Electroweak Baryogenesis

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Baryon Asymmetry of the Universe

$$\frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s} \simeq (0.37 - 0.88) \times 10^{-10}$$

Big Bang Nucleosynthesis

$$Y = \frac{2n}{n+p} \quad (= 0.25 \leftrightarrow \frac{n}{p} = \frac{1}{7})$$

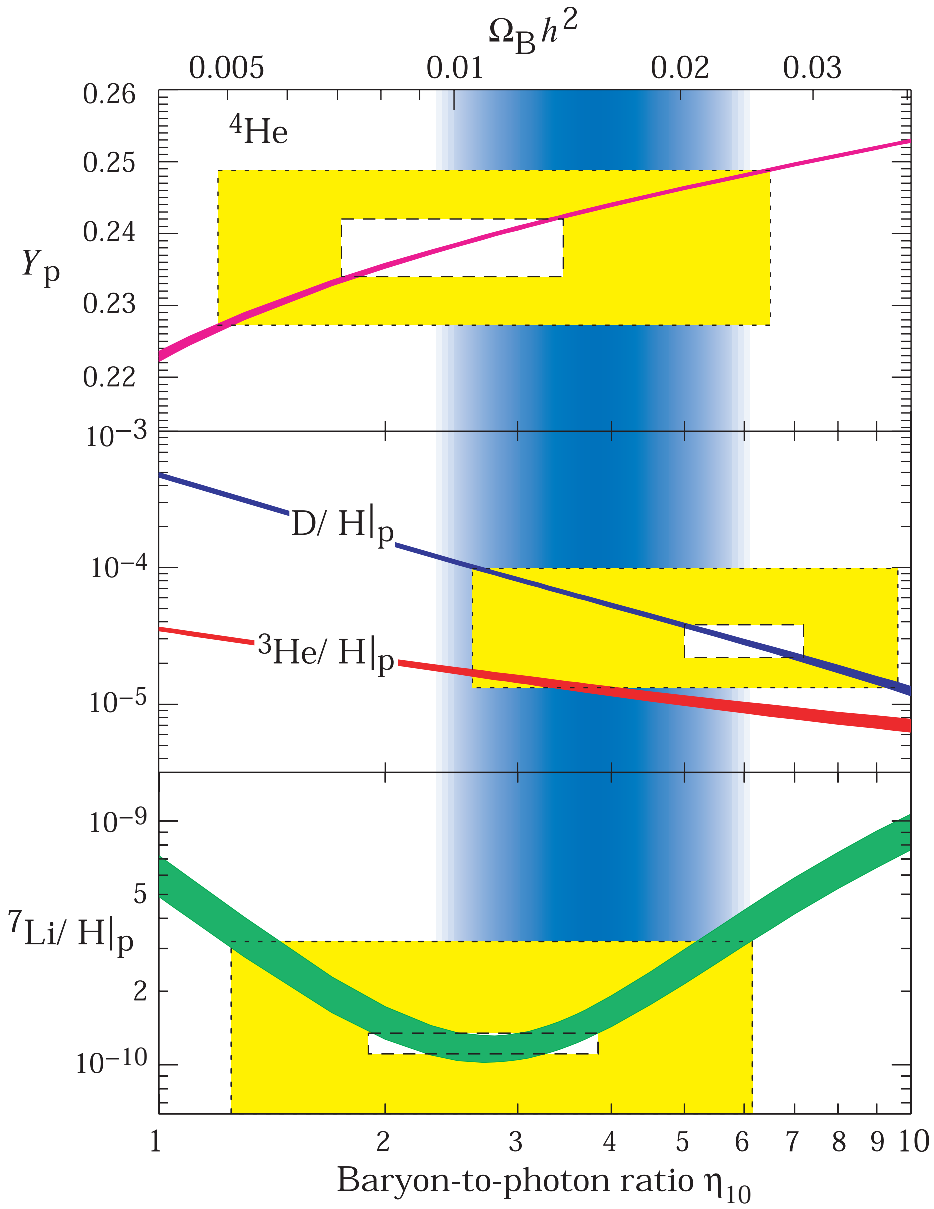
- $T \gg 1\text{MeV}$: $n + \nu_e \leftrightarrow p + e \Rightarrow \frac{n}{p} = 1$

- $T = T_F \simeq 1\text{MeV}$: $\Gamma_{n\leftrightarrow p}(T_F) \simeq H$

$$\left.\frac{n}{p}\right|_{\text{freeze-out}} = e^{-(m_n - m_p)/T_F} \simeq \frac{1}{6}$$

- $T = 0.3 - 0.1\text{MeV}$

$$\frac{n}{p} \to \frac{1}{6} - \frac{1}{7}$$ dependeing on $\eta = \frac{n_B}{n_\gamma}$



cf. $s \simeq 7n_\gamma$ today

evidence of BAU

[Steigman, Ann.Rev.Astron.Astrop.**14**('76)]

1. no anti-matter in cosmic rays from our galaxy
   some anti-matter consistent as secondary products

2. nearby clusters of galaxies are stable
   a cluster: $(1 \sim 100) M_{\text{galaxy}} \simeq 10^{12 \sim 14} M_\odot$

Starting from a B-symmetric universe ...

$$\frac{n_b}{s} \simeq \frac{n_{\bar{b}}}{s} \sim 8 \times 10^{-11} \quad \text{at } T = 38\text{MeV}$$

$$\sim 7 \times 10^{-20} \quad \text{at } T = 20\text{MeV} \qquad N\bar{N}\text{-annihilation decouple}$$

At $T = 38$MeV, mass within a causal region $= 10^{-7} M_\odot \ll 10^{12} M_\odot$.

We must have the BAU $\dfrac{n_B}{s} = (0.37 - 0.88) \times 10^{-10}$
before the universe was cooled down to $T \simeq 38$MeV

- **Introduction**
  - **Big Bang Cosmology**
  - **Saharov's conditions**
  - **Baryogenesis in GUTs and the others**
- **Sphaleron Process**
- **Electroweak Baryogenesis**
  - **EW Phase Transition vs Higgs mass**
  - **CP Violation**
- **Discussions**

Introduction

⋆ Review of the Big Bang Cosmology

Friedmann Universe — uniform and isotropic space

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

$a(t)$: scale factor in the comoving coordinate
$k = 1, 0, -1$: closed, flat, open space

Einstein eq. $\Rightarrow$
$$\begin{cases} H^2 = \left(\dfrac{\dot{a}}{a}\right)^2 = \dfrac{8\pi G}{3}\rho - \dfrac{k}{a^2} + \dfrac{\Lambda}{3}, \\ \dfrac{\ddot{a}}{a} = -\dfrac{4\pi G}{3}(\rho + 3p) + \dfrac{\Lambda}{3} \end{cases}$$

energy cons. $\Rightarrow a^3\dfrac{dp}{dt} = \dfrac{d}{dt}[a^3(\rho + p)] \Rightarrow \dfrac{d}{dt}\left(\rho a^{3(1+\gamma)}\right) = 0$

ρ = energy density, p = isotropic pressure

$$p = \gamma\rho \quad \text{with} \quad \begin{cases} \gamma = 1/3 & \text{(RD universe)} \\ \gamma \ll 1 & \text{(MD universe)} \end{cases}$$

For RD universe,

$$\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{|\boldsymbol{k}|}{e^{|\boldsymbol{k}|/T} \mp 1} = \begin{cases} \dfrac{\pi^2}{30} T^4, \\ \dfrac{7}{8} \dfrac{\pi^2}{30} T^4, \end{cases} \Rightarrow \rho(T) = \frac{\pi^2}{30} g_* T^4 \quad \text{with} \quad g_* \equiv \sum_B g_B + \frac{7}{8} \sum_F g_F$$

For the EW theory with N_f generations and m Higgs doublets,

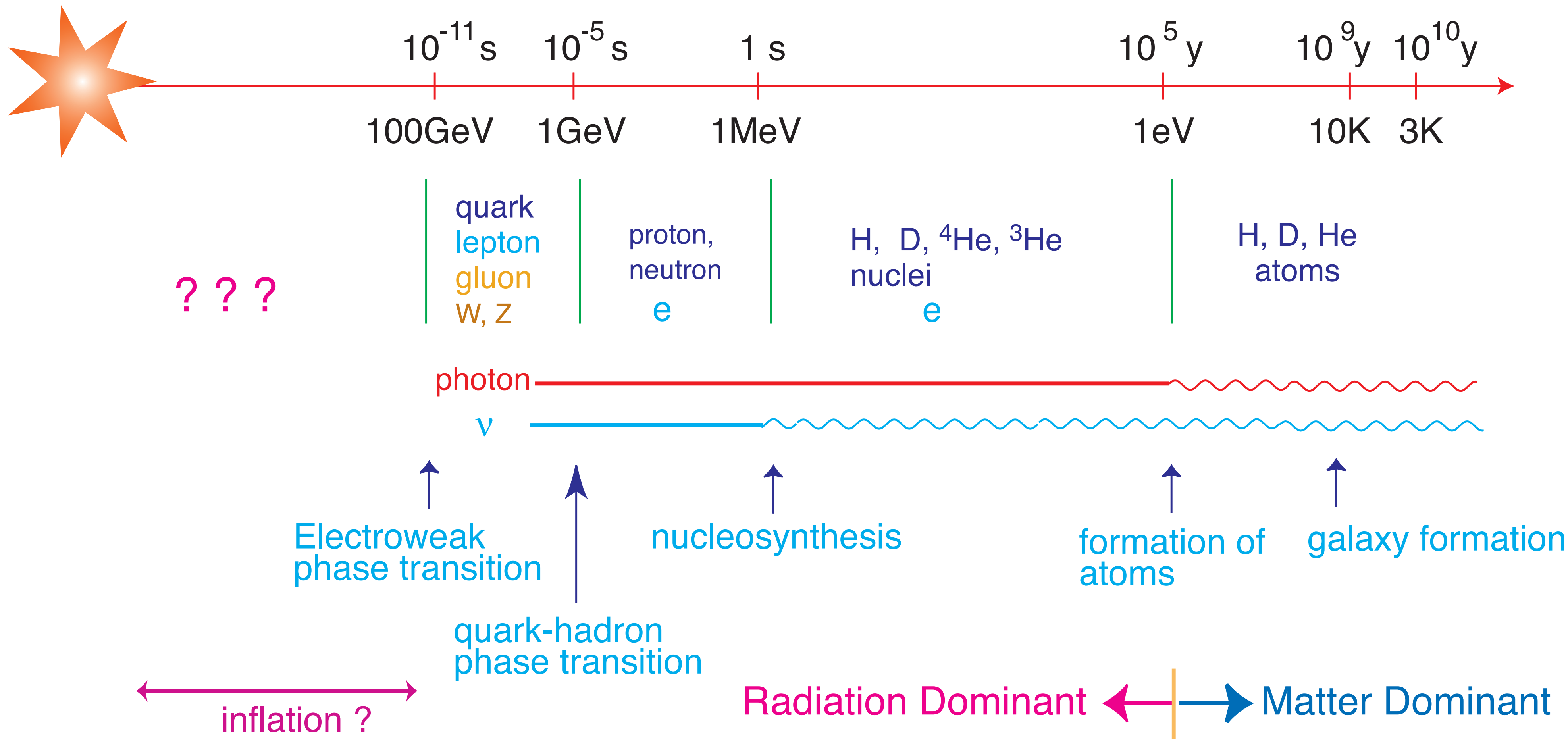
$$g_* = 24 + 4m + \frac{7}{8} \times 30 N_f$$

so that $g_* = 106.75$ for the Minimal SM.

In RD universe, neglecting Λ,

$$H \simeq \sqrt{\frac{8\pi G}{3}\rho} \simeq 1.66\sqrt{g_*}\frac{T^2}{m_{Pl}}$$

$$m_{Pl} = G^{-1/2} = 1.22 \times 10^{19}\text{GeV}$$



3 requirements for generation of the BAU [Sakharov, '67]

(1) baryon number violation

(2) C and CP violation

(3) departure from equilibrium

$\because$ (2) If C *or* CP is conserved, no B is generated: $\Leftarrow$ B is odd under C and CP.

indeed . . .

ρ_0 : baryon-symmetric initial state of the universe $s.t.$

$$\langle n_B \rangle_0 = \mathrm{Tr}[\rho_0 n_B] = 0$$

time evolution of $\rho \Longleftrightarrow$ Liouville eq.: $i\hbar \dfrac{\partial \rho}{\partial t} + [\rho, H] = 0$

If H is C- *or* CP-invariant, $[\rho, \mathcal{C}] = 0$ *or* $[\rho, \mathcal{CP}] = 0$ [spont. CP viol. $\Rightarrow [\rho, \mathcal{CP}] \neq 0$]

Since $\mathcal{C}B\mathcal{C}^{-1} = -B$ and $\mathcal{CP}B(\mathcal{CP})^{-1} = -B$ [$i.e.$, B is vectorlike, odd under C.]

$$\langle n_B \rangle = \mathrm{Tr}[\rho n_B] = \mathrm{Tr}[\rho\,\mathcal{C} n_B \mathcal{C}^{-1}] = -\mathrm{Tr}[\rho n_B]$$

$$or$$

$$\langle n_B \rangle = \mathrm{Tr}[\rho\,\mathcal{CP} n_B (\mathcal{CP})^{-1}] = -\mathrm{Tr}[\rho n_B]$$

$\therefore$ Both C and CP must be violated to have $\langle n_B \rangle \neq 0$, starting from $\langle n_B \rangle_0 = 0$.

possiblities ?

- B violation
  - explicit violation — GUTs
  - spontaneous viol. — $\langle \text{squark} \rangle \neq 0$
  - chiral anomaly — sphaleron process

It must be suppressed at present for protons not to decay.

- C violation $\Longleftarrow$ chiral gauge interactions (EW, GUTs)

- CP violation
  - KM phase in the MSM
  - beyond the SM — SUSY, extended Higgs sector

- out of equilibrium
  - expansion of the universe — $\Gamma_{\Delta B \neq 0} \simeq H(T)$
  - first-order phase transition
  - reheating (or preheating) after inflation

the first example — GUTs

[Yoshimura, PRL '78]

$SU(5)$ model:

matter: $\left\{ \begin{array}{lcl} \mathbf{5}^* : \psi^i_L & \ni & d^c_R, l_L \\ \mathbf{10} : \chi_{[ij]L} & \ni & q_L, u^c_R, e^c_R \end{array} \right.$ gauge: $A_\mu = \begin{pmatrix} G_\mu, B_\mu & X^{a\alpha}_\mu \\ X^{a\alpha}_\mu & W_\mu, B_\mu \end{pmatrix}$

$i = 1-5 \ \rightarrow (\alpha = 1-3, a = 1,2)$

$$\mathcal{L}_{\rm int} \ni g\bar{\psi}\gamma^\mu A_\mu \psi + g{\rm Tr}\left[\bar{\chi}\gamma^\mu\{A_\mu,\chi\}\right]$$
$$\ni gX^a_{\alpha\mu}\left[\varepsilon^{\alpha\beta\gamma}\bar{u}^c_{R\gamma}\gamma^\mu q_{L\beta a} + \epsilon_{ab}\left(\bar{q}_{L\alpha b}\gamma^\mu e^c_R + \bar{l}_{Lb}\gamma^\mu d^c_{R\alpha}\right)\right]$$

in the decay of X-$\bar{X}$ pairs

process	br. ratio	ΔB
$X \longrightarrow qq$	r	$2/3$
$X \longrightarrow \bar{q}\bar{l}$	$1-r$	$-1/3$
$\bar{X} \longrightarrow \bar{q}\bar{q}$	$\bar{r}$	$-2/3$
$\bar{X} \longrightarrow q, l$	$1-\bar{r}$	$1/3$

$$\langle \Delta B \rangle = \frac{2}{3}r - \frac{1}{3}(1-r) - \frac{2}{3}\bar{r} + \frac{1}{3}(1-\bar{r}) = r - \bar{r}$$

$\therefore$ C *or* CP is conserved$(r = \bar{r})$

$\Longrightarrow \Delta B = 0$

If the inverse process is suppressed, $B \propto r - \bar{r}$ is generated.

