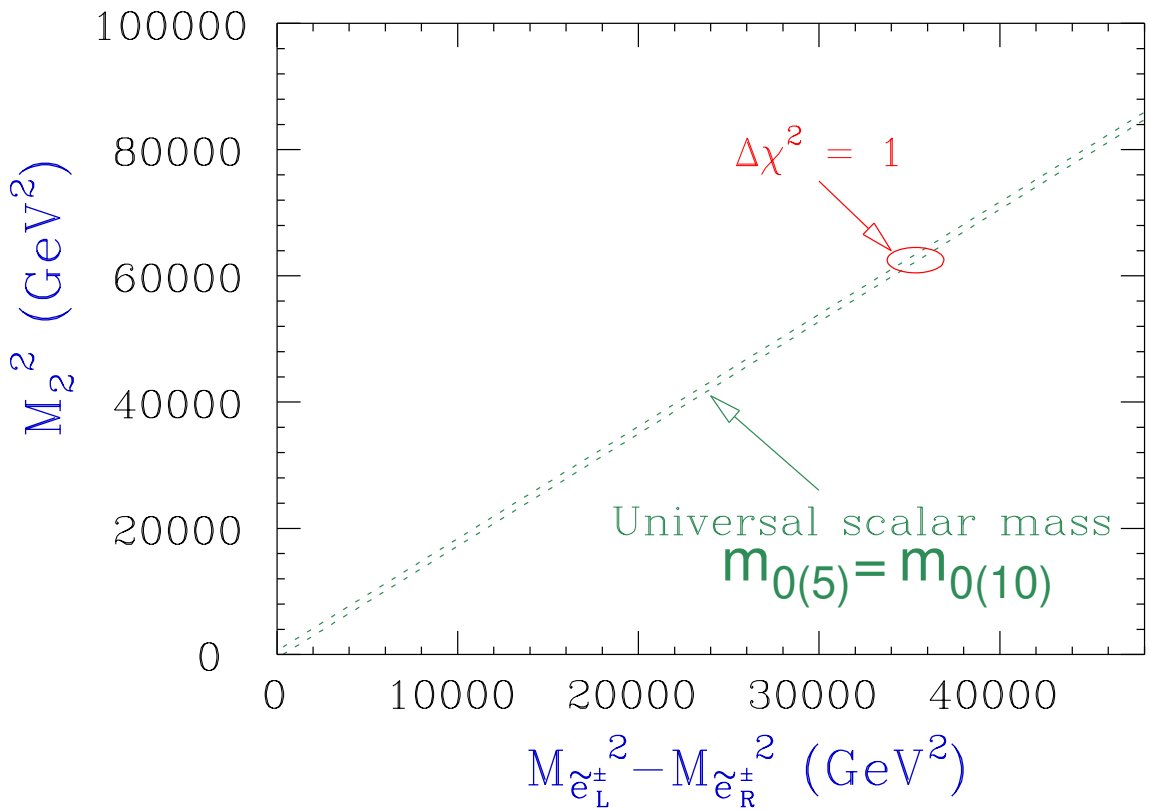
Test of Universal Scalar Mass

Sfermion Mass Formula

$$m^2_{\tilde{e}_L} - m^2_{\tilde{e}_R} = m^2_{0(5)} - m^2_{0(10)} + 0.53\, M_2^2 - \underbrace{0.44\, D}_{\text{Small}} \approx m_Z^2 \cos 2\beta < 0$$

