3/5, 2004 at KEK

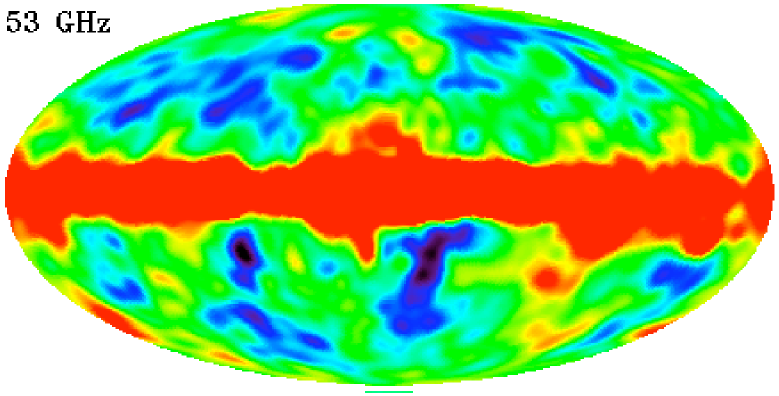
WMAPが開く量子重力的宇宙像

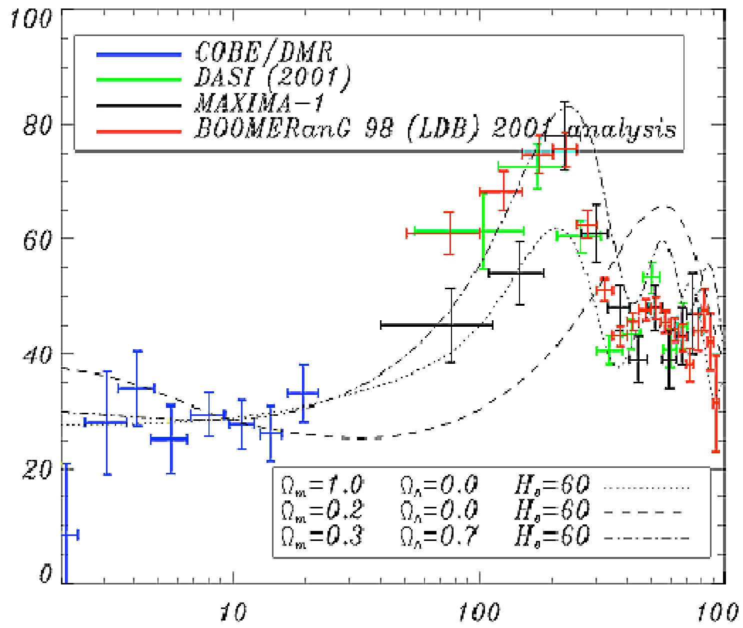
KEK 浜田賢二

Based on
CMB Anisotropies Reveal Quantized Gravity
By 湯川 哲之 (総研大) and K.H.
astro-ph/0401070

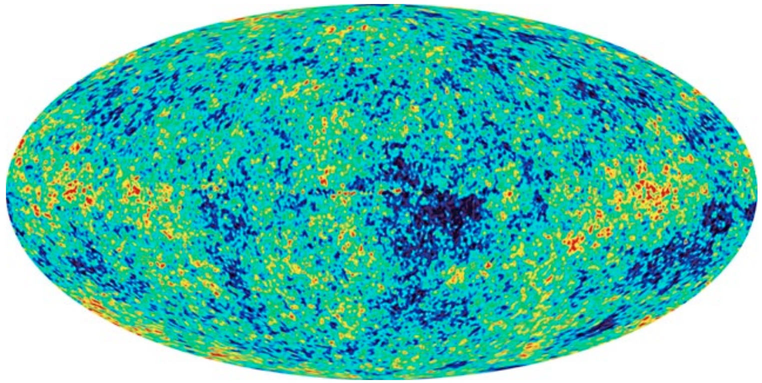
Anisotropies in the CMB

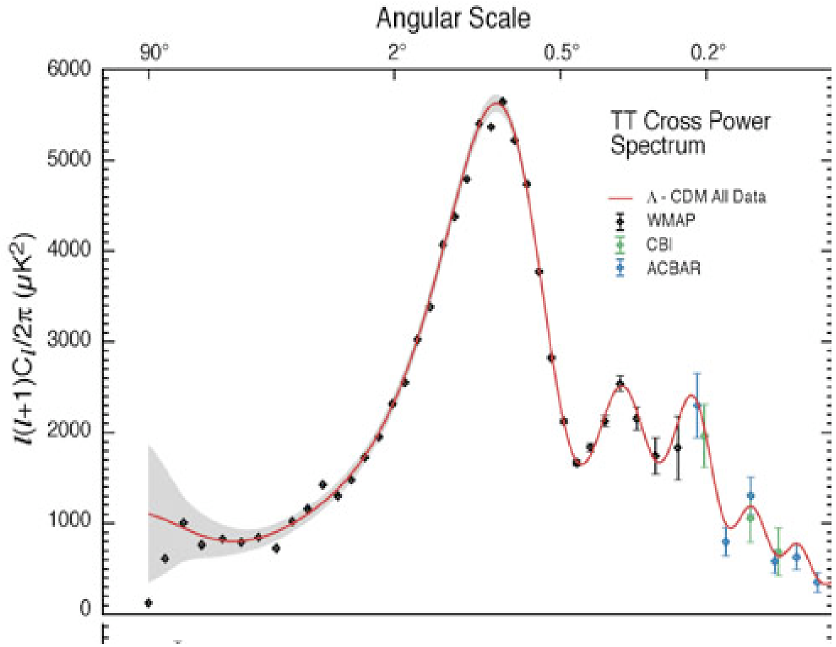
COBE (1996)