At $T \simeq m_X$, decay rate of X $= \Gamma_D \simeq \alpha m_X$ $\quad$ $\alpha \sim 1/40$ for gauge boson,

$\Gamma_D \simeq H(T \simeq m_X) \Longrightarrow$ decay and production of $X\bar{X}$ are out of equilibrium

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The $SU(5)$ GUT model conserves $B - L$. $\quad$ *i.e.* $(B + L)$-genesis

$\Downarrow$

Any B is washed-out by the sphaleron process, as we shall see later

$\downarrow$

new varieties of baryogenesis

e.g. Leptogenesis $\Rightarrow$ BAU $\quad$ $B = -L$

- $^{\exists}$Majorana neutrino $\Rightarrow$ L-violating interaction [Fukugita & Yanagida, PL174B]

$$\left.\begin{array}{c}\text{decoupling of heavy-}\nu\text{ decay}\\ CP\text{ violation in the heavy -}\nu\text{ sector}\end{array}\right\} \Rightarrow \text{Leptogenesis} \overset{\text{sphaleron}}{\Longrightarrow} \text{BAU}$$

[review: Buchmüller et al., hep-ph/0401240]

- Affleck-Dine mechanism in a SUSY model [A-D, NPB249; Dine, et al., NPB458]

  $\langle \tilde{q} \rangle \neq 0$ or $\langle \tilde{l} \rangle \neq 0$ along (nearly) flat directions, at high temperature
  coherent motion of complex $\langle \tilde{q} \rangle$, $\langle \tilde{l} \rangle \neq 0$ B, C, CP viol.
  $\Longrightarrow$ B- and/or L-genesis

- Electroweak Baryogenesis

(1) $\Delta(B+L) \neq 0 \begin{cases}\text{enhanced by sphaleron at } T > T_C \\ \text{suppressed by instanton at } T = 0\end{cases}$
(2) C-violation (chiral gauge); CP-violation: KM phase or extension of the MSM
(3) first-order EWPT with expanding bubble walls

- topological defects
  EW string, domain wall $\sim$ EW baryogenesis
  effective volume is too small, mass density of the universe

Sphaleron Process

⋆ Anomalous fermion number nonconservation ⟸ axial anomaly in the SM

$$\partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2}[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}]$$

$$\partial_\mu j^\mu_{B-L} = 0$$

N_f = number of the generations

$\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$

integrating these equations,

$$\begin{aligned} B(t_f) - B(t_i) &= \frac{N_f}{32\pi^2}\int_{t_i}^{t_f} d^4x \left[g^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right] \\ &= N_f\,[N_{CS}(t_f) - N_{CS}(t_i)] \end{aligned}$$

where N_{CS} is the Chern-Simons number:
in the $A_0 = 0$ gauge,

$$N_{CS}(t) = \frac{1}{32\pi^2}\int d^3x\,\epsilon_{ijk}\left[g^2 \mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}gA_iA_jA_k\right) - g'^2 B_{ij}B_k\right]_t$$

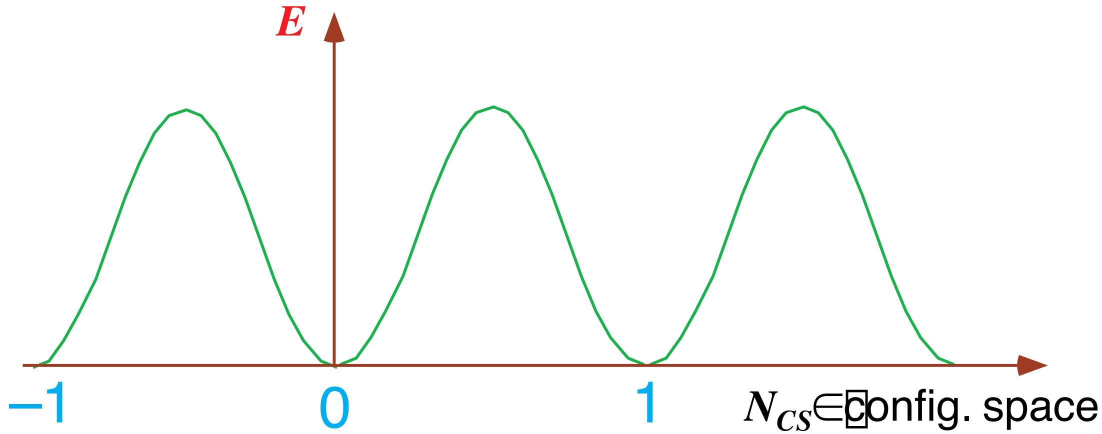
classical vacua of the gauge sector: $\mathcal{E} = \frac{1}{2}(\boldsymbol{E}^2 + \boldsymbol{B}^2) = 0 \iff F_{\mu\nu} = B_{\mu\nu} = 0$

$\iff A = iU^{-1}dU$ and $B = dv$ with $U \in SU(2)$

$\therefore \quad U(\boldsymbol{x}) : S^3 \ni \boldsymbol{x} \longrightarrow U \in SU(2) \simeq S^3$

$\pi_3(S^3) \simeq \mathbf{Z} \Rightarrow U(\boldsymbol{x})$ is classified by an integer N_{CS}.

energy functional vs configuration space

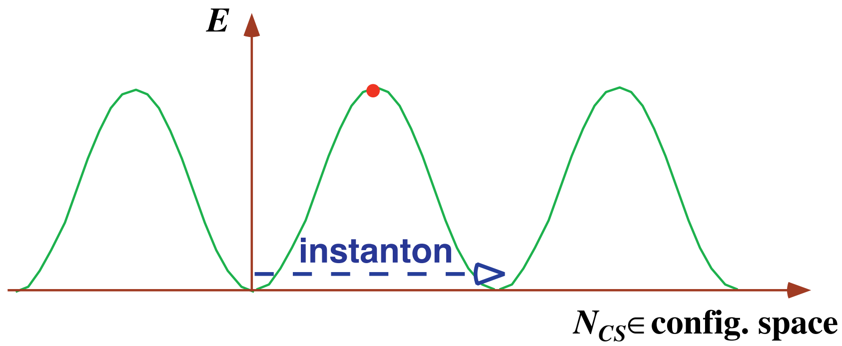


background U changes with $\Delta N_{CS} = 1$
$\Longrightarrow \Delta B = 1$ $(\Delta L = 1)$ in each (left-) generation

$\iff$ fermion:
- level crossing
- index theorem

Transition of the field config. with $\Delta B \neq 0$

▷ quantum tunneling	low temperature
▷ thermal activation	high temperature



transition rate with $N_{CS} = 1 \Longleftarrow$ WKB approx.

At $T = 0$,

$$\text{tunneling amplitude} \simeq e^{-S_{\text{instanton}}} = e^{-4\pi^2/g^2}$$

instanton

- ⋆ stable
- ⋆ 4d solution with finite euclidean action
- ⋆ integer Pontrjagin index $\sim \int d^4x_E \, F_{\mu\nu}\tilde{F}^{\mu\nu}$

What is Sphaleron ?

sphaleros : $\sigma\varphi\alpha\lambda\epsilon\rho o\sigma$ = 'ready to fall'

a saddle-point solution of 4d $SU(2)$ gauge-Higgs system
[Klinkhammer & Manton, PRD30 ('84)]

$$E_{\rm sph} = 8 - 14 \text{ TeV}$$

- ⋆ unstable
- ⋆ static (3d) solution with finite energy
- ⋆ Chern-Simons No. = "$1/2$"

$\Longrightarrow$ over-barrier transition at finite temperature

$$\Gamma_{\rm sph} \sim e^{-E_{\rm sph}/T}$$

cf. for EW theory

$$\Gamma_{\rm tunneling} \sim e^{-2S_{\rm instanton}} = 10^{-164}$$

⋆ Transition rate

[Arnold and McLerran, P.R.D36('87)]

♣ $\frac{\omega_-}{(2\pi)} \lesssim T \lesssim T_C$ ω_-:negative-mode freq. around the sphaleron

$$\Gamma_{\rm sph}^{(b)} \simeq k \mathcal{N}_{\rm tr} \mathcal{N}_{\rm rot} \frac{\omega_-}{2\pi} \left(\frac{\alpha_W(T)T}{4\pi} \right)^3 e^{-E_{\rm sph}/T}$$

$\mathcal{N}_{\rm tr} = 26$, $\mathcal{N}_{\rm rot} = 5.3 \times 10^3 \leftarrow$ zero modes

$\omega_-^2 \simeq (1.8 \sim 6.6) m_W^2$ for $10^{-2} \leq \lambda/g^2 \leq 10$, $k \simeq O(1)$

♣ $T \gtrsim T_C$ symmetric phase — no mass scale

$$\Gamma_{\rm sph}^{(s)} \simeq \kappa(\alpha_W T)^4$$

▷ Monte Carlo simulation $\langle N_{CS}(t) N_{CS}(0) \rangle = \langle N_{CS} \rangle^2 + A e^{-2\Gamma V t}$ as $t \to \infty$

$\kappa > 0.4$	$SU(2)$ gauge-Higgs system	[Ambjørn, et al. N.P.B353('91)]
$\kappa = 1.09 \pm 0.04$	$SU(2)$ pure gauge system	[Ambjørn and Krasnitz, P.L.B362('95)]

'sphaleron transition' even in the symmetric phase.

B and L in the Hot Universe

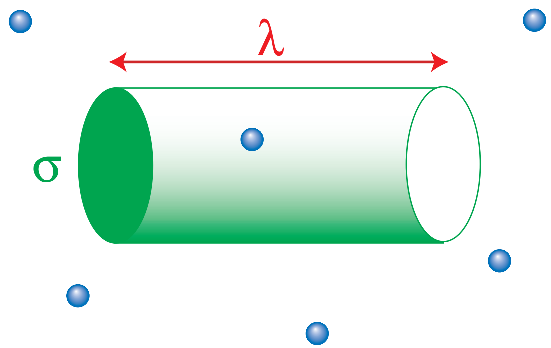
reaction rate: $\Gamma(T) > H(t) \Longleftrightarrow$ the process is in chemical equilibrium

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} \simeq \sqrt{\frac{8\pi G}{3}\rho} \simeq 1.66\sqrt{g_*}\frac{T^2}{m_{Pl}}$$

$\Gamma(T) \longrightarrow$ time scale of interactions

mean free path : $\lambda \cdot \sigma = \dfrac{1}{n}$

$m \ll T \Rightarrow \lambda \simeq \bar{t} =$ mean free time



$$n = g\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{e^{\omega_k/T} \mp 1} \overset{m\ll T}{\simeq} g\begin{cases} \dfrac{\zeta(3)}{\pi^2}T^3 \\ \dfrac{3}{4}\dfrac{\zeta(3)}{\pi^2}T^3 \end{cases}$$

$$\overset{m\gg T}{\simeq} g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$\zeta(3) = 1.2020569\cdots$

For relativistic particles at T, $\sigma \simeq \frac{\alpha^2}{s} \simeq \frac{\alpha^2}{T^2}$, we have $\lambda \simeq \frac{10}{gT^3}\left(\frac{\alpha^2}{T^2}\right)^{-1} = \frac{10}{g\alpha^2 T}$.

For $T = 100\text{GeV}$, $H^{-1} \simeq 10^{14}\text{GeV}^{-1}$,

$$\lambda_s \simeq \frac{1}{{\alpha_s}^2 T} \sim 1\,\text{GeV}^{-1} \qquad \text{for strong interactions}$$

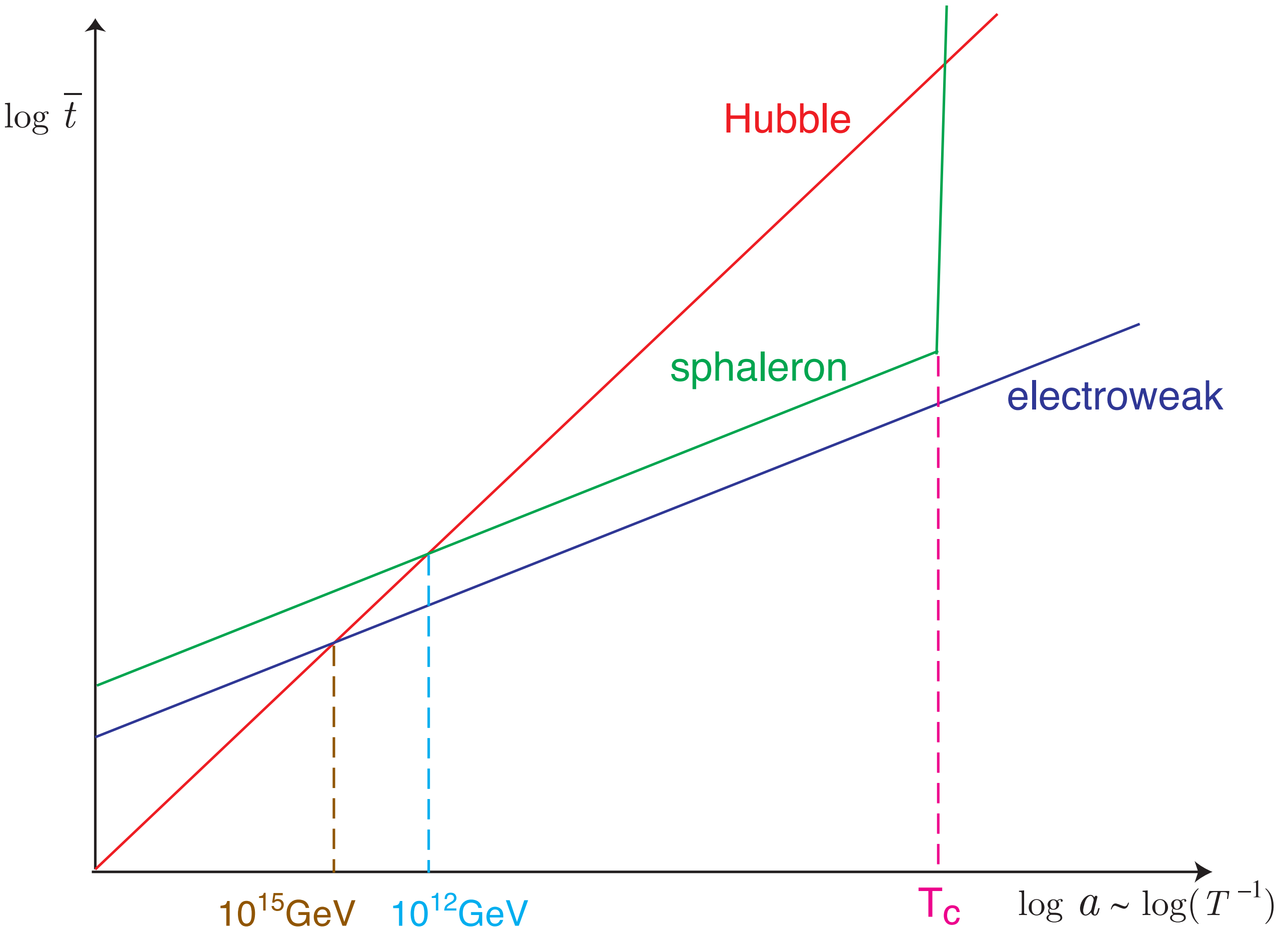
$$\lambda_{EW} \simeq \frac{1}{{\alpha_W}^2 T} \sim 10\,\text{GeV}^{-1} \qquad \text{for EW interactions}$$

$$\lambda_Y \simeq \left(\frac{m_W}{m_f}\right)^4 \lambda_{EW} \qquad \text{for Yukawa interactions}$$

time scale of sphaleron process

$$\bar{t}_{\text{sph}} = (\Gamma_{\text{sph}}/n)^{-1} \simeq \begin{cases} 10^6 T^{-1}\,\text{GeV}^{-1} & (T > T_C) \\ 10^5 T^{-1} e^{E_{\text{sph}}/T}\,\text{GeV}^{-1} & (T < T_C) \end{cases}$$

[*cf.* $E_{\text{sph}} \simeq 10\text{TeV}$ for $v_0 = 246\text{GeV}$]



If $v(T_C) \ll 200\text{GeV}$ (eg. 2nd order EWPT), ${}^{\exists}T_{\rm dec}$, $s.t.$

$$T_{\rm dec} < T < T_C \implies \Gamma^{(b)}_{\rm sph}(T) > H(T)$$

wash-out of $B + L$ even in the broken phase

$\star$ Quantum numbers in equilibrium

Q_i: conserved quantum number $\qquad [H, Q_i] = 0$

equilibrium partition function: $Z(T, \mu) \equiv \mathrm{Tr}\left[e^{-(H - \sum_i \mu_i Q_i)/T}\right]$

$$\Rightarrow \langle Q_i \rangle (T, \mu) = T \frac{\partial}{\partial \mu_i} \log Z(T, \mu)$$

$\longrightarrow$ relations among μ's $\Longleftrightarrow$ relations among Q's

In the SM, $Q_i = \frac{1}{N} B - L_i$ without lepton-flavor mixing.