$m^2_{\tilde{e}_L}$: 5 $\quad$ $m^2_{\tilde{e}_R}$: 10

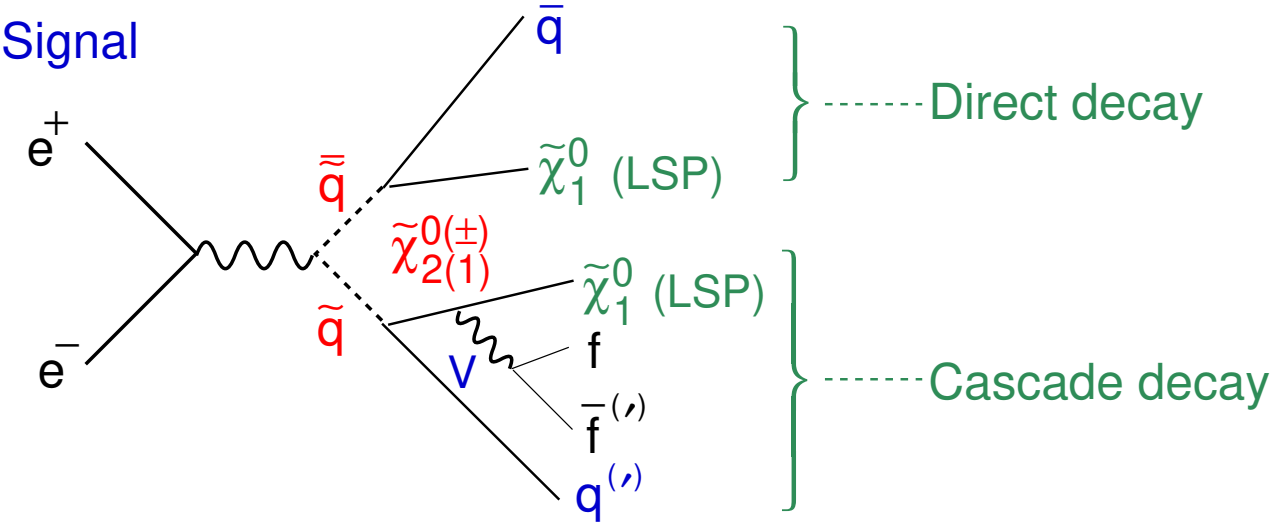
Test of Representation Independence



Once the GUT relation is confirmed and the universal scalar mass hypothesis is verified in the slepton sector, we can set, with confidence, the next target energy for $\tilde{q}$ pair production !

$\tilde{q}_G$ Study (G=1,2)

Signal



⇢ Acoplanar 2 jets (+ leptons / V=W/Z)

BG

WW
eνW
ννZ
$\tilde{\chi}^+_1\tilde{\chi}^-_1$
$\tilde{\chi}^0_i\tilde{\chi}^0_j$

⇢ Negligible for $m_{q\bar{q}} > 100$ GeV, $\not{P}_T > 35$ GeV, $\theta_A > 30°$

$m_{q\bar{q}} = m_{W^{(*)}/Z^{(*)}} < 100$ GeV !

⇢ We can use both e^-_L and e^-_R beams !

Chirality Selection

Much better than the slepton case !

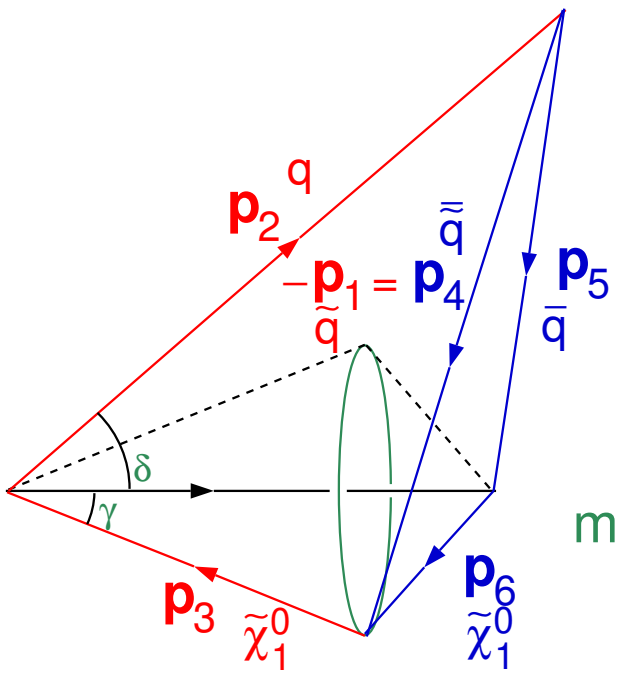
$$\sigma_R(\tilde{q}_R\bar{\tilde{q}}_R) : \sigma_R(\tilde{q}_L\bar{\tilde{q}}_L) \cong \sigma_L(\tilde{q}_L\bar{\tilde{q}}_L) : \sigma_L(\tilde{q}_R\bar{\tilde{q}}_R) \cong 10 : 1$$

if $m_{\tilde{q}_R} \cong m_{\tilde{q}_L}$

Mass Determination

We can use the endpoint method as with $\tilde{l}^{\pm}$ and $\tilde{\chi}_1^{\pm}$

Better quantity proposed by Feng-Finnell



$m_{\tilde{\chi}_1^0}$: known

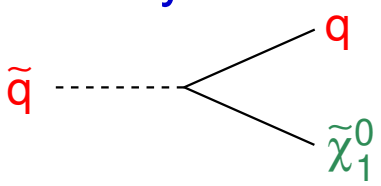
Minimal Kinematically Allowed Squark Mass