WMAP (2003)

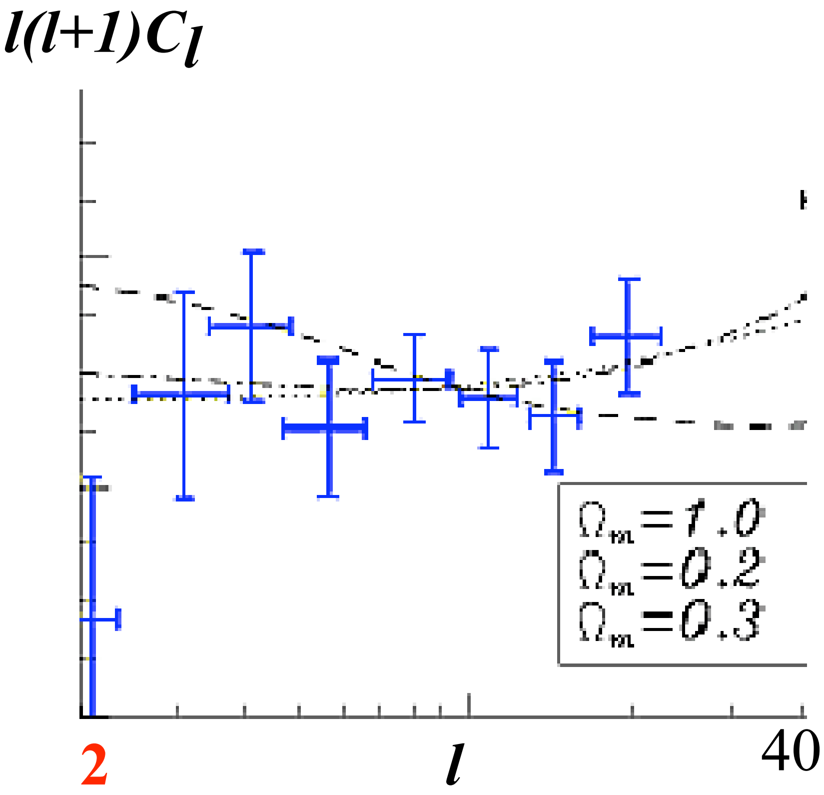


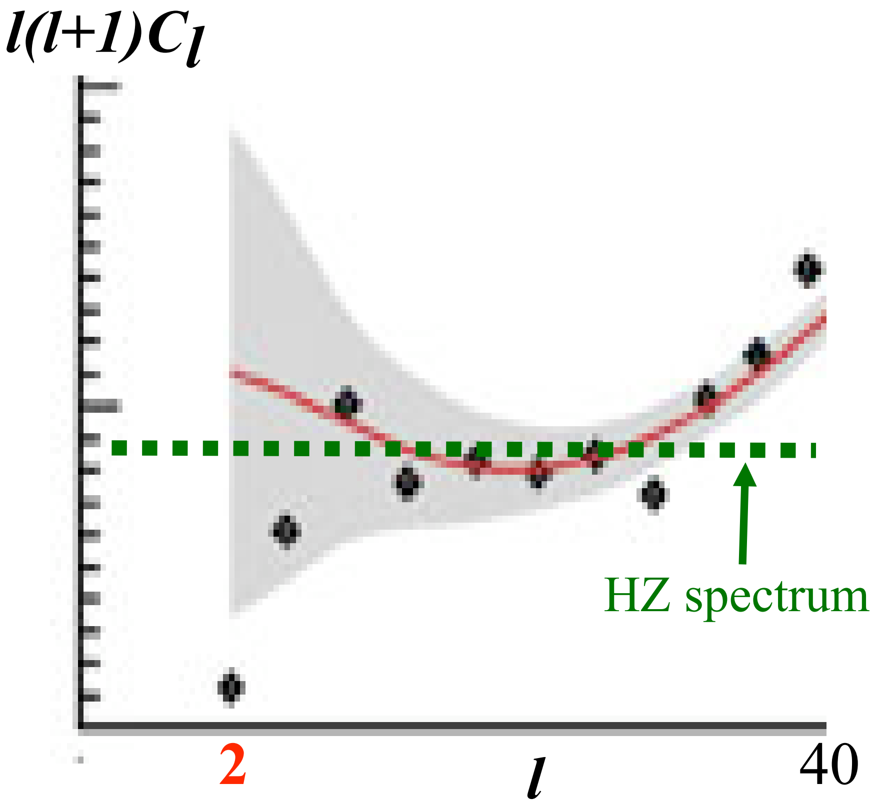


Angular Power Spectrum

Angular Power Spectrum at Ultra Super-Horizon (l<40) Region

Sachs-Wolfe effect with Harrison-Zel'dovich spectrum





Large deviation at l=2

No scale (except deSitter scale)

$\Longleftrightarrow$ Harrison-Zel'dovich spectrum

Dynamical scales deform HZ spectrum

c_s sound speed $\longrightarrow$ acoustic peak

K spatial curvature

Sharpe damping at large angle can be explained by cosmic variance ?
No, it indicates new dynamical scale!