1st-principle calculation of $Z(T, \mu)$

- path integral over *all* fields
- *nonperturbative* $B + L$ violation

$\Downarrow$

- perturbation $\qquad$ [Shaposhnikov, et al, PLB387 ('96); PRD61 ('00)]
- free-field approximation $\qquad$ chemical potentials of the particles

number density of free particles (per degree of freedom)

$$\langle N \rangle = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\frac{1}{e^{(\omega_k-\mu)/T}\mp 1}-\frac{1}{e^{(\omega_k+\mu)/T}\mp 1}\right]$$

$$\overset{m\ll T}{\simeq} \frac{T^3}{2\pi^2}\int_0^\infty dx\left[\frac{x^2}{e^{x-\mu/T}\mp 1}-\frac{x^2}{e^{x+\mu/T}\mp 1}\right] \overset{|\mu|\ll T}{\simeq} \begin{cases} \frac{T^3}{3}\cdot\frac{\mu}{T}, & \text{(bosons)} \\ \frac{T^3}{6}\cdot\frac{\mu}{T}, & \text{(fermions)} \end{cases}$$

Quantum number densities in terms of μ

[Harvey & Turner, PRD42 ('90)]

SM with N generations and N_H Higgs doublets $(\phi^0\,\phi^-)$

W^-	$u_{L(R)}$	$d_{L(R)}$	$e_{iL(R)}$	ν_{iL}	ϕ^0	ϕ^-
μ_W	$\mu_{u_{L(R)}}$	$\mu_{d_{L(R)}}$	$\mu_{iL(R)}$	μ_i	μ_0	μ_-

gauge int., Yukawa int, quark mixings are in equilibrium.

$$\mu_\gamma = \mu_Z = \mu_{\rm gluon} = 0$$

$$\Downarrow$$

$(3N+7)$ μ's

$$\text{gauge} \quad \Leftrightarrow \quad \mu_W = \mu_{d_L} - \mu_{u_L} = \mu_{iL} - \mu_i = \mu_- + \mu_0$$

$$\text{Yukawa} \quad \Leftrightarrow \quad \mu_0 = \mu_{u_R} - \mu_{u_L} = \mu_{d_L} - \mu_{d_R} = \mu_{iL} - \mu_{iR}$$

$2(N+2)$ relations $\Rightarrow$ $N+3$ independent μ's: $(\mu_W, \mu_0, \mu_{u_L}, \mu_i)$

sphaleron process in equilibrium: $|0\rangle \leftrightarrow \prod_i (u_L d_L d_L \nu_L)_i \Leftrightarrow N(\mu_{u_L} + 2\mu_{d_L}) + \sum_i \mu_i = 0$

Quantum number densities [in unit of $T^2/6$]

$$\begin{aligned}
B &= N(\mu_{u_L} + \mu_{u_R} + \mu_{d_L} + \mu_{d_R}) = 4N\mu_{u_L} + 2N\mu_W, \\
L &= \sum_i (\mu_i + \mu_{iL} + \mu_{iR}) = 3\mu + 2N\mu_W - N\mu_0 \\
Q &= \frac{2}{3}N(\mu_{u_L} + \mu_{u_R}) \cdot 3 - \frac{1}{3}N(\mu_{d_L} + \mu_{d_R}) \cdot 3 - \sum_i (\mu_{iL} + \mu_{iR}) - 2 \cdot 2\mu_W - 2N_H\mu_- \\
&= 2N\mu_{u_L} - 2\mu - (4N + 4 + 2N_H)\mu_W + (4N + 2N_H)\mu_0 \\
I_3 &= -(2N + N_H + 4)\mu_W \qquad\qquad \mu \equiv \sum_i \mu_i
\end{aligned}$$

• $T \gtrsim T_C$ (symmetric phase) We require $Q = I_3 = 0$. $(\mu_W = 0)$

$$B = \frac{8N + 4N_H}{22N + 13N_H}(B - L), \qquad L = -\frac{14N + 9N_H}{22N + 13N_H}(B - L)$$

• $T \lesssim T_C$ (broken phase) $Q = 0$ and $\mu_0 = 0$ ($\because$ ϕ^0 condensates.)

$$B = \frac{8N + 4(N_H + 2)}{24N + 13(N_H + 2)}(B - L), \qquad L = -\frac{16N + 9(N_H + 2)}{24N + 13(N_H + 2)}(B - L)$$

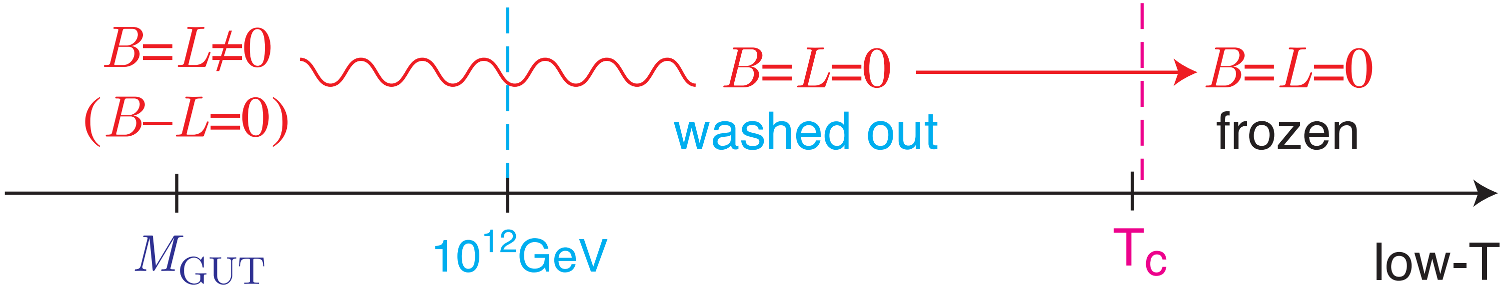
In any case, $B = L = 0$, if $(B - L)_{\text{primordial}} = 0$.

To have nonzero BAU,

(i) we must have $B - L$ before the sphaleron process decouples, or

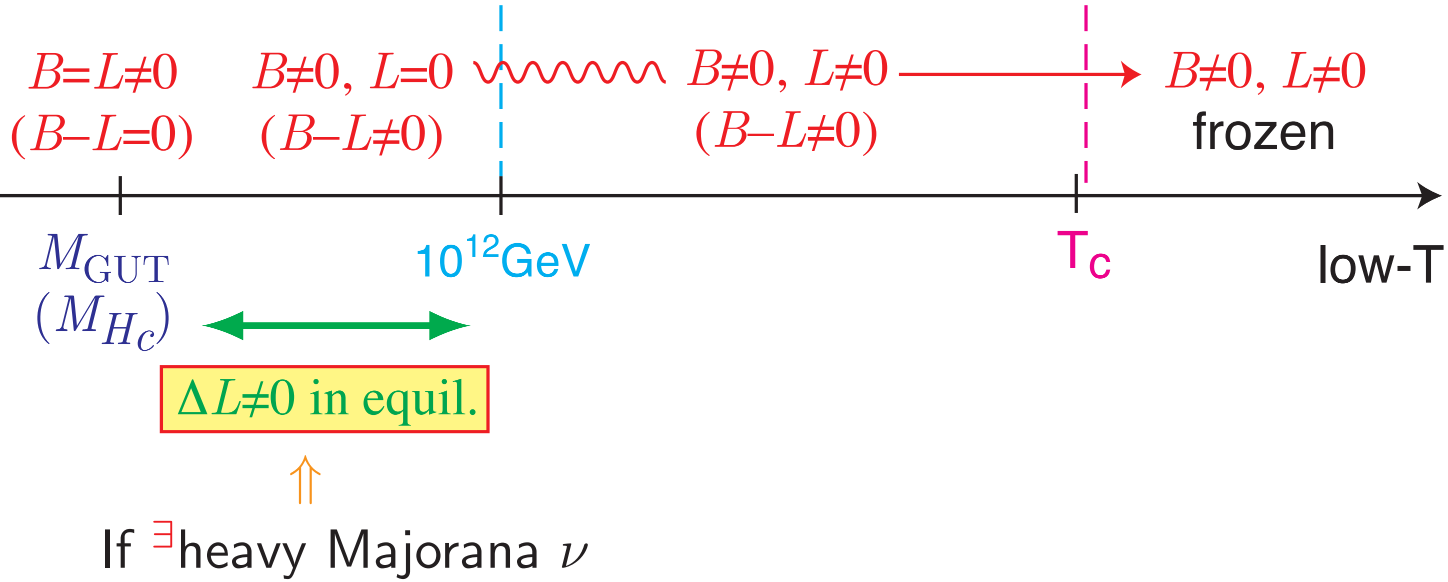
(ii) $B + L$ must be created at the first-order EWPT, *and* the sphaleron process must decouple immediately after that.

Wash-out of B and L in $(B-L)$-conserving GUTs



Resurrection of $(B-L)$-conserving GUT Baryogenesis

[Fukugita & Yanagida, PRL 89]



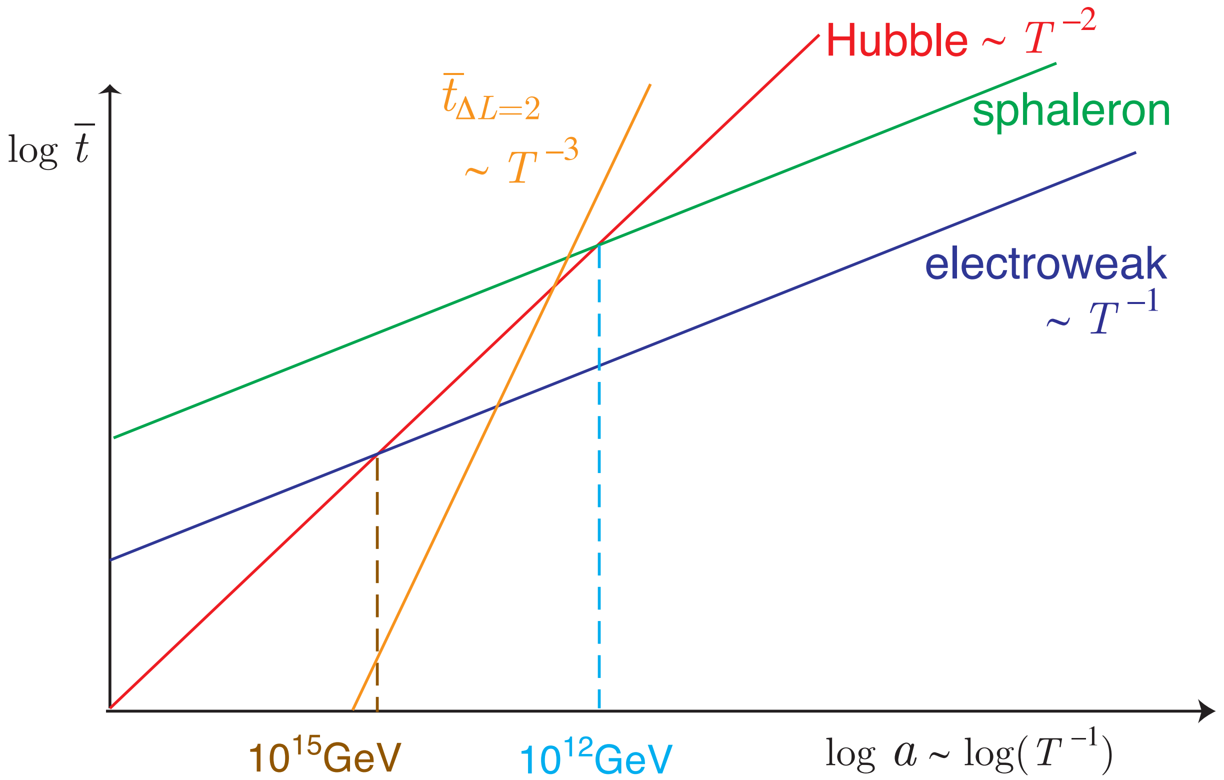
We must require that the processes decouple before T lowers to 10^{12}GeV.

otherwise, $B = L = 0$!!

e.g.,

$$\mathcal{L}_{\rm eff} = \frac{g_i^2}{m_{N_i}} l_i \phi\, l_i \phi \implies \Gamma_{\Delta L=2} \simeq \frac{0.12 g_i^4 T^3}{4\pi m_{N_i}^2} < H(T) \text{ at } T < 10^{12}\text{GeV}$$

$\implies$ lower bound on $m_{N_i} \iff m_{\nu_i} < 0.8$eV



Electroweak Baryogenesis

review articles:

- KF, Prog.Theor.Phys. 96 (1996) 475
- Rubakov and Shaposhnikov, Phys.Usp. 39 (1996) 461-502 (hep-ph/9603208)
- Riotto and Trodden, Ann. Rev. Nucl. Part. Sci. 49 (1999) 35 (hep-ph/9901362)
- Bernreuther, Lect.Notes Phys. 591 (2002) 237 (hep-ph/0205279)

$$T \simeq 100\text{GeV} \Rightarrow \quad H^{-1}(T) \simeq 10^{14}\text{GeV}^{-1} \ll \bar{t}_{EW} \simeq \frac{1}{\alpha_W T^2} \sim 10\text{GeV}^{-1}$$

$\therefore$ All the particles of the SM are in equilibrium.

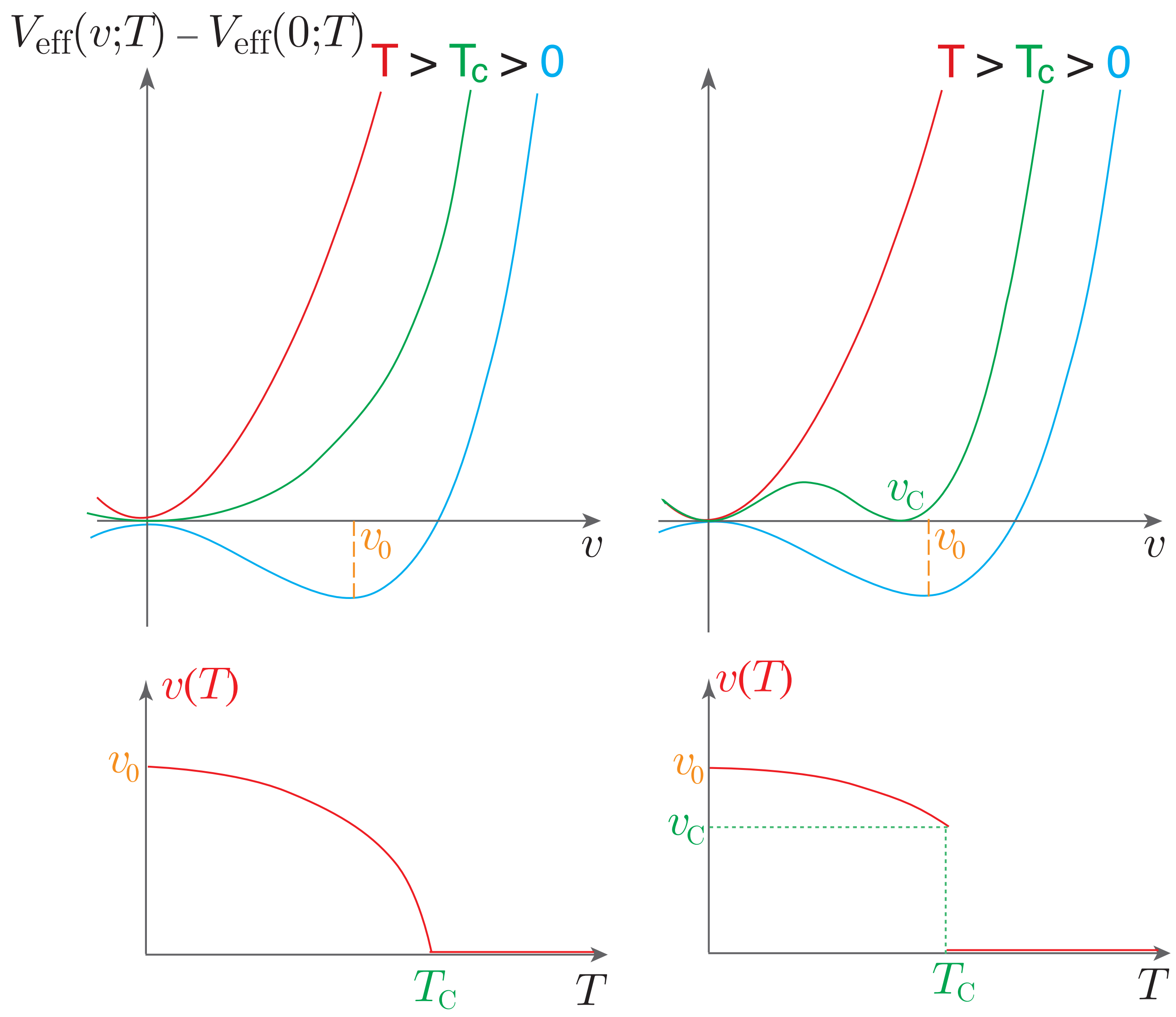
nonequilibrium state $\Longleftarrow$ **1st order EW phase transition**

study of the EWPT

⋆ static properties $\Leftarrow$ effective potential $=$ free energy density

$$V_{\text{eff}}(v;T) = -\frac{1}{V} T \log Z = -\frac{1}{V} \log \text{Tr}\left[e^{-H/T}\right]_{\langle\phi\rangle = v}$$