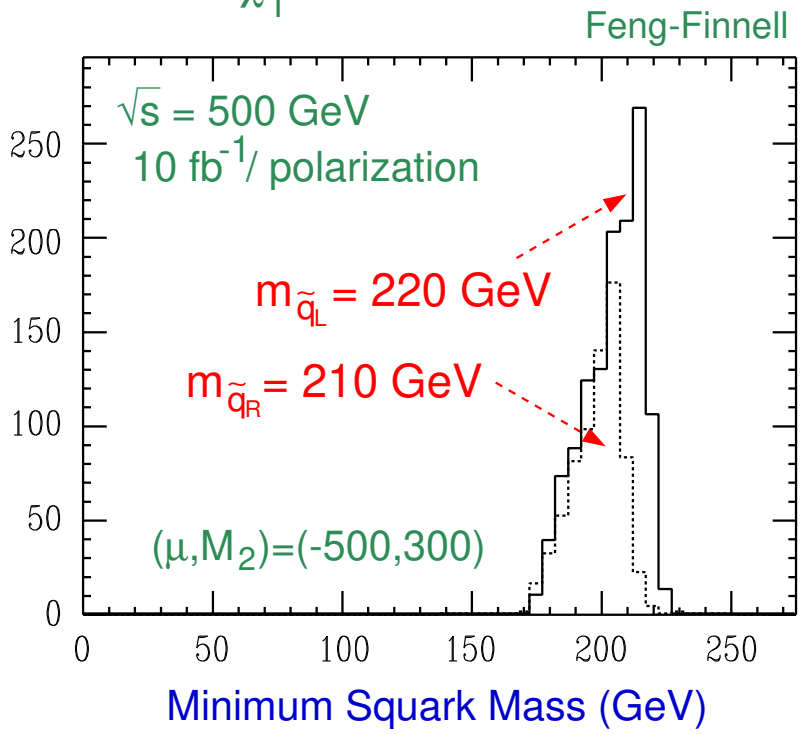
$$(m_{\tilde{q}}^{min})^2 = E_{BM}^2 - |\mathbf{p}_1|_{max}^2$$
$$= E_{BM}^2 - |\mathbf{p}_2|^2 - |\mathbf{p}_3|^2 + 2\,|\mathbf{p}_2| \cdot |\mathbf{p}_3| \cos(\delta + \gamma)$$

Use the information contained in

the relative configuration of q and $\bar{q}$!

Direct Decays

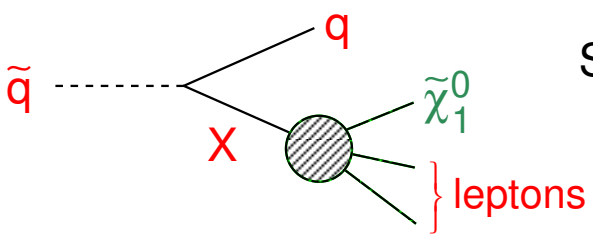




$\Delta m_{\tilde{q}} \cong$ 1 GeV at 95% CL

if $m_{\tilde{\chi}_1^0}$: known

Cascade Decays



Select

2 q-jets + leptons

$m_X \cong m_{\tilde{\chi}_1^\pm} \cong m_{\tilde{\chi}_2^0}$ ⟶ Replace $\tilde{\chi}_1^0$ by X = $\tilde{\chi}_1^\pm$/ $\tilde{\chi}_2^0$

$m_{\tilde{q}}^{min}$ ⟶ $\Delta m_{\tilde{q}} \cong$ 2.4 GeV at 95% CL

(μ, M_2)=(-500,200)

if $m_{\tilde{\chi}_1^\pm} \cong m_{\tilde{\chi}_2^0}$: known

Summary

Pol e^- Beam

Powerful tool to

i) suppress SM BG and enhance SIGNAL !

ii) sort out sparticle mixings !

Precision Measurements

$$\dashrightarrow \begin{cases} \Delta m/m \lesssim O(1\%) \text{ !} \\ d\sigma/d\Omega \end{cases}$$

$\therefore$ We can learn a lot from the FSP alone !

$\dashrightarrow$ Systematic and Consistent Searches and Studies

FSP $\dashrightarrow$ NSP $\dashrightarrow$ ..... $\dashrightarrow$ N^nSP

S-parameters

$\dashrightarrow$ $\mathcal{L}_{\text{S-breaking}} \dashrightarrow \mathcal{L}_{\text{Planck?}}$!

First realistic step toward GUT/Planck scale physics !