Because cosmic variance is based on Ergodic Hypothesis

(2l+1) Ensemble of sub-Universe

→ statistical error of $C_l : \pm 1/\sqrt{2l+1}$

But, at super-horizon region two points causally disconnected

↓

Not Ergodic

Initial quantum fluctuations will be preserved for super-horizon separation

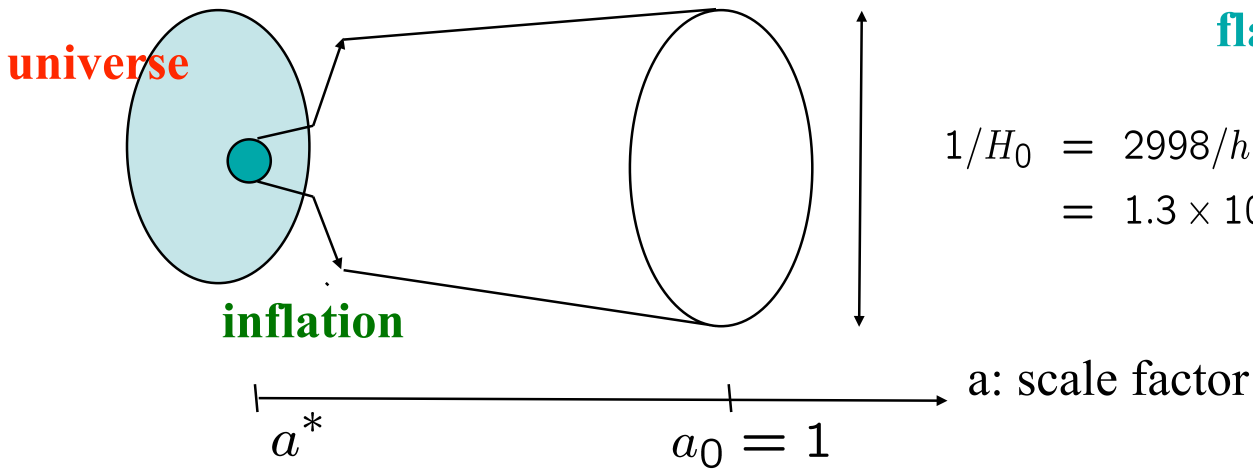
If we believe the idea of inflation, CMB anisotropies provide us information about dynamics beyond the Planck scale.

⬇

Dynamical scale of Quantum Gravity

NB. Trans-Plankian problem: $L_P = a^* \times (1/H_0)$ ➞ $a^* = 6.3 \times 10^{-59}$

Necessary to solve flatness problem



$$\begin{aligned} 1/H_0 &= 2998/h \text{ Mpc } (h = 0.72) \\ &= 1.3 \times 10^{28} \text{ cm} \end{aligned}$$

Contents

2. Why Quantum Gravity is necessary

- **Singularity in BH and early universe**

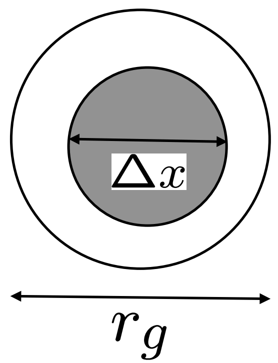
  → **divergence/non-calculability**
  information loss

- **Breakdown of particle picture**

  A excitation with the Planck mass → **BH**

Heisenberg uncertainty **Schwarzshild radius**

$$\Delta x \sim \frac{\hbar}{M_{\mathsf{P}} c} = L_{\mathsf{P}} \quad < \quad r_g = \frac{2 G M_{\mathsf{P}}}{c^2} = 2 L_{\mathsf{P}}$$



cf. Proton mass, m

$$\Delta x \sim \frac{\hbar}{mc} = 10^{-14}\ (\mathrm{cm}) \quad > \quad r_g = \frac{2Gm}{c^2} = 10^{-52}\ (\mathrm{cm})$$

Spacetime fluctuates greatly

→ **loss of the concept of distance**
= background-metric independence

No scales /No singularity

Dynamical scale generates classical spacetime

量子重力は特定の時空の上の場の量子論ではなく、
時空のゆらぎそのものを記述するものでなければならない。
同時に、いわゆる我々の時空を生成するダイナミクス
を含むものでなければならない。

3. The Renormalizable Model

Limitation of Einstein theory

- **non-renormalizable**
- **singular spacetime configs. cannot be removed**

$$\exp\left(-\int d^4x\sqrt{g}R\right) \sim O(1) \quad \text{for } R=0$$

(Here, Euclidean is considered)

Conformal weight

$$\exp\left(-\int d^4x\sqrt{g}C^2_{\mu\nu\lambda\sigma}\right) \to 0 \quad \text{for } C_{\mu\nu\lambda\sigma}=\infty$$

Weyl tensor

$$C_{\mu\nu\lambda\sigma} = R_{\mu\nu\lambda\sigma} + \cdots$$

Singularity removed !

The Renormalizable Gravity

$$I=\int d^4x\sqrt{-g}\left\{\underbrace{-\frac{1}{t^2}C^2_{\mu\nu\lambda\sigma}-bG_4}_{\text{conformal invariant 4-derivative actions}}+\frac{M_{\mathrm{P}}}{2}R-\Lambda_{\mathrm{cos}}\right\}+I_{\mathrm{M}}$$

sgn=(-1,1,1,1)

The metric fields in strong gravity phase

$$g_{\mu\nu}=\underbrace{e^{2\phi}}_{\text{conformal mode}}\left(\hat{g}_{\mu\nu}+t\underbrace{h_{\mu\nu}}_{\text{traceless mode}}+\cdots\right), \qquad tr(h)=0$$

t : The coupling constant for traceless mode

$C_{\mu\nu\lambda\sigma}$: Weyl tensor = field strength of traceless mode

G_4 : Topological density

I_{M} : Mater actions (scalar N_{X}, fermion N_{W} ,vector N_{A})