⋆ dynamics — formation and motion of the bubble wall when 1st order PT



2nd order PT

1st order PT

Minimal SM

order parameter:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}$$

$\therefore$ 1st order EWPT

$\Updownarrow$

$$v_C \equiv \lim_{T \uparrow T_C} \varphi(T) \neq 0$$

Minimal SM — perturbation at the 1-loop level

$$V_{\rm eff}(\varphi;T) = -\frac{1}{2}\mu^2\varphi^2 + \frac{\lambda}{4}\varphi^4 + 2Bv_0^2\varphi^2 + B\varphi^4\left[\log\left(\frac{\varphi^2}{v_0^2}\right) - \frac{3}{2}\right] + \bar{V}(\varphi;T)$$

where $B = \frac{3}{64\pi^2 v_0^4}(2m_W^4 + m_Z^4 - 4m_t^4)$,

$$\bar{V}(\varphi;T) = \frac{T^4}{2\pi^2}\left[6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_t)\right], \qquad (a_A = m_A(\varphi)/T)$$

$$I_{B,F}(a^2) \equiv \int_0^\infty dx\, x^2 \log\left(1 \mp e^{-\sqrt{x^2+a^2}}\right).$$

high-temperature expansion $[m/T \ll 1]$

$$I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{4\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right)\frac{a^4}{2} + O(a^6)$$

$$I_F(a^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{16}\log\frac{\sqrt{a^2}}{\pi} - \frac{a^4}{16}\left(\gamma_E - \frac{3}{4}\right) + O(a^6)$$

For $T > m_W, m_Z, m_t$,

$$V_{\rm eff}(\varphi;T) \simeq D(T^2 - T_0^2)\varphi^2 - E\,T\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

where

$$D = \frac{1}{8v_0^2}(2m_W^2 + m_Z^2 + 2m_t^2), \qquad E = \frac{1}{4\pi v_0^3}(2m_W^3 + m_Z^3) \sim 10^{-2}$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_0^4}\left(2m_W^4 \log\frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log\frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log\frac{m_t^2}{\alpha_F T^2}\right)$$

$$T_0^2 = \frac{1}{2D}(\mu^2 - 4Bv_0^2), \qquad \log\alpha_{F(B)} = 2\log(4)\pi - 2\gamma_E$$

At T_C, $^{\exists}$degenerate minima: $\varphi_C = \dfrac{2E\,T_C}{\lambda_{T_C}}$

$$\Gamma_{\rm sph}^{(b)}/T_C^3 < H(T_C) \iff \frac{\varphi_C}{T_C} \gtrsim 1$$

$\Longrightarrow$ upper bound on λ $\quad [m_H = \sqrt{2}\lambda v_0]$

$$m_H \lesssim 46\text{GeV}$$

$\Longrightarrow$ MSM is excluded

⋆ Monte Carlo simulations [MSM]

effective fermion mass : $m_f(T) \sim O(T) \leftarrow$ nonzero modes

$\therefore$ simulation only with the bosons

QFT on the lattice $\begin{cases} \text{scalar fields:} & \phi(x) \text{ on the sites} \\ \text{gauge fields:} & U_\mu(x) \text{ on the links} \end{cases}$

$$Z = \int [d\phi\, dU_\mu] \exp\{-S_E[\phi, U_\mu]\}$$

- 3-dim. $SU(2)$ system with a Higgs doublet and a triplet time-component of U_μ
  [Laine & Rummukainen, hep-lat/9809045]

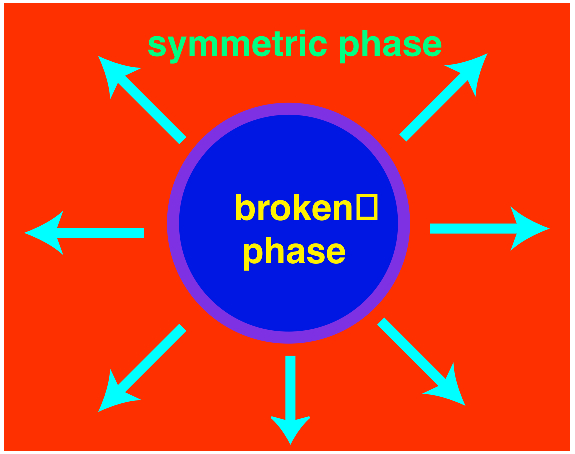
- 4-dim. $SU(2)$ system with a Higgs doublet [Csikor, hep-lat/9910354]
  EWPT is first order for $m_h < 66.5 \pm 1.4$GeV

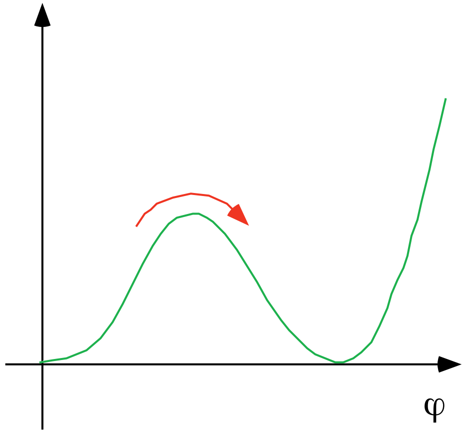
Both the simulations found end-point of EWPT at

$$m_h = \begin{cases} 72.3 \pm 0.7 \text{ GeV} \\ 72.1 \pm 1.4 \text{ GeV} \end{cases} \Rightarrow \boxed{\text{no PT (cross-over) in the MSM !}}$$

⋆ Dynamics of the phase transition

first-order EWPT accompanying
bubble nucleation/growth





nucleation rate per unit time and unit volume: $I(T) = I_0 e^{-\Delta F(T)/T}$
where

$$\Delta F(T) = \frac{4\pi}{3} r^3 [p_s(T) - p_b(T)] + 4\pi r^2 \sigma,$$

$$p_s(T) = -V_{\rm eff}(0;T), \qquad p_b(T) = -V_{\rm eff}(\varphi(T);T)$$

supercooling $\longrightarrow$ $p_s(T) < p_b(T)$

$\sigma \simeq \int dz (d\varphi/dz)^2$: surface energy density

radius of the critical bubble : $r_*(T) = \dfrac{2\sigma}{p_b(T) - p_s(T)}$

How the EWPT proceeds ?

[Carrington and Kapsta, P.R.D47('93)]

$f(t)$: fraction of space converted to the broken phase

$$f(t) = \int_{t_C}^{t} dt' \, I(T(t'))[1 - f(t')]V(t', t)$$

where

$V(t', t)$: volume of a bubble at t which was nucleated at t'

$$V(t', t) = \frac{4\pi}{3} \left[r_*(T(t')) + \int_{t'}^{t} dt'' \, v(T(t'')) \right]^3$$

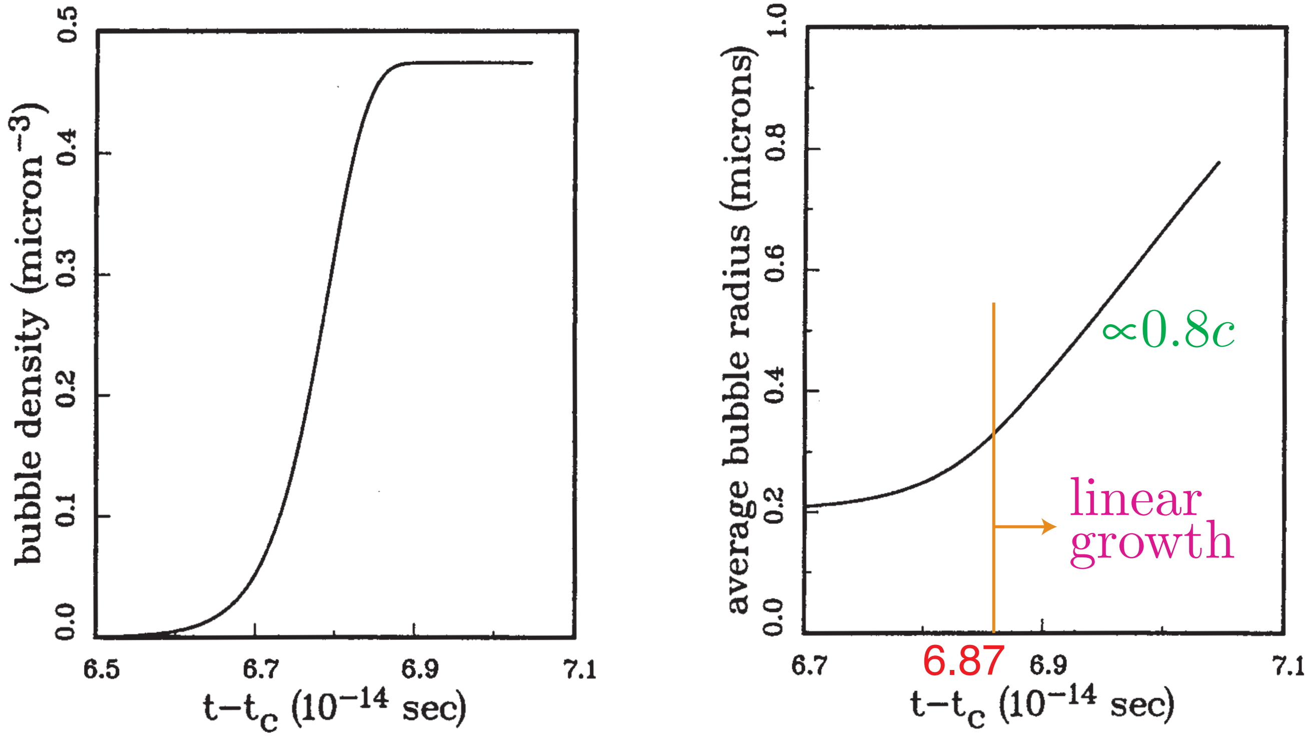
$T = T(t) \Leftarrow \rho = (\pi^2/30) g_* T^4 \propto R^{-4}$ for RD universe

$v(T)$: wall velocity

- one-loop $V_{\rm eff}$ of MSM with $m_H = 60$GeV and $m_t = 120$GeV

  At $t = 6.5 \times 10^{-14}$ sec, bubbles began to nucleate.

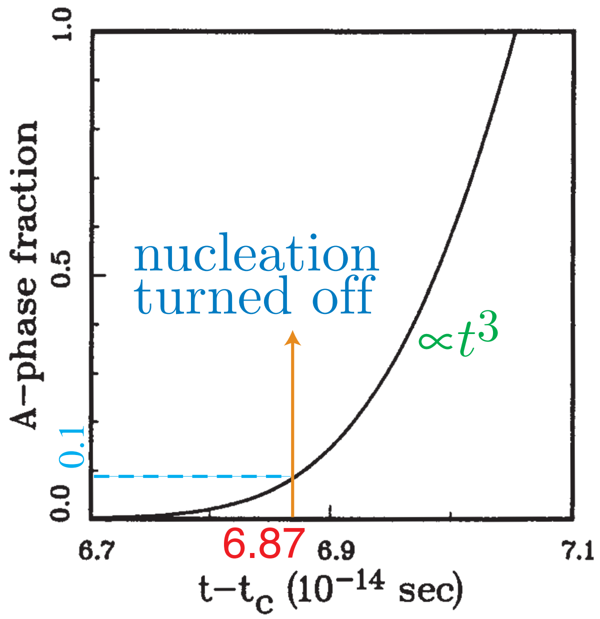
  [A characteristic time scale of the EW processes is $O(10^{-26})$sec.]

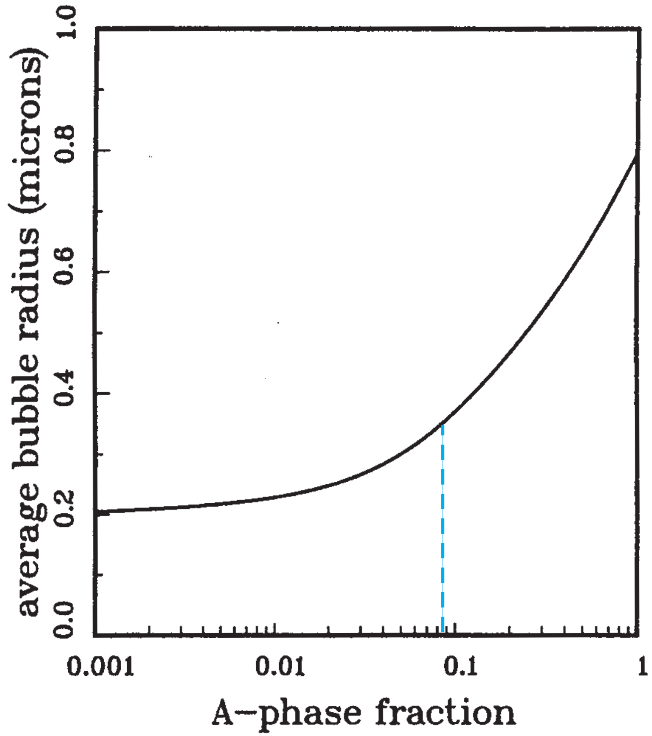


horizon size : $H^{-1} \simeq 7.1 \times 10^{12}$ GeV$^{-1} = 0.14$ cm

$r = 0.3\mu$m $\Rightarrow$ #(bubbles within a horizon) $\simeq 3 \times 10^{11}$

very small supercooling : $(T_C - T_N)/T_C \simeq 2.5 \times 10^{-4}$





weakly first order $\Longleftrightarrow$ small v_C and/or lower barrier height

$\Longrightarrow$ { nucleation dominance over growth
thick bubble wall
large fluctuation between the two phases

Mechanism of the baryogenesis

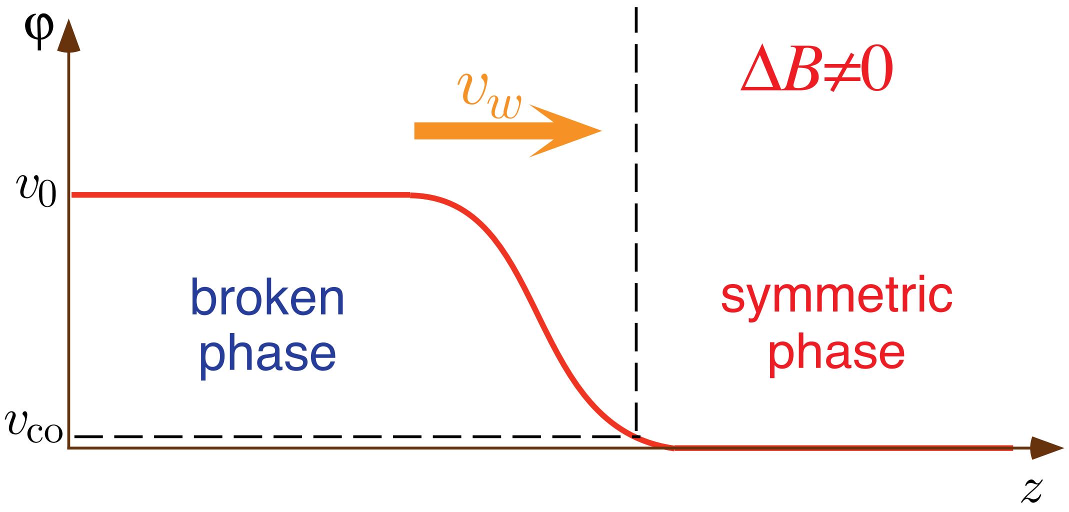
$$\bar{t}_s \simeq 0.1\text{GeV}^{-1} \ll \bar{t}_{EW} \simeq 1\text{GeV}^{-1} \ll \bar{t}_{\text{sph}} \simeq 10^5\text{GeV}^{-1} \ll H^{-1} \simeq 10^{14}\text{GeV}^{-1}$$

EW bubble wall motion: $t_{\text{wall}} = \dfrac{l_w}{v_w} = \dfrac{(1-40)/T}{0.1-0.9} = (0.01-4)\text{GeV}^{-1}$

1. All the particles are in *kinetic equilibrium* at the same temperature, because of $H^{-1} \gg \bar{t}_{EW}$, far from the bubble wall.

2. Since $\lambda_Y > \lambda_{EW} \gg l_w$, the leptons and some of the quarks propagate almost freely before and after the scattering off the bubble wall.

3. Because of $t_{\text{wall}} \ll \bar{t}_{\text{sph}}$, the sphaleron process is out of *chemical equilibrium* near the bubble wall.



$v_{co} \simeq 0.01 v_0 \Leftarrow E_{\rm sph}/T_C \simeq 1$

bubble wall $\Leftarrow$ classical config. of the gauge-Higgs system

- interactions between the particles and the bubble wall
- accumulation of **chiral charge** in the symmetric phase

$\Downarrow$

generation of baryon number through sphaleron process

$\Downarrow$

decoupling of sphaleron process in the broken phase

2 scenarios:
- ○ spontaneous baryogenesis + diffusion — classical, adiabatic
- • charge transport scenario — quantum mechanical, nonlocal

Both need CP violation other than KM matrix ⇒ extension of the MSM

2HDM, MSSM, . . .

⋆ Charge transport mechanism

CP violation in the Higgs sector or in the mass matrix of $\tilde{\chi}, \tilde{q}$

[spacetime-dependent CP violation]

⇓

difference in reflections of chiral fermions and antifermions

⇓

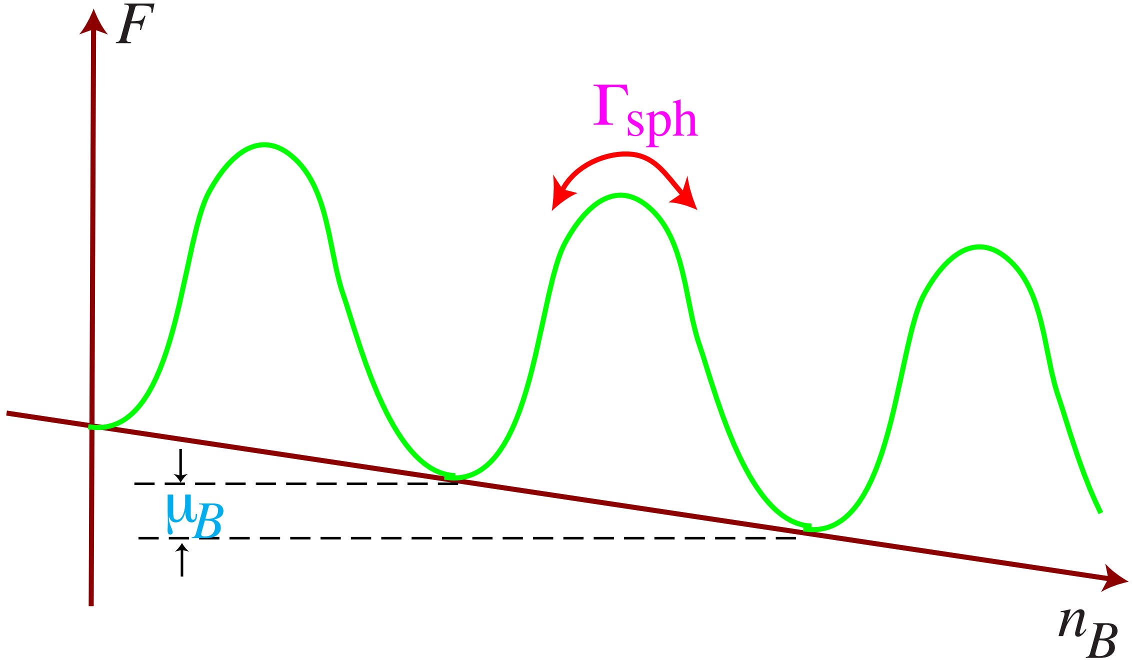
net chiral charge flux into the symmetric phase

⇓

change of distribution functions by the chiral charge
with the sphaleron process in equilibrium

$v_w \simeq$ const. $\Rightarrow$ stationary nonequilibrium : bias on free energy along B

$$\dot{n}_B \simeq -\frac{\mu_B \Gamma_{\rm sph}}{T}$$



According to the relations among the chemical potentials (sphaleron is excluded),

$$\mu_B = \frac{Y}{2(m+5/3)T^2}$$

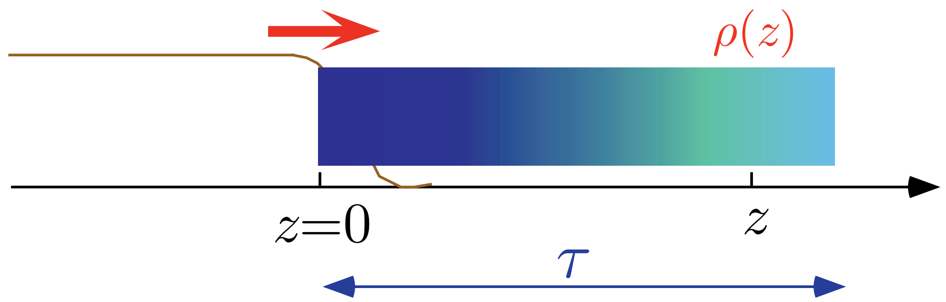
$m = \#$(Higgs doublets)

BAU by electroweak baryogenesis

$$n_B = -\frac{\Gamma_{\text{sph}}}{T} \int dt\, \mu_B = -\frac{\Gamma_{\text{sph}}}{2(m+5/3)T^3} \int dt\, Y$$

where

$$\int dt\, Y = \int_{-\infty}^{z/v_w} dt\, \rho_Y(z - v_w t) = \frac{1}{v_w} \int_0^{\infty} dz\, \rho_Y(z) \simeq \frac{F_Y \tau}{v_w}$$



τ = transport time within which the scattered fermions are captured by the wall

$$\frac{n_B}{s} \simeq \mathcal{N}\frac{100}{\pi^2 g_*} \cdot \kappa\alpha_W^4 \cdot \frac{F_Y}{v_w T^3} \cdot \tau T$$

$$\mathcal{N} \sim O(1), \qquad \tau T \simeq \begin{cases} 1 & \text{for quarks} \\ 10^2 \sim 10^3 & \text{for leptons} \end{cases}$$

MC simulation $\Longrightarrow$ forward scattering enhanced :

for top quark $\tau T \simeq 10 \sim 10^3$ max. at $v_w \simeq 1/\sqrt{3}$

for this optimal case

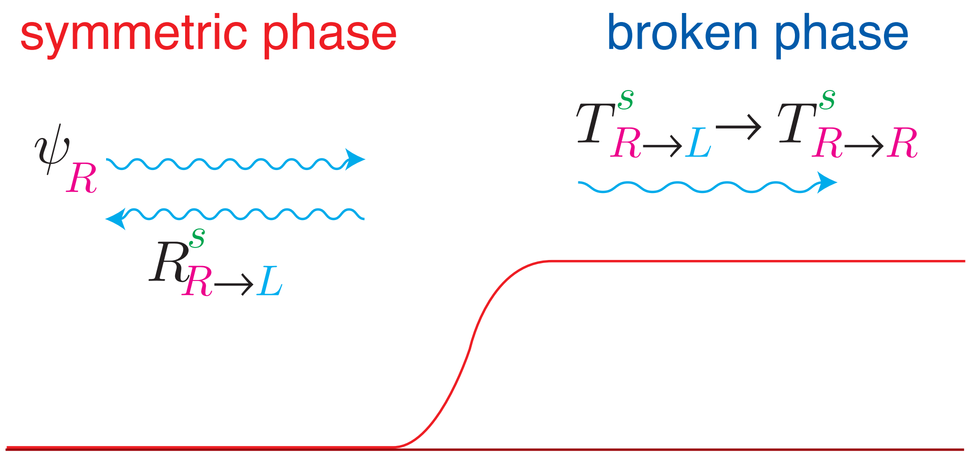
$$\frac{n_B}{s} \simeq 10^{-3} \cdot \frac{F_Y}{v_w T^3}$$

$F_Y/(v_w T^3) \sim O(10^{-7})$ would be sufficient to explain the BAU.

Calculation of the chiral charge flux

$$i\partial\!\!\!/\psi(x) - m(x)P_R\,\psi(x) - m^*(x)P_L\,\psi(x) = 0$$

where $\quad -f\langle\phi(x)\rangle = m(x) \;\in \mathbf{C} \quad$ through the Yukawa int.



$Q^i_{L(R)}$: charge of a L(R)-handed fermion of species i

${R^s}_{R\to L}$: reflection coeff. for the R-handed fermion incident from the symmetric phase region

${\bar{R}^s}_{R\to L}$: the same as above for the R-handed antifermion

⟨injected charge into symmetric phase⟩ brought by the fermions and antifermions in the symmetric phase :

$$\begin{aligned}\Delta Q_i{}^s = & [(Q_R{}^i - Q_L{}^i) R^s{}_{L\to R} + (-Q_L{}^i + Q_R{}^i)\bar{R}^s{}_{R\to L} \\ & +(-Q_L{}^i)(T^s{}_{L\to L} + T^s{}_{L\to R}) - (-Q_R{}^i)(\bar{T}^s{}_{R\to L} + \bar{T}^s{}_{R\to R})] f^s{}_{Li} \\ & +[(Q_L{}^i - Q_R{}^i) R^s{}_{R\to L} + (-Q_R{}^i + Q_L{}^i)\bar{R}^s{}_{L\to R} \\ & +(-Q_R{}^i)(T^s{}_{R\to L} + T^s{}_{R\to R}) - (-Q_L{}^i)(\bar{T}^s{}_{L\to L} + \bar{T}^s{}_{L\to R})] f^s{}_{Ri}\end{aligned}$$

the same brought by transmission from the broken phase :

$$\begin{aligned}\Delta Q_i{}^b = & Q_L{}^i (T^b{}_{L\to L} f^b{}_{Li} + T^b{}_{R\to L} f^b{}_{Ri}) + Q_R{}^i (T^b{}_{L\to R} f^b{}_{Li} + T^b{}_{R\to R} f^b{}_{Ri}) \\ & +(-Q_L{}^i)(\bar{T}^b{}_{R\to L} f^b{}_{Li} + \bar{T}^b{}_{L\to L} f^b{}_{Ri}) + (-Q_R{}^i)(\bar{T}^b{}_{R\to R} f^b{}_{Li} + \bar{T}^b{}_{L\to R} f^b{}_{Ri})\end{aligned}$$

by use of

unitarity: $R^s{}_{L\to R} + T^s{}_{L\to L} + T^s{}_{L\to R} = 1,$ etc.

reciprocity: $T^s{}_{R\to L} + T^s{}_{R\to R} = T^b{}_{L\to L} + T^b{}_{R\to L},$ etc.

$$f^{s(b)}_{iL} = f^{s(b)}_{iR} \equiv f^{s(b)}_i$$

$$\Delta Q^s{}_i + \Delta Q^b{}_i = ({Q_L}^i - {Q_R}^i)({f^s}_i - {f^b}_i)\Delta R$$

where

$$\Delta R \equiv R^s{}_{R\to L} - \bar{R}^s{}_{R\to L}$$

which depends on

- profile of the bubble wall — wall thickness, height, CP phase
- momentum of the incident particle

total flux injected into the *symmetric phase* region

$$F^i{}_Q = \frac{{Q_L}^i - {Q_R}^i}{4\pi^2\gamma}\int_{m_0}^{\infty} dp_L \int_0^{\infty} dp_T\, p_T \left[{f_i}^s(p_L, p_T) - {f_i}^b(-p_L, p_T)\right] \Delta R(\frac{m_0}{a}, \frac{p_L}{a})$$

$$\begin{aligned} {f_i}^s(p_L, p_T) &= \frac{p_L}{E}\frac{1}{\exp[\gamma(E - v_w p_L)/T] + 1} \\ {f_i}^b(-p_L, p_T) &= \frac{p_L}{E}\frac{1}{\exp[\gamma(E + v_w\sqrt{p_L^2 - m_0^2})/T] + 1} \end{aligned}$$

$1/a =$ wall width, $m_0 =$ mass in the broken phase, $E = \sqrt{p_L^2 + p_T^2}$

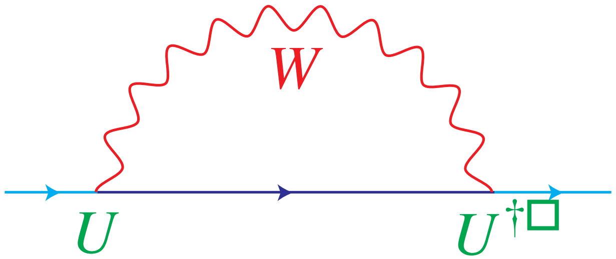
available charge : $\left.\begin{array}{c} Q_L - Q_R \neq 0 \\ \text{conserved in the symmetric phase} \end{array}\right\} \Longrightarrow$ $Y, \quad I_3$

N.B. For B, no F_B is generated, since it is vectorlike.

CP violation effective for ΔR

- Minimal SM — KM phase

dispersion relation of the fermion $\sim O(\alpha_W)$ [Farrar and Shaposhnikov, PRD,'94]



— decoherence by QCD effects (short range)

[Gavela, et al., NPB '94]

- CP violation in mass or mass matrix

tree-level quantum scattering by the bubble wall

⋆ relative phase of 2 Higgs doublets $\Rightarrow m(x) = -g\,|\phi(x)|\,e^{i\theta(x)}$

⋆ relative phases of the complex parameters in Supersymmetric SM

$\Rightarrow$ mass matrices of chargino, neutralino, sfermions

$$M_{\chi^\pm} = \begin{pmatrix} M_2 & -\frac{i}{\sqrt{2}} g_2 v_u e^{-i\theta} \\ -\frac{i}{\sqrt{2}} g_2 v_d & -\mu \end{pmatrix}$$

x-dependent v_d and v_u $\Rightarrow$ effectively x-dependent phases

⋆ **Example** [Nelson et al., NPB, '92]

$$m(z) = m_0 \frac{1+\tanh(az)}{2} \exp\left(-i\pi \frac{1-\tanh(az)}{2}\right)$$

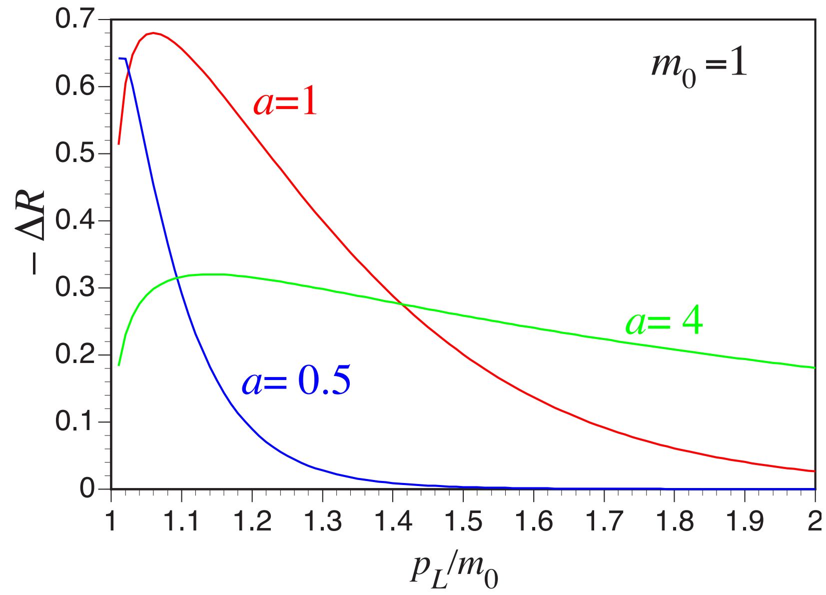
— no CP violation in the broken phase $[z \sim \infty]$