Conformal mode is treated non-perturbatively

The partition function

$$
\begin{aligned}
Z &= \int [dg \cdots]_{\underline{g}} \exp(iI) \\
&= \int [d\phi dh \cdots]_{\underline{\hat{g}}} \exp(i\underline{S(\phi)} + iI)
\end{aligned}
$$

Jacobian = Wess-Zumino action

Dynamics of conformal mode is induced from the measure:

$$
S(\phi) = -\frac{b_1}{(4\pi)^2} \int d^4x \sqrt{-\hat{g}} \left\{ 2\phi \hat{\Delta}_4 \phi + \left(G_4 - \frac{2}{3} \hat{\nabla}^2 \hat{R} \right) \phi \right\} + O(\phi^3)
$$

Higher order of the coupling t

➡ Conformal Field Theory (CFT)

conformal inv : $\phi \to \phi + \omega$**, and thus**

$$
Z(e^{2\omega}\hat{g}) = Z(\hat{g})
$$

Dynamics of the Traceless Mode

Asymptotically Free

$$\beta_t = -\beta_0 t_r^3 + \cdots \qquad (\beta_0 > 0)$$

Dynamical Scale of Gravity

$$\alpha_G = \frac{t_r^2(p)}{4\pi} = \frac{1}{4\pi\beta_0} \frac{1}{\log(p^2/\Lambda_{\mathrm{QG}}^2)}$$

Running coupling constant

At very high energies $E \gg \Lambda_{\mathrm{QG}}$, the coupling vanishes and conformal mode dominates => CFT

※This also implies that background-metric independence for traceless mode is less important.

Renormalization (QED + gravity)

Beta functions

$$\beta_t = -\left(\frac{n_F}{40}+\frac{10}{3}\right)\frac{t_r^3}{(4\pi)^2} - \frac{7n_F}{72}\frac{e_r^2 t_r^3}{(4\pi)^4} + o(t_r^5)$$

$$\beta_e = \frac{4n_F}{3}\frac{e_r^3}{(4\pi)^2} + \left(4n_F - \frac{8}{9}\frac{n_F^2}{b_1}\right)\frac{e_r^5}{(4\pi)^4} + o(e_r^3 t_r^2)$$

where $b_1 = \frac{11n_F}{360} + \frac{40}{9}$: coeff. of WZ action of type $\phi\hat{\Delta}_4\phi$

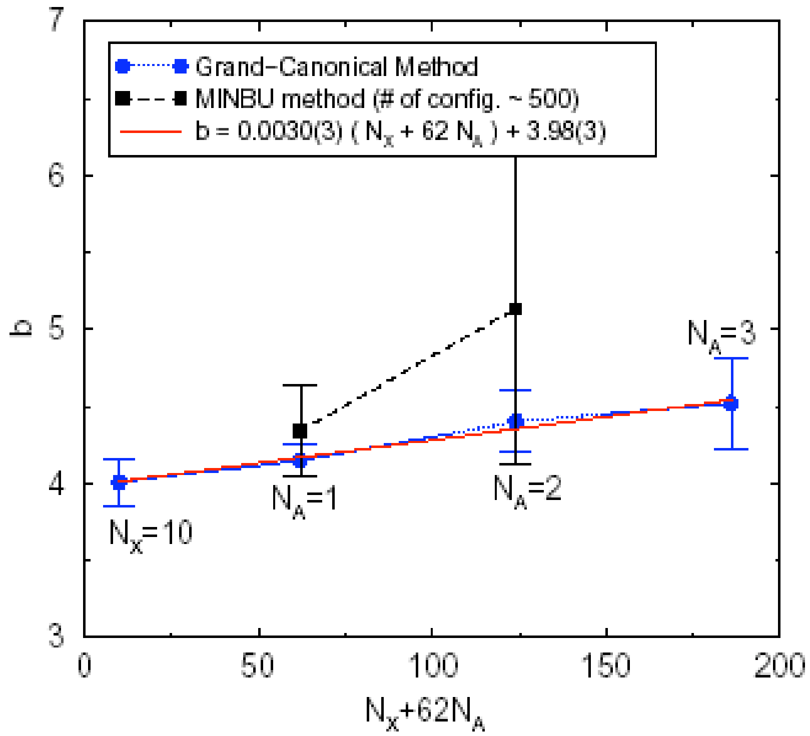
New WZ actions (=new vertices) like $\phi^n F_{\mu\nu}F^{\mu\nu}$ are induced at higher orders

Conformal mode is not renormalized : $Z_\phi = 1$

K.H. hep-th/0203250

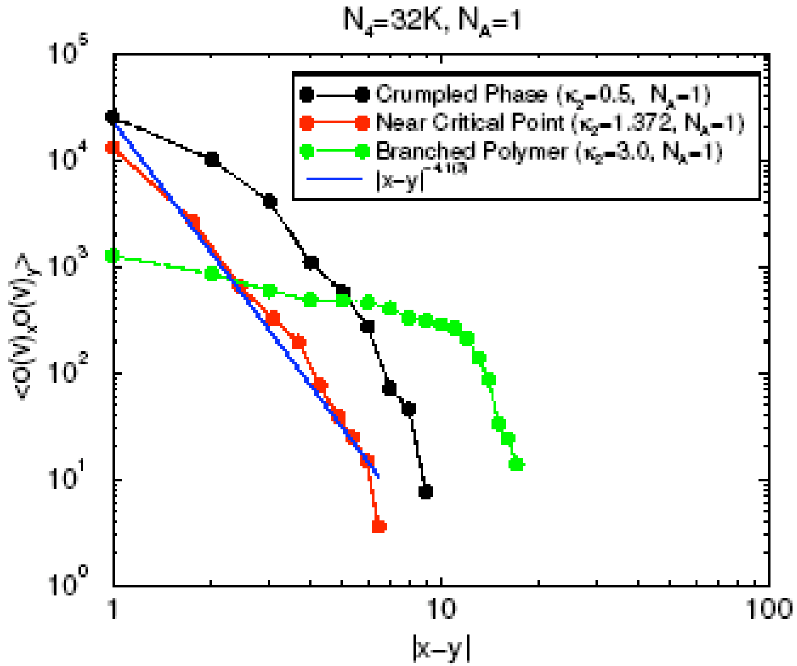
Simplicial Quantum Gravity support Renormalizable Gravity