- $\Delta R \equiv {R^s}_{R\to L} - {\bar{R}^s}_{R\to L}$ [FKOTT, PRD'94; PTP'96]

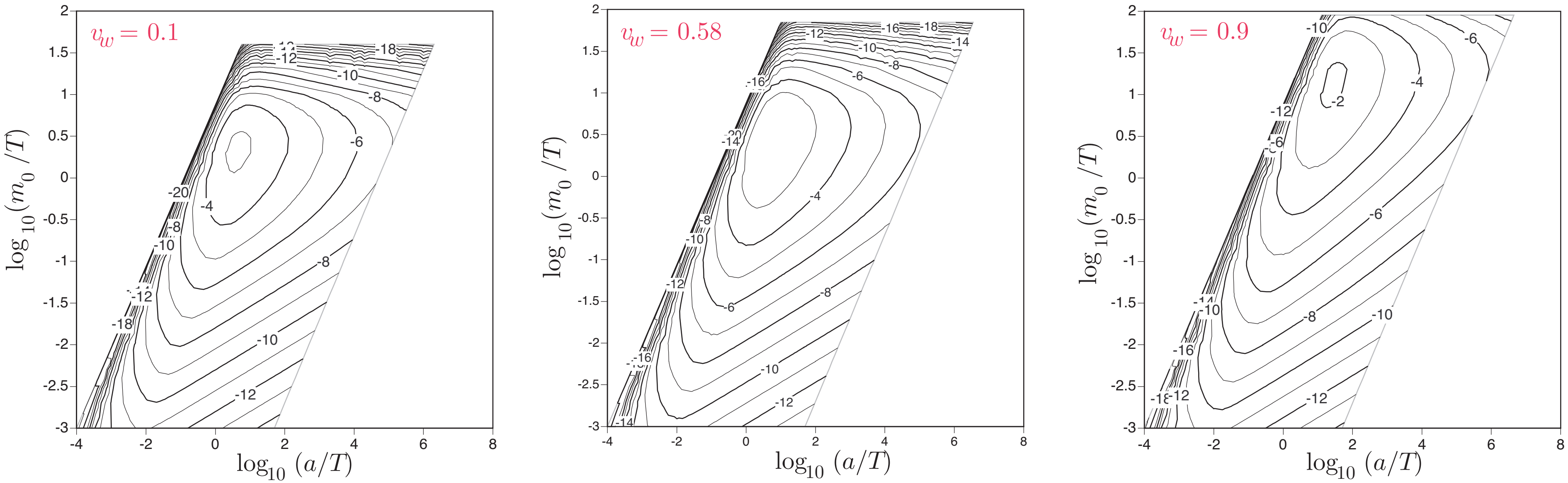
  wall width $\simeq$ wave length of the carrier $\Rightarrow$ $\Delta R \sim O(1)$

  $\Downarrow$

  stronger Yukawa coupling does *not* always implies larger flux



- chiral charge flux normalized as $\dfrac{F_Q}{T^3(Q_L - Q_R)}$ [dimensionless] at $T = 100\text{GeV}$



$$\frac{n_B}{s} \simeq \mathcal{N}\frac{100}{\pi^2 g_*} \cdot \kappa\alpha_W^4 \cdot \frac{F_Y}{v_w T^3} \cdot \tau T \simeq 10^{-3} \cdot \frac{F_Y}{v_w T^3}$$

for an optimal case

EW baryogenesis in the MSSM

- EW Phase Transition

  3 order parameters: $\langle\Phi_d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \langle\Phi_u\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 + i v_3 \end{pmatrix}$

- CP Violation

  complex parameters: $\mu,\ M_{3,2,1},\ A,\ \mu B = m_3^2$

  $v_3 \neq 0$ — $v_3 = 0$ at the tree level

- sphaleron solution

  $\left\{\begin{array}{l} \text{2HDM} \\ \text{squarks vs sphaleron} \end{array}\right.$

  2HDM [Peccei, et al, PLB '91]

  squarks vs sphaleron [Moreno, et al, PLB '97]

⋆ Electroweak phase transition

light stop scenario [de Carlos & Espinosa, NPB '97]

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + \left(\frac{g_1^2}{24} - \frac{g_2^2}{8}\right)(v_u^2 - v_d^2) + \frac{y_t^2}{2} v_u^2 & \frac{y_t}{\sqrt{2}} (\mu v_d + A(v_2 - i v_3)) \\ * & m_{\tilde{t}_R}^2 - \frac{g_1^2}{6}(v_u^2 - v_d^2) + \frac{y_t^2}{2} v_u^2 \end{pmatrix}$$

$m_{\tilde{t}_L}^2 = 0$ or $m_{\tilde{t}_R}^2 = 0 \Longrightarrow$ smaller eigenvalue: $m_{\tilde{t}_1}^2 \sim O(v^2)$

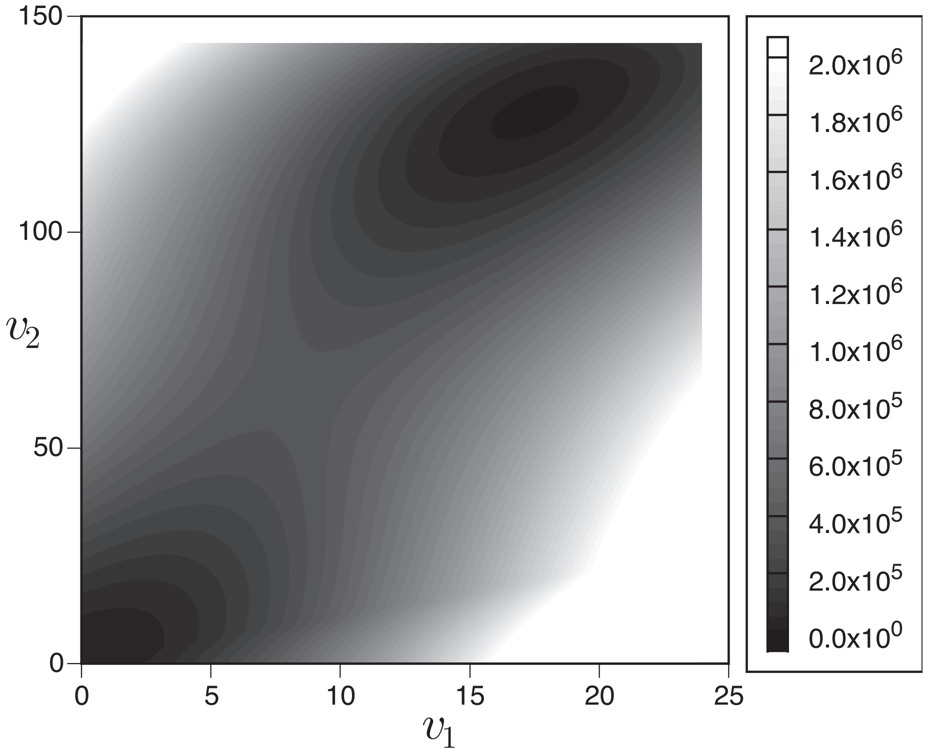
$\therefore$ high-T expansion

$$\bar{V}_{\tilde{t}}(\boldsymbol{v}; T) \Rightarrow -\frac{T}{6\pi}(m_{\tilde{t}_1}^2)^{3/2} \sim T v^3$$

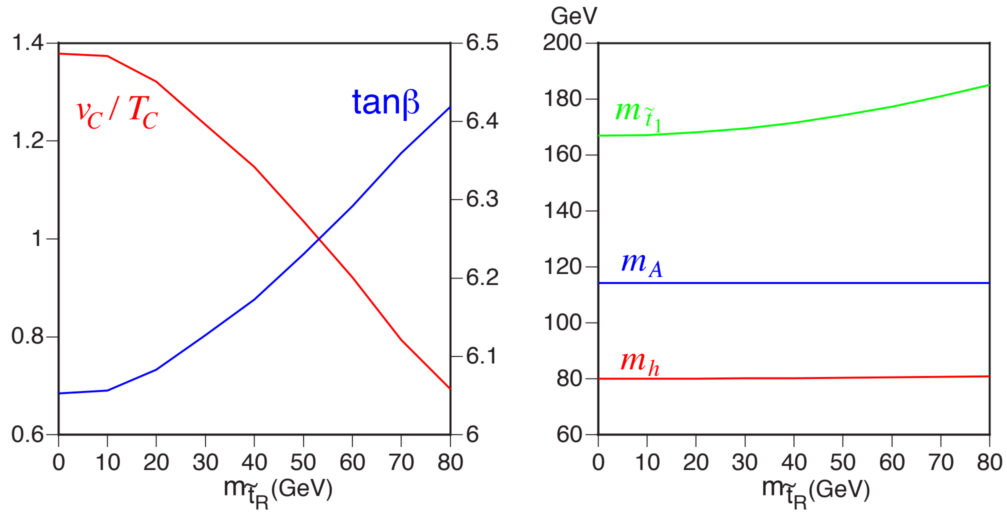
$\longrightarrow$ stronger 1st order PT

effective for larger y_t — smaller $\tan\beta$

An example: $\tan\beta = 6$, $m_h = 82.3$GeV, $m_A = 118$GeV, $m_{\tilde{t}_1} = 168$GeV
$T_C = 93.4$GeV, $v_C = 129$GeV [KF, PTP101]



$V_{\rm eff}(v_1, v_2, v_3 = 0; T_C)$



m_{t_R}-dependence $(\tan\beta = 6)$

⋆ Lattice MC studies

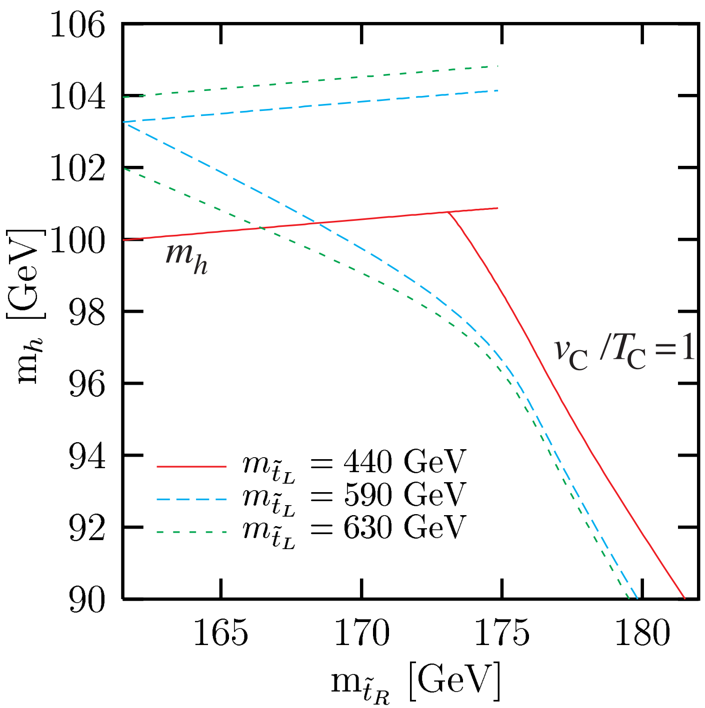
- 3d reduced model [Laine et al. hep-lat/9809045]

  strong 1st order for $m_{\tilde{t}_1} \lesssim m_t$ and $m_h \leq 110\text{GeV}$

- 4d model [Csikor, et al. hep-lat/0001087]

  with $SU(3)$, $SU(2)$ gauge bosons, 2 Higgs doublets, stops, sbottoms

  $A_{t,b} = 0$, $\tan\beta \simeq 6$ $\longrightarrow$ agreement with the perturbation theory within the errors

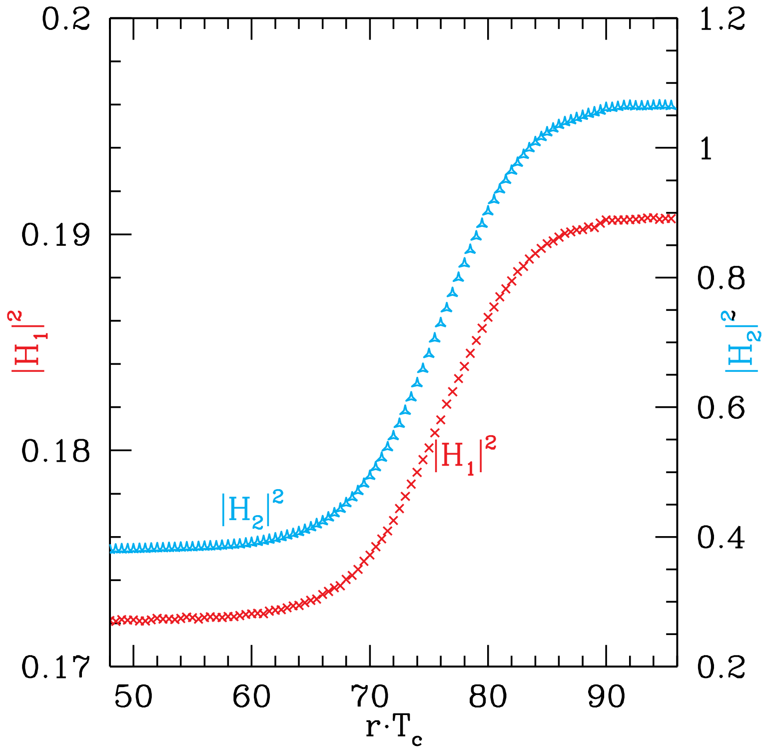


$m_A = 500$ GeV

$v_C/T_C > 1$
below the steeper lines

$\Downarrow$

max. $m_h = 103 \pm 4$ GeV
for $m_{\tilde{t}_L} \simeq 560$ GeV

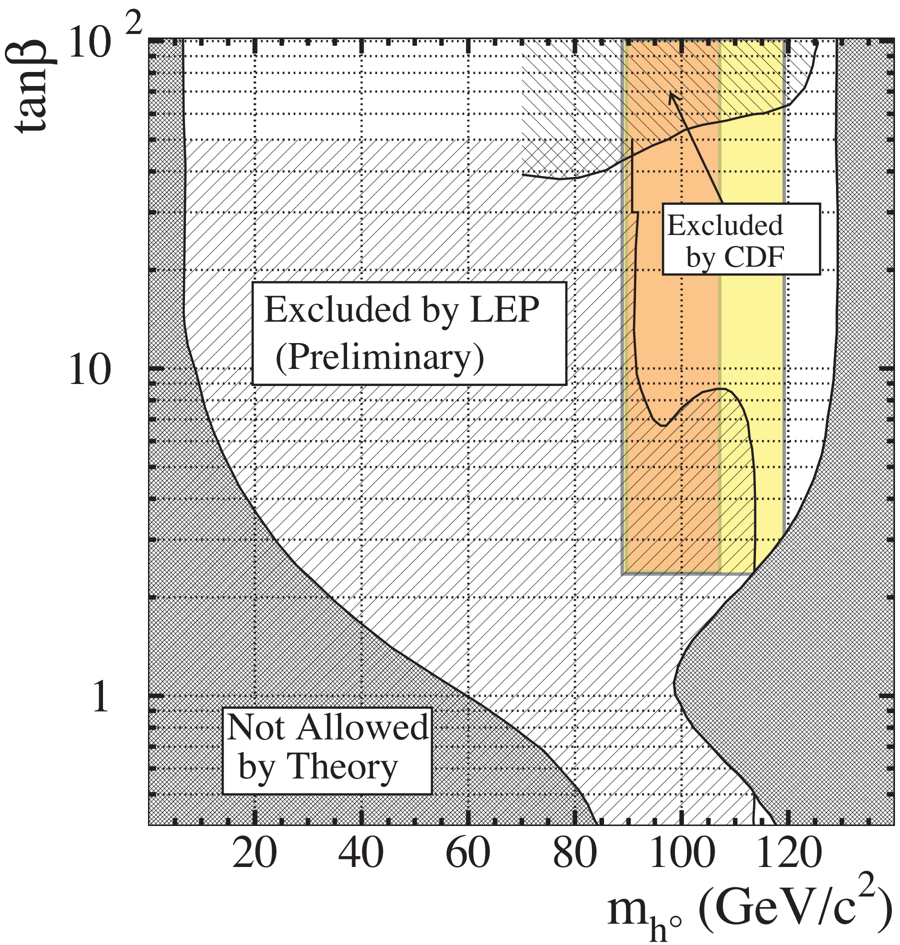


bubble-wall profile

$\Delta\beta = 0.0061 \pm 0.0003$

$\Rightarrow$ $\beta \simeq$ const.

wall width $\simeq \dfrac{11}{T_C}$



[PDG, http://ccwww.kek.jp/pdg/]

light stop: $m_{t_R} = 0$

negative soft mass^2: $m_{t_R}^2 > -(65\text{GeV})^2$

[Laine & Rummukainen, NPB597]

⋆ CP Violation

⋆ relative phases of μ, M_2, M_1, A_t

chargino, neutralino, stop transport [Huet & Nelson, PRD '96; Aoki, et al. PTP '97]

⋆ relative phase $\theta = \theta_1 - \theta_2$ of the two Higgs doublets

quarks and leptons $\Longleftarrow$ Yukawa coupl. $\propto \rho_i e^{i\theta_i}$

chargino, neutralino, stop mass matrix

[Nelson et al. NPB '92; FKOTT, PRD '94, PTP '96]

θ is induced by the loops of SUSY particle.

$\Uparrow \leftarrow \mathrm{Arg}(\mu M_2)$, $\mathrm{Arg}(\mu M_1)$, $\mathrm{Arg}(\mu A_t)$

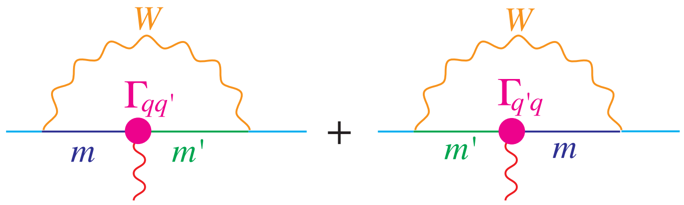
minimum of $V_{\mathrm{eff}}(\rho_i, \theta; T = 0)$

CP violation at $T = 0$ is constrained by experiments (B factory, nEDM, etc)

CP violation at T_C near the bubble wall is relevant to baryogenesis bounds from the EDM : $|d_n| < 0.63 \times 10^{-25} e \cdot \text{cm}$ [Kizukuri & Oshimo, PRD '92]

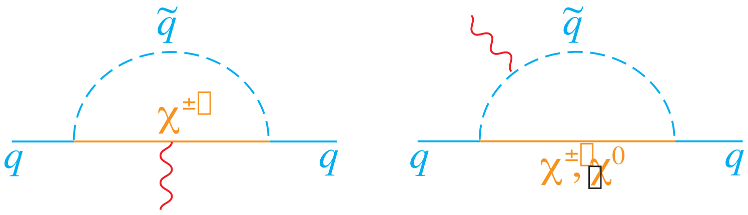
MSM contribution:

CP-odd of [



] $< 10^{-33} e \cdot \text{cm}$

MSSM contribution:



$\cdots$ etc.

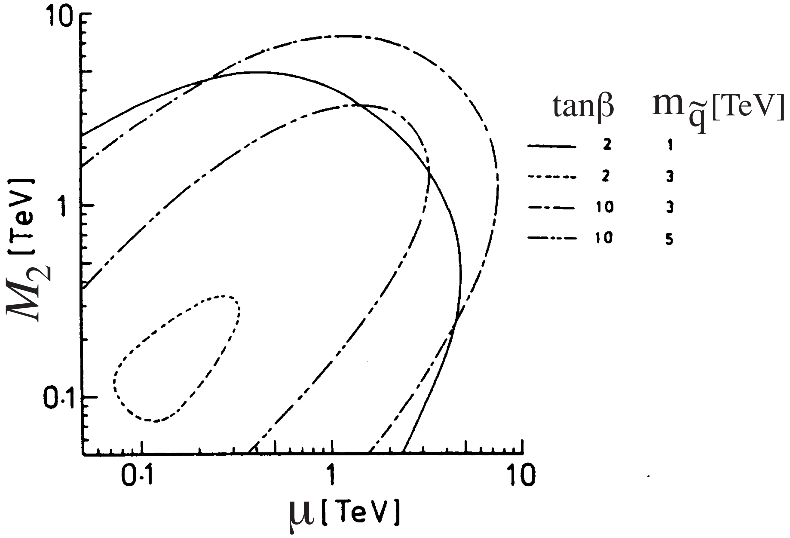


FIG. 4. The parameter regions compatible with the present experimental upper bound on the neutron EDM.

$\theta + \delta_\mu + \delta_2 = \pi/4$
$\mathrm{Arg}A = \pi/4$

inside is excluded

- $\theta + \delta_\mu + \delta_2 = O(1) \Longrightarrow m_{\tilde{q}}, m_{\tilde{l}} \gtrsim 10\text{TeV}$
- $m_{\tilde{q}}, m_{\tilde{l}} \lesssim 1\text{TeV} \Longrightarrow \theta + \delta_\mu + \delta_2 \lesssim 10^{-3}$

CP violation relevant to Baryogenesis — $\theta(x)$ in the bubble wall

Eqs. of motion for $(\rho_i(x), \theta(x))$ with $V_{\rm eff}(\rho_i, \theta; T_C)$

with B.C. determined by the min. of $V_{\rm eff}(T_C)$

$\rho(x)$: 0 (sym. phase) $\longrightarrow v_C$ (br. phase) — kink-like

$\theta(x)$: $-{\rm Arg}(m_3^2)$ (sym. phase) $\longrightarrow \theta_C$ (br. phase) $\overset{\rm loop}{\Longleftarrow}$ explicit CP violation

bubble wall $\sim$ macroscopic, static $\rightarrow$ 1d system $\Rightarrow$ numerical solution

possible CP violations

- $\theta(x)$ near the wall $\sim O(\theta_C)$
- transitional CP violation — $\theta(x) = O(1)$ near the wall, even if $\theta_C \ll 1$

Suppose that at $T \simeq T_C$, without explicit CP violation, $[\theta = \theta_1 - \theta_2]$

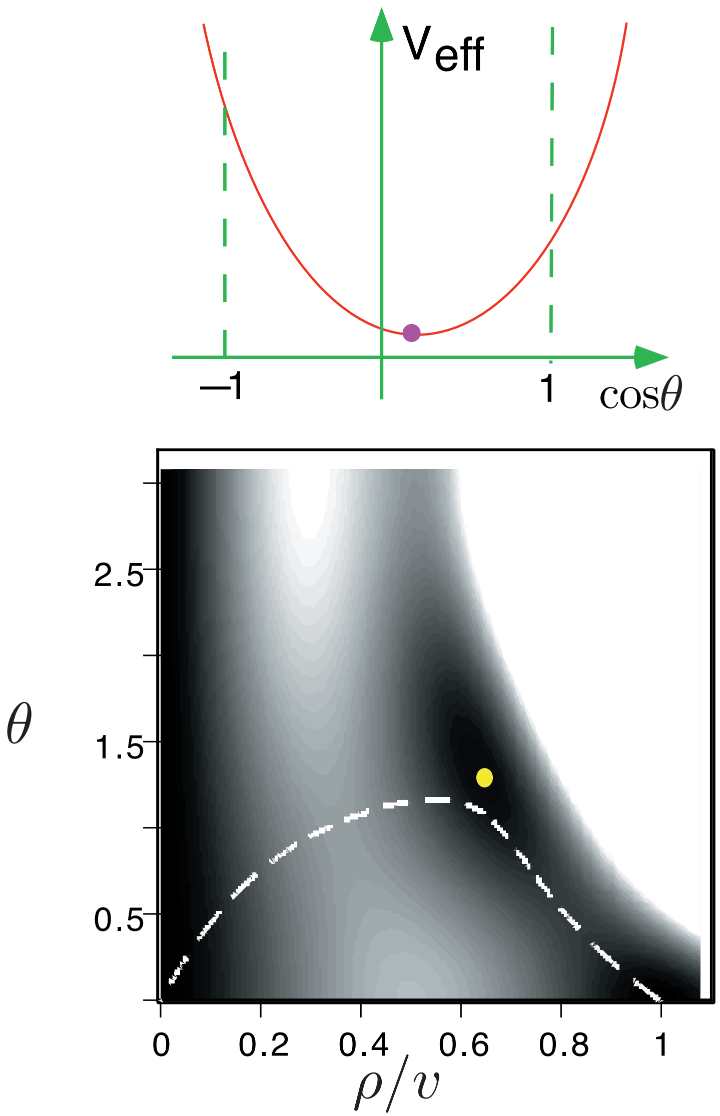
$$
\begin{aligned}
V_{\rm eff}&(\rho_i, \theta) \\
&= \frac{1}{2}\bar{m}_1^2\rho_1^2 + \frac{1}{2}\bar{m}_2^2\rho_2^2 - \bar{m}_3^2\rho_1\rho_2\cos\theta + \frac{\lambda_1}{8}\rho_1^4 + \frac{\lambda_2}{8}\rho_2^4 \\
&+ \frac{\lambda_3+\lambda_4}{4}\rho_1^2\rho_2^2 + \frac{\lambda_5}{4}\rho_1^2\rho_2^2\cos 2\theta - \frac{1}{2}(\lambda_6\rho_1^2 + \lambda_7\rho_2^2)\rho_1\rho_2\cos\theta \\
&- [A\rho_1^3 + \rho_1^2\rho_2(B_0 + B_1\cos\theta + B_2\cos 2\theta) + \rho_1\rho_2^2(C_0 + C_1\cos\theta + C_2\cos 2\theta) + D\rho_2^3] \\
&= \left[\frac{\lambda_5}{2}\rho_1^2\rho_2^2 - 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2)\right]\left[\cos\theta - \frac{2\bar{m}_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 - 8(B_2\rho_1 + C_2\rho_2)}\right]^2 \\
&\qquad +\theta\text{-independent terms}
\end{aligned}
$$

where all the parameters are real

conditions for spontaneous CP violation for a given (ρ_1, ρ_2)

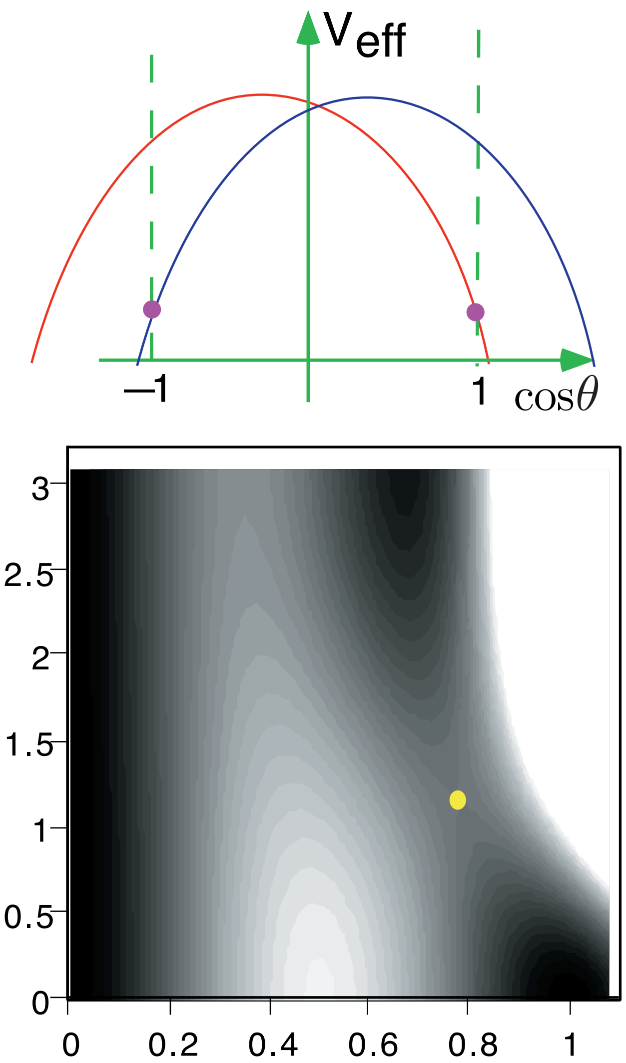
$$
\begin{aligned}
F(\rho_1, \rho_2) &\equiv \frac{\lambda_5}{2}\rho_1^2\rho_2^2 - 2(B_2\rho_1^2\rho_2 + C_2\rho_1\rho_2^2) > 0, \\
-1 < G(\rho_1, \rho_2) &\equiv \frac{2\bar{m}_3^2 + \lambda_6\rho_1^2 + \lambda_7\rho_2^2 + 2(B_1\rho_1 + C_1\rho_2)}{2\lambda_5\rho_1\rho_2 - 8(B_2\rho_1 + C_2\rho_2)} < 1
\end{aligned}
$$

$F(\rho_1, \rho_2) > 0$



CP-violating local min.

$F(\rho_1, \rho_2) < 0$



CP-violating saddle point

An Example

[KF, Otsuki & Toyoda, PTP '99]

input parameters

$\tan\beta_0$	m_3^2	μ	A_t	$M_2 = M_1$	$m_{\tilde{t}_L}$	$m_{\tilde{t}_R}$
6	8110 GeV^2	-500 GeV	60 GeV	500 GeV	400 GeV	0

mass spectrum

m_h	m_A	m_H	$m_{\tilde{t}_1}$	$m_{\chi_1^\pm}$	$m_{\chi_1^0}$
82.28 GeV	117.9 GeV	124.0 GeV	167.8 GeV	457.6 GeV	449.8 GeV

results: $T_C = 93.4$ GeV, $v_C = 129.17$ GeV, $\tan\beta = 7.292$,

inverse wall width: $a = 13.23$GeV

$\Longrightarrow$ BAU: $\dfrac{n_B}{s} \sim 10^{-(12-10)}$ $[v_w = 0.1, \mathrm{Arg}(m_3^2) = 10^{-3}]$

— τ-dominant

Effects of CP violation on the EWPT

[KF, Tao & Toyoda, PTP 109]

EWPT in the light-stop scenario $[m_{\tilde{t}_R} = 10\text{GeV}]$

$\text{Im}(\mu A_t e^{i\theta}) \neq 0 \Rightarrow$

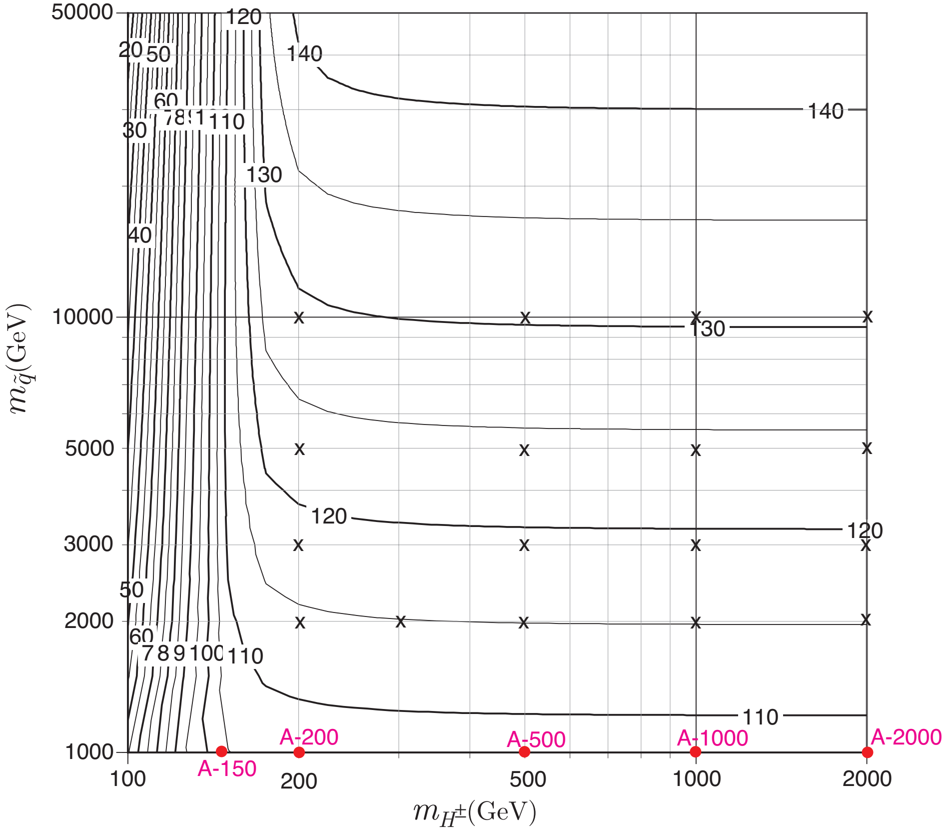
- ▷ scalar-pseudoscalar mixing [Carena, et al., NPB586]
- ▷ induces $\delta = \text{Arg}(m_3^2)$
- • weakens the EWPT