String susceptibility



$b(\text{theor.}) = 0.0028(N_X + 62N_A) + 4.27 - o(t^2)$

Two Point Correlation



Horata, Egawa, Yukawa,
hep-lat/0209004

4. New Dynamical Scenario of Inflation

Order of Mass Scales

$$M_{\mathrm{P}} \gg \Lambda_{\mathrm{QG}} \gg \Lambda_{\mathrm{COS}}^{1/4}$$

At very high energies $E \gg M_{\mathrm{P}}$ **:**

$$C_{\mu\nu\lambda\sigma} \to 0 \ \ (t \to 0) \quad \Longrightarrow \quad g_{\mu\nu} = e^{2\phi}\hat{g}_{\mu\nu}$$

Conformal mode dominates => CFT

Wess-Zumino action (=Jacobian)

$$S(\phi)|_{t=0} = -\frac{b_1}{(4\pi)^2}\int d^4x\sqrt{-\hat{g}}\left\{2\phi\hat{\Delta}_4\phi + \left(G_4 - \frac{2}{3}\hat{\nabla}^2\hat{R}\right)\phi\right\}$$

where $$b_1 = \frac{1}{360}\left(N_{\mathrm{X}} + \frac{11}{2}N_{\mathrm{W}} + 62N_{\mathrm{A}}\right) + \frac{769}{180}$$

conf. inv. 4-th order op. : $\sqrt{-\hat{g}}\hat{\Delta}_4 = \sqrt{-\hat{g}}(\hat{\nabla}^4 + \cdots)$

For $M_{\mathrm{P}} \geq E \geq \Lambda_{\mathrm{QG}}$: CFT + Einstein term

Conformal mode fluctuates greatly around inflating solution of E.O.M:

$$b_1 \partial_\eta^4 \phi_{\mathrm{cl}} - 24\pi^2 M_{\mathrm{P}}^2 e^{2\phi_{\mathrm{cl}}} \left\{ \partial_\eta^2 \phi_{\mathrm{cl}} + (\partial_\eta \phi_{\mathrm{cl}})^2 \right\} = 0$$

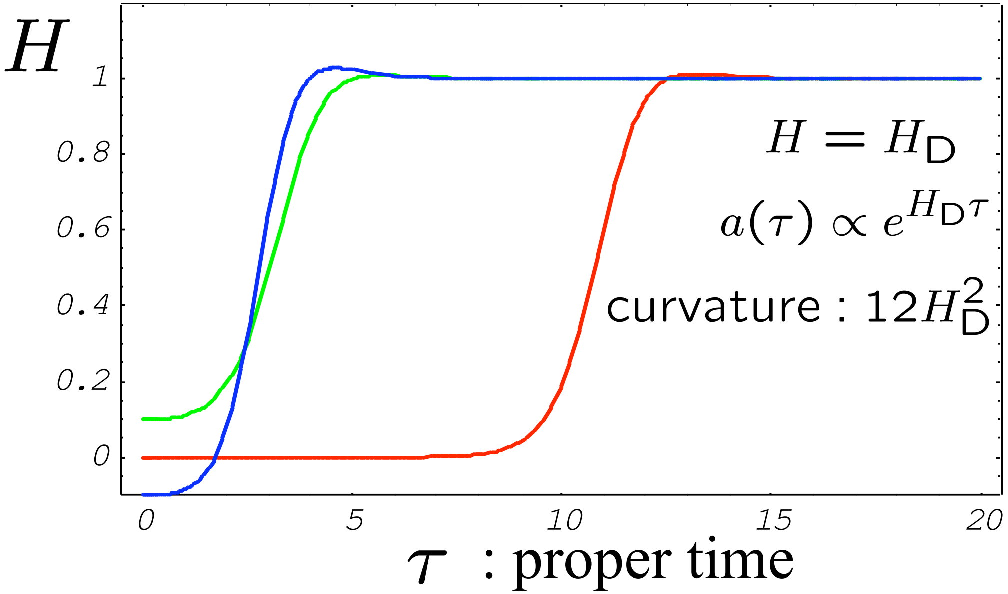
$$\Rightarrow \quad b_1 \left(\dddot{H} + 7H\ddot{H} + 4\dot{H}^2 + 18H^2\dot{H} + 6H^4 \right) - 24\pi^2 M_{\mathrm{P}}^2 \left(\dot{H} + 2H^2 \right) = 0$$

where $a(\tau) = e^{\phi_{\mathrm{cl}}(\tau)}$

$$d\tau = a d\eta$$

$$H(\tau) = \frac{\dot{a}(\tau)}{a(\tau)}$$

$$H_{\mathrm{D}} = \sqrt{\frac{8\pi^2}{b_1}} M_{\mathrm{P}}$$



For $E \leq \Lambda_{\mathsf{QG}}$:

The coupling diverges : $\alpha_G \to \infty$

Interactions become short range and effective action changes drastically

At Λ_{QG} , inflation terminates and Einstein spacetime is generated

The number of e-foldings :

$$\mathcal{N}_e = \log \frac{a(\tau_{\mathsf{P}})}{a(\tau_{\Lambda})} = H_{\mathsf{D}}(\tau_{\Lambda} - \tau_{\mathsf{P}}) \simeq \frac{H_{\mathsf{D}}}{\Lambda_{\mathsf{QG}}}$$

Large number of e-foldings can be obtained

Summary of the model

E

Background-metric independent spacetime
No singularity
Described by CFT (long range)

Quantum spacetime

M_{P}

Inflation era (still long range)

Λ_{QG}

The traceless-mode coupling α_G gets large,
and correlation becomes short range: $1/\Lambda_{\mathrm{QG}}$

Friedmann era

ordinary particles are described
as excitations around classical spacetime:
graviton, photons, fermions.

Classical spacetime

$\Lambda_{\mathrm{cos}}^{1/4}$

De Sitter again

5. Primordial Power Spectrum

WMAP observes quantum fluctuations of scalar curvature just before quantum spacetime transits to classical spacetime at $\tau_\Lambda = 1/\Lambda_{\mathrm{QG}}$

$$\frac{\delta R}{R}\Big|_{\tau_\Lambda^-} \longrightarrow \frac{\delta\rho}{\rho}\Big|_{\tau_\Lambda^+} \longrightarrow \frac{\delta T}{T}\Big|_{\mathrm{today}}$$

transition to classical spacetime

spectrum deformed by scales

Power spectrum = Two-point correlation

$$\left\langle \frac{\delta T}{T}\frac{\delta T}{T} \right\rangle \sim \left\langle\left\langle \frac{\delta R}{R}\frac{\delta R}{R} \right\rangle\right\rangle$$

Two-point correlation of scalar curvature is calculated by CFT

Scaling dimension of curvature

$$\gamma_n = 2b_1\left(1-\sqrt{1-\frac{4-n}{b_1}}\right)$$

$$\begin{aligned}\Delta_R &= 4-4\frac{\gamma_2}{\gamma_0} \\ &= 2+\underbrace{1/b_1+2/b_1^2+o(1/b_1^3)}_{\text{anomalous dimensions by CFT}}\end{aligned}$$

Annomalous dimension by the traceless mode dynamics

$$\bar{\Delta}_R = \Delta_R + u\alpha_G, \qquad (u>0)$$

Two-point correlation function:

$$\left\langle\left\langle \frac{\delta R}{R}(\tau_\Lambda, \mathbf{r})\frac{\delta R}{R}(\tau_\Lambda, \mathbf{r}')\right\rangle\right\rangle \sim \left(H_{\mathrm{D}}|\mathbf{r}-\mathbf{r}'|\right)^{-2\bar{\Delta}_R}$$

Physical distance

Angular Power Spectrum: C_l

Legendre polynomials

$$c_2(\theta) = \left\langle \frac{\delta T}{T}(\mathbf{n}) \frac{\delta T}{T}(\mathbf{n}') \right\rangle = \frac{1}{4\pi} \sum_l C_l (2l+1) P_l(\underbrace{\mathbf{n}\cdot\mathbf{n}'}_{\cos\theta})$$

Using the relations:

$$\delta T/T \simeq \Phi(\mathbf{x}_{\mathrm{ls}})/3$$

Sachs-Wolfe relation

$$\vec{\nabla}^2 \Phi = 4\pi G \delta\rho$$

Poisson eq.

$$H^2 = 8\pi G \rho/3$$

Friedmann eq.

and our proposal: $\delta\rho/\rho|_{\tau_{\mathrm{rec}}} \sim \delta R/R|_{\tau_\Lambda}$

$$c_2(\theta) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{4} \left(\frac{m_{\mathrm{rec}}}{k} \right)^4 \left\langle\left\langle \frac{\delta R}{R}(\mathbf{k}) \frac{\delta R}{R}(-\mathbf{k}) \right\rangle\right\rangle e^{i\mathbf{k}\cdot(\mathbf{n}-\mathbf{n}')\mathbf{x}_{\mathrm{ls}}}$$

$k = |\mathbf{k}|$ **: comoving wavenumber at** τ_Λ **, or** $\mathbf{k} = a(\tau_\Lambda)\mathbf{p}$

$$m_{\mathrm{rec}} = a(\tau_{\mathrm{rec}}) H(\tau_{\mathrm{rec}})$$

We finally obtain $C_l = \int_\lambda^\infty \frac{dk}{k} j_l^2(kx_{\mathrm{ls}}) P(k)$ **with**

$$P(k) = A \left(\frac{k}{m_\lambda}\right)^{n-1+\frac{v}{\log(k/\lambda)^2}} \qquad A = \frac{A_{\mathrm{CFT}}}{2\pi}\left(\frac{m_{\mathrm{rec}}}{m_\lambda}\right)^4$$

running coupling effect

where

$$\begin{aligned} n &= 2\Delta_R - 3 \\ &= 1 + 2/b_1 + 4/b_1^2 + O(1/b_1^3) \end{aligned}$$ **:spectral index by CFT**