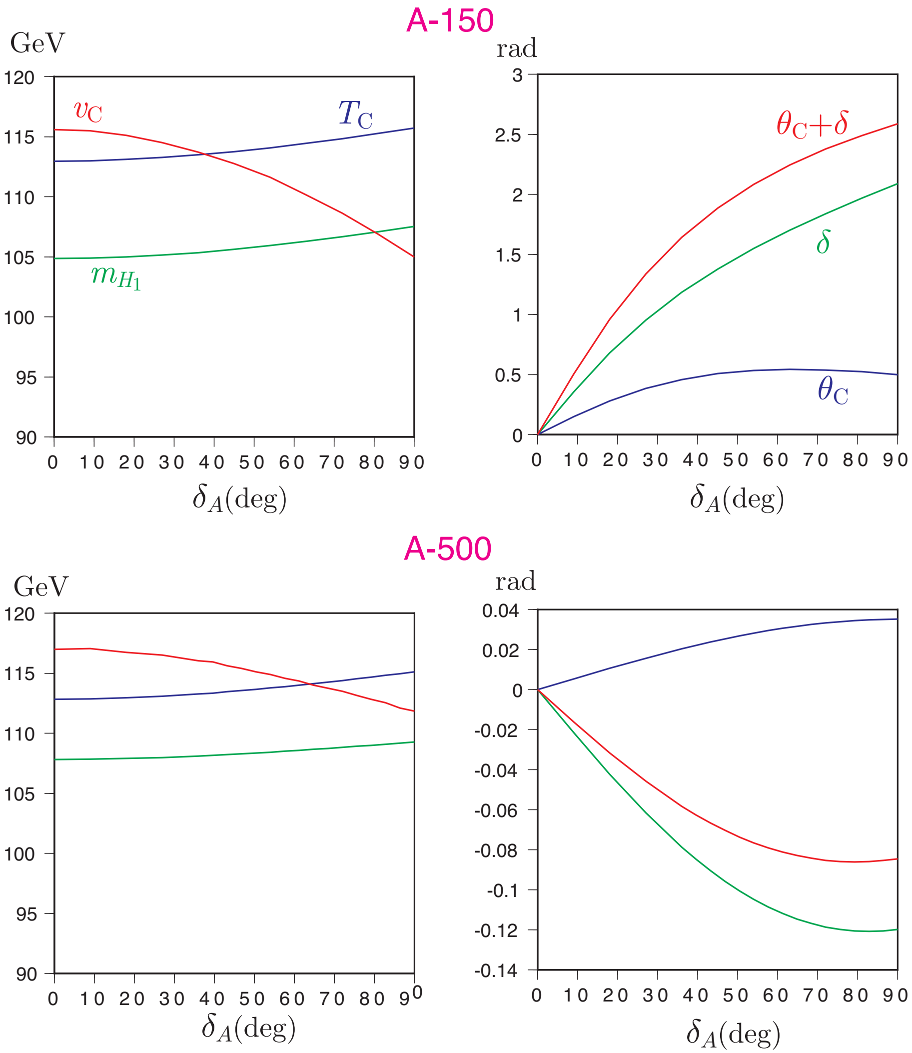
field-dependent mass^2 of the lighter stop:

$$\bar{m}_{\tilde{t}_1}^2 = \frac{1}{2}\left[m_{\tilde{q}}^2 + m_{\tilde{t}_R}^2 + y_t^2 v_u^2 + \frac{g_2^2 + g_1^2}{4}(v_d^2 - v_u^2) - \sqrt{\left(m_{\tilde{q}}^2 - m_{\tilde{t}_R}^2 + \frac{x_t}{2}(v_d^2 - v_u^2) \right)^2 + y_t^2 \left| \mu v_d - A_t^* e^{-i\theta} v_u \right|^2} \right]$$

$\tan\beta = 10,\ \mu = 1500\text{GeV},\ |A| = 150\text{GeV}$

mass of the lightest Higgs boson (CP conserving)





Discussions

Baryogenesis requires

1. baryon number violation
2. C and CP violation
3. departure from equilibrium

$\Downarrow$

rare^{3}

Electroweak Baryogenesis

- **based on a testable model** $\longleftrightarrow$ stringent constraints
- **free from proton decay problem**

other attemps:

⋆ GUTs

⋆ Leptogenesis

⋆ Affleck-Dine

⋆ Gravitational Baryogeneisis

[Alexander, Peskin & Sheikh-Jabbari, hep-th/0405214]

⋆ Inflationary Baryogenesis [KF, Kakuto, Otsuki & Toyoda, PTP 105]

[Nanopoulos & Rangarajan, PRD 64]

viable models for EW baryogenesis

- Minimal SM — excluded !! ✕ $\left\{ \begin{array}{l} \text{strongly 1st-order EWPT (with acceptable } m_h\text{)} \\ \text{sufficient } CP \text{ violation} \end{array} \right.$

- MSSM

  ⋆ $\left. \begin{array}{c} m_h \leq 110\text{GeV} \ \text{ and } \ m_{\tilde{t}_1} \leq m_t \\ m_h \leq 120\text{GeV} \ \text{ if } \ m^2_{\tilde{t}_R} < 0\,? \end{array} \right\}$ $\Longrightarrow$ 1st-order EWPT with $\dfrac{v_C}{T_C} > 1$

  ⋆ many sources of CP violation

  complex parameters: $\mu,\ M_2,\ M_1,\ A;\ \theta$ — $\mathrm{Im}(\mu A_t e^{i\theta})$ weakens the EWPT

- Other extensions of the MSM

  ▷ non-SUSY : 2HDM — many parameters not so constrained

  ▷ Next-to-MSSM (NMSSM) $=$ MSSM $+$ Singlet chiral superfield

  [KF & Tao, in preparation]

NMSSM

$$W = \epsilon_{ij} \left(y_b H_d^i Q^j B - y_t H_u^i Q^j T + y_l H_d^i L^j E - \lambda N H_d^i H_u^j \right) - \frac{\kappa}{3} N^3$$

$\lambda \langle N \rangle \sim \mu$ in the MSSM

$$\mathcal{L}_{\rm soft} \ni -m_N^2 n^* n + \left[\lambda A_\lambda n H_d H_u + \frac{\kappa}{3} A_\kappa n^3 + \text{h.c.} \right].$$

order parameters: $\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad \langle \Phi_u \rangle = \frac{e^{i\theta}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle n \rangle = \frac{e^{i\varphi}}{\sqrt{2}} v_n$

tree-level Higgs potential:

$$\begin{aligned} V_0 \quad = \quad & m_1^2 \Phi_d^\dagger \Phi_d + m_2^2 \Phi_u^\dagger \Phi_u + m_N^2 n^* n - \left(\lambda A_\lambda \epsilon_{ij} n \Phi_d^i \Phi_u^j + \frac{\kappa}{3} A_\kappa n^3 + \text{h.c.} \right) \\ & + \frac{g_2^2 + g_1^2}{8} \left(\Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u \right)^2 + \frac{g_2^2}{2} \left(\Phi_d^\dagger \Phi_u \right) \left(\Phi_u^\dagger \Phi_d \right) \\ & + |\lambda|^2 n^* n \left(\Phi_d^\dagger \Phi_d + \Phi_u^\dagger \Phi_u \right) + \left| \lambda \epsilon_{ij} \Phi_d^i \Phi_u^j + \kappa n^2 \right|^2 \end{aligned}$$

1. 5 neutral and 1 charged scalars (3 scalar and 2 pseudoscalar when CP cons.)

   The lightest Higgs scalar can escape from the lower bound 114GeV, because of small coupling to Z caused by large mixing among 3 scalars. [Miller, et al. NPB 681]

2. CP violation at the tree level: $\mathrm{Im}\left(\lambda A_\lambda e^{i(\theta+\varphi)}\right)$, $\mathrm{Im}\left(\kappa A_\kappa e^{3i\varphi}\right)$, $\mathrm{Im}\left(\lambda\kappa^* e^{i(\theta-2\varphi)}\right)$.

3. $v_n \to \infty$ with λv_n and κv_n fixed $\Longrightarrow$ MSSM [Ellis, et al, PRD 39]

   $\longrightarrow$ new features expected for $v_n = O(100)$GeV

the parameters are restricted by requiring

- all the mass^2 of the scalars be positive
- the vacuum $(v_0, \tan\beta, v_n)$ be the absolute minimum of $V_{\rm eff}$

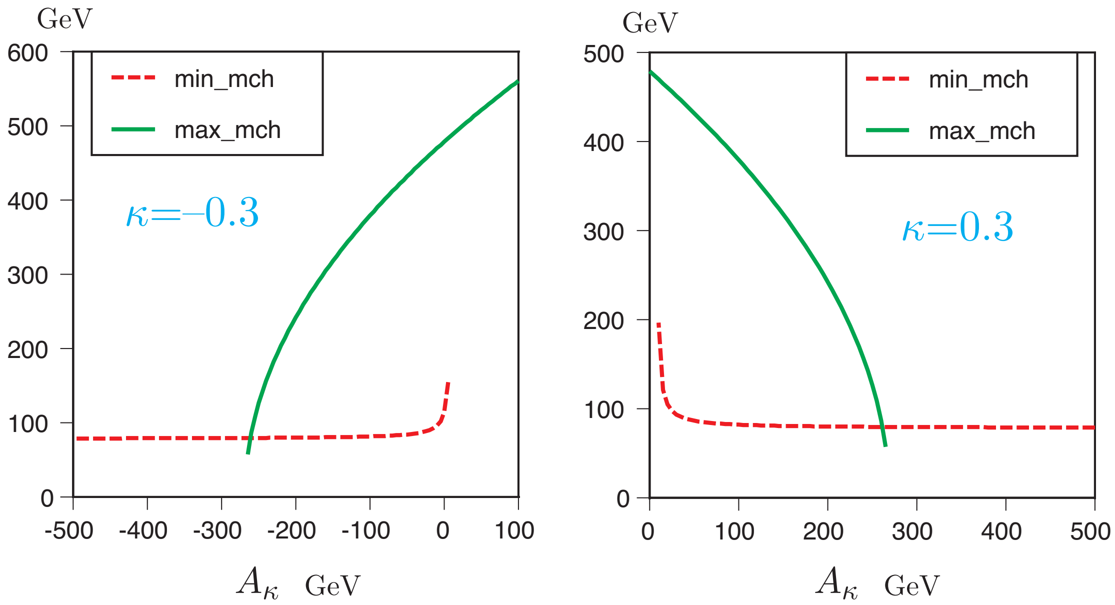
Without CP violation, at the tree level,

$$m_{H^\pm}^2 = m_W^2 - \frac{1}{2}\lambda^2 v_0^2 + (2R_\lambda - \lambda\kappa v_n)\frac{v_n}{\sin 2\beta}, \qquad R_\lambda = \frac{1}{\sqrt{2}}\lambda A_\lambda$$

- $m_{A_1}^2 > 0 \Longrightarrow$ lower bound on $m_{H^\pm}$ $\qquad$ [$m_{H^\pm}^2 = m_W^2 + m_A^2$ in the MSSM]
- $V_0(v_0, \tan\beta, v_n) < V_0(0) \Longrightarrow$ upper bound on $m_{H^\pm}$

[this bound $\to \infty$ in the MSSM limit]

$\tan\beta = 5, \;\; v_n$=300GeV



$\Rightarrow \kappa A_k > 0$ is favored

EWPT in the NMSSM

[Pietroni, NPB402]

order parameters :
$$\begin{cases} v_d = v\cos\beta(T) = y\cos\alpha(T)\cos\beta(T), \\ v_u = v\sin\beta(T) = y\cos\alpha(T)\sin\beta(T), \\ v_n = y\sin\alpha(T) \end{cases}$$

$$\begin{aligned} V_0 &= \frac{1}{2}\left((m_1^2\cos^2\beta + m_2^2\sin^2\beta)\cos^2\alpha + m_N^2\sin^2\alpha\right) y^2 \\ &\qquad - \left(R_\lambda\cos^2\alpha\sin\alpha\cos\beta\sin\beta + \frac{1}{3}R_\kappa\sin^3\alpha\right) y^3 + \cdots. \end{aligned}$$

$$R_\lambda = \frac{1}{\sqrt{2}}\lambda A_\lambda, \quad R_\kappa = \frac{1}{\sqrt{2}}\kappa A_\kappa \in \mathbf{R}$$

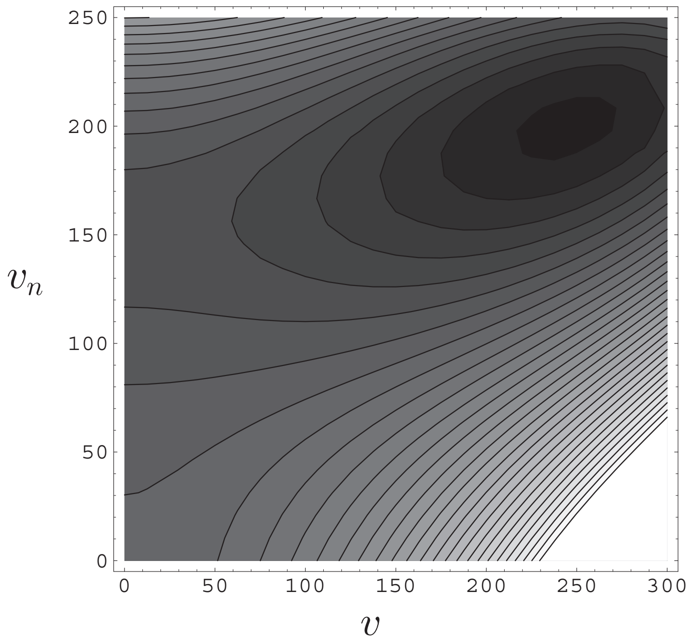
$\longrightarrow$ y^3-term even at the tree level !

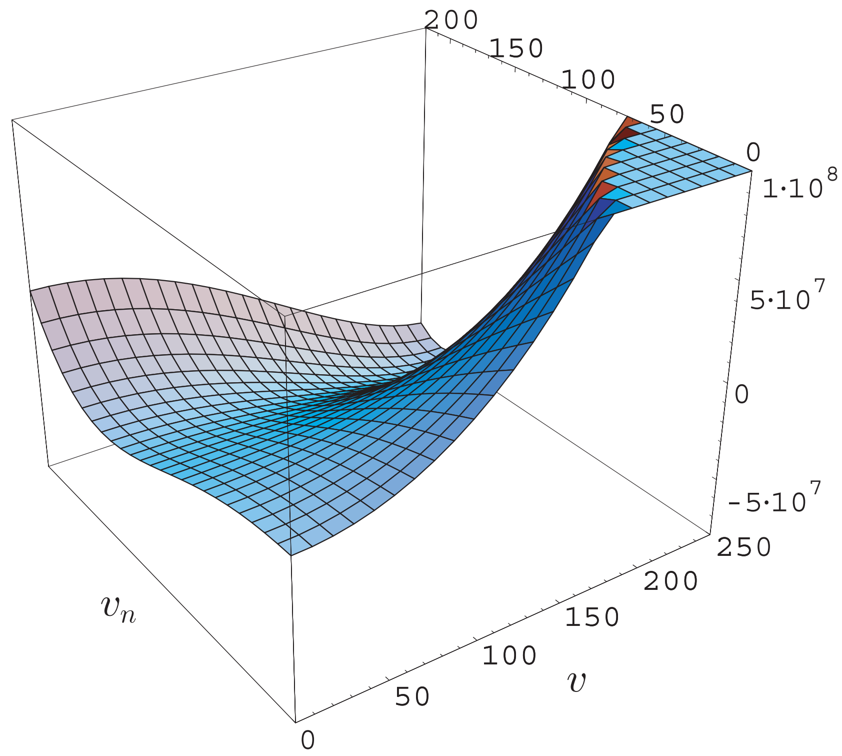
$\Longrightarrow$ stronger 1st-order PT ?

our study of the EWPT

$\beta(T_C) = $ const., but $\alpha(T_C) \neq$ const. along the least $V_{\rm eff}$ path

"reduced effective potential": $\tilde{V}_{\rm eff}(v, v_n; T) \equiv V_{\rm eff}(v \cos\beta(T), v \sin\beta(T), v_n; T)$





work in progress $\cdots$