$\lambda = a(\tau_\Lambda)\Lambda_{\mathrm{QG}}$ **: comoving dynamical scale**

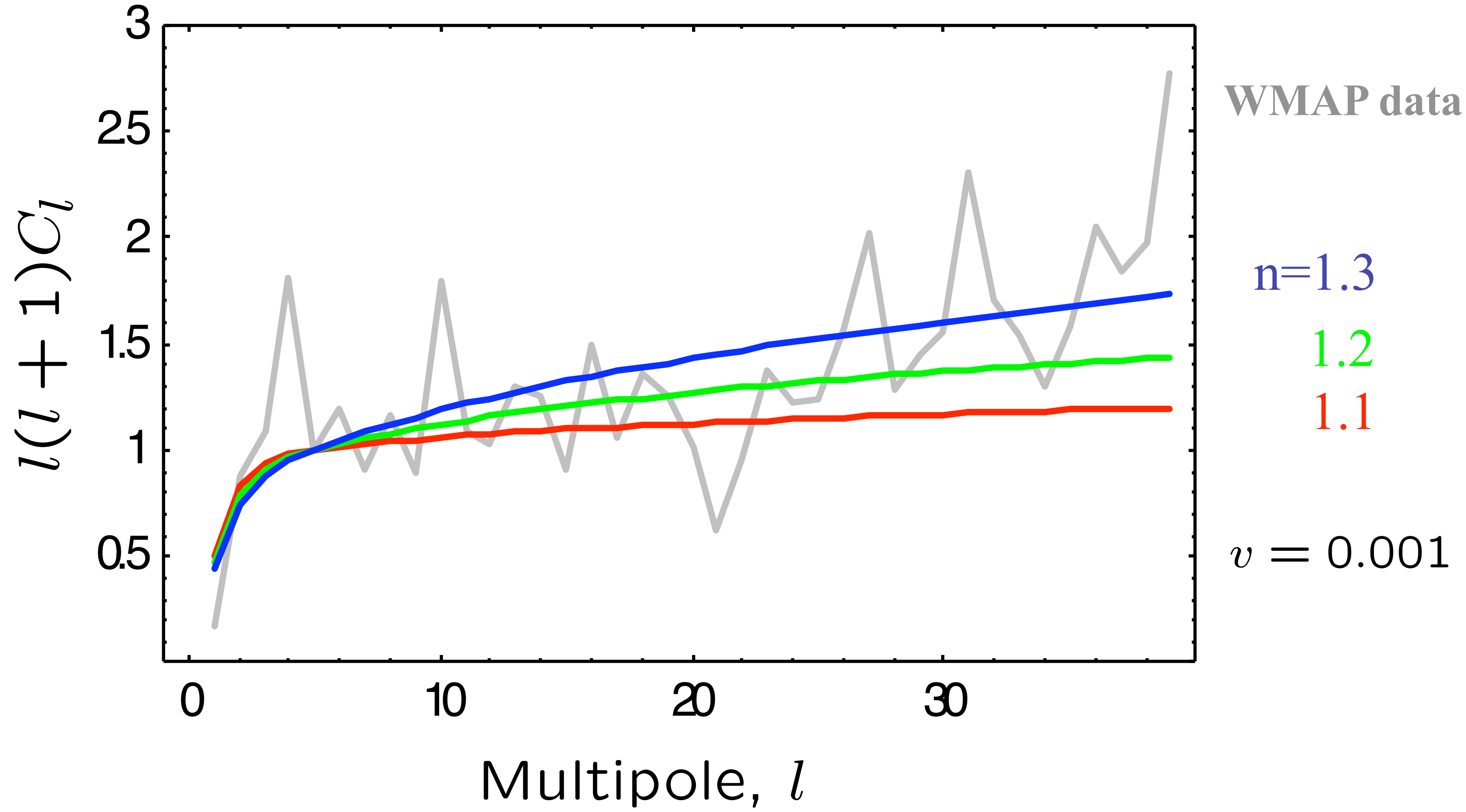
$m_\lambda = a(\tau_\Lambda) H_{\mathrm{D}}$ **: comoving Planck constant**

Number of e-foldings : $\mathcal{N}_e \simeq \frac{H_{\mathrm{D}}}{\Lambda_{\mathrm{QG}}} = \frac{m_\lambda}{\lambda}$

Sharp damping at l=3 → comoving dynamical scale

$$\lambda = 3/x_{\mathrm{ls}} = 0.0002\ \mathrm{Mpc}^{-1}\ (x_{\mathrm{ls}} = 14000\ \mathrm{Mpc})$$

If we take $\mathcal{N}_e = 100$



Scales

Planck scale: $H_\mathrm{D} = \sqrt{8\pi^2/b_1} M_\mathrm{P} \simeq 10^{19}\ \mathrm{GeV}$

Large number of e-foldings is necessary for solving the flatness problem

If $\mathcal{N}_e \simeq H_\mathrm{D}/\Lambda_\mathrm{QG} = 100$

Dynamical scale : $\Lambda_\mathrm{QG} \sim 10^{17}\ \mathrm{GeV}$

scale factor :

$$\begin{aligned} a(\tau_\Lambda) &\simeq 10^{-59} \\ a(\tau_\mathrm{P}) &\simeq 10^{-102} \end{aligned}$$

WMAP suggests blue spectrum (n > 1) at large angle

Quantum gravity scenario predicts large blue spectrum at large angle

$$n = 1.41$$ **: Standard Model** $(N_\mathsf{A} = 12,\ N_\mathsf{W} = 45)$

$$b_1 = \frac{1}{360}\left(N_\mathsf{X} + \frac{11}{2}N_\mathsf{W} + 62N_\mathsf{A}\right) + \frac{769}{180}$$

This value seems to large compared with observed spectrum

$\longrightarrow$ **Extra fields/dark matters?**

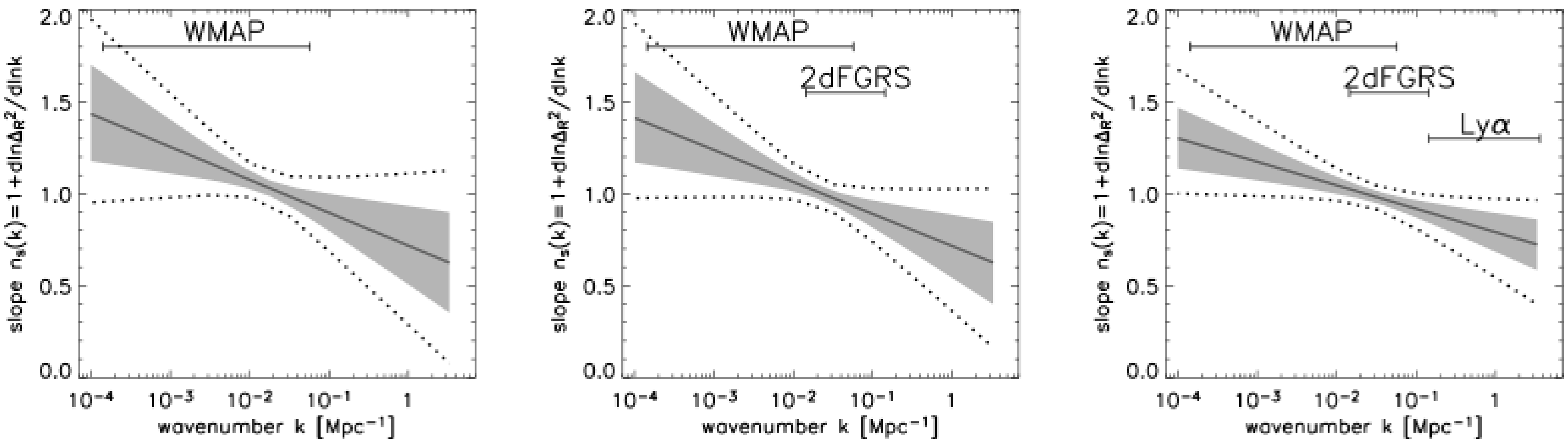
- **GUT (n=1.28 for SU(5))**
- **SUSY**
- **etc**

Violation of background-metric indep.
=violation of superconformal sym.?

WMAP suggests red (n < 1) for small angle

We have assumed that the unique decoupling time, τ_Λ , for entire momentum range.
However, if there is a time lag in the phase transition, short scale delay will change m_λ to be an increasing function of the comoving wavenumber.

$$m_\lambda \to m_\lambda(k)$$



Peiris et al astro-ph/0302225

6. Further Discussions about Λ_{QG}

How did the Big Bang start?

The traceless mode coupling becomes large and
Interactions turn to short range $1/\Lambda_{QG}$.
Then, the conformal mode freezes to classical spacetime.

↓

The field fluctuation percolates to localized object:
graviball with m Λ_{QG}

↓ decay to ordinary matters

Thermal equilibrium / Big bang

Model for the strong α_G interactions

At higher order, new interactions (like $\phi^n \hat{\Delta}_4 \phi$, $\phi^n C^2_{\mu\nu\lambda\sigma}$) are induced from the measure

$$\Downarrow$$

Effective action is modified
and
Inflation terminates to Friedmann universe !

How to determine effective action ?

Initial condition = CFT
Final condition = Friedmann
Sharp transition at τ_Λ :continuum or discrete
etc.

How the information loss problem is solved?

⇕

Unitarity problem in strong gravity phase

Physical excitations in strong gravity/CFT phase
are no longer ordinary particles.

The physical observable is not S-matrix,
but statistical exponents of correlation functions.
→ Power spectrum

Unitarity implies the positivity of the statistical weight.
To prove unitarity,
need detailed structure of physical states.
→ Classification of rep. of conformal algebra

Modification of Einstein theory

After conformal mode freezes to classical spacetime, weak field approximation (expansion by G) becomes effective.

(Note that $R = \mathrm{conf.mode} + t_r^2 h \partial^2 h + \cdots$

Here, take $t_r h_{\mu\nu} \to h_{\mu\nu}$)

Full propagator of the traceless mode :

$$\frac{4}{M_\mathrm{P}^2} \times \frac{1}{p^2 K(p^2)} \qquad p^2 < \Lambda_\mathrm{QG}^2 \qquad \left(\Lambda_\mathrm{QG}^2 < \frac{m_\mathrm{P}^2}{4\beta_0} e \right)$$

No tachyon condition

normalization removed **graviton pole** **correction (no ghost pole)**

$$K(p^2) = 1 + \frac{4\beta_0}{M_\mathrm{P}^2} p^2 \log \frac{p^2}{\Lambda_\mathrm{QG}^2}$$

[$K(0) = 1$ and t_r independent]

cf. gauge theories

$$K(p^2) = \log \frac{p^2}{\Lambda_\mathrm{QCD}^2}$$

$$p^2 > \Lambda_\mathrm{QCD}^2$$

Low energy effective action

$$\frac{1}{\Lambda_{\mathsf{QG}}^2} \int d^4x \sqrt{-g} (\nabla_\mu R) J_{\mathsf{M}}^\mu$$

$$\frac{1}{\Lambda_{\mathsf{QG}}^2} \int d^4x \sqrt{-g} R_{\mu\nu} T_{\mathsf{M}}^{\mu\nu}$$

$$\vdots$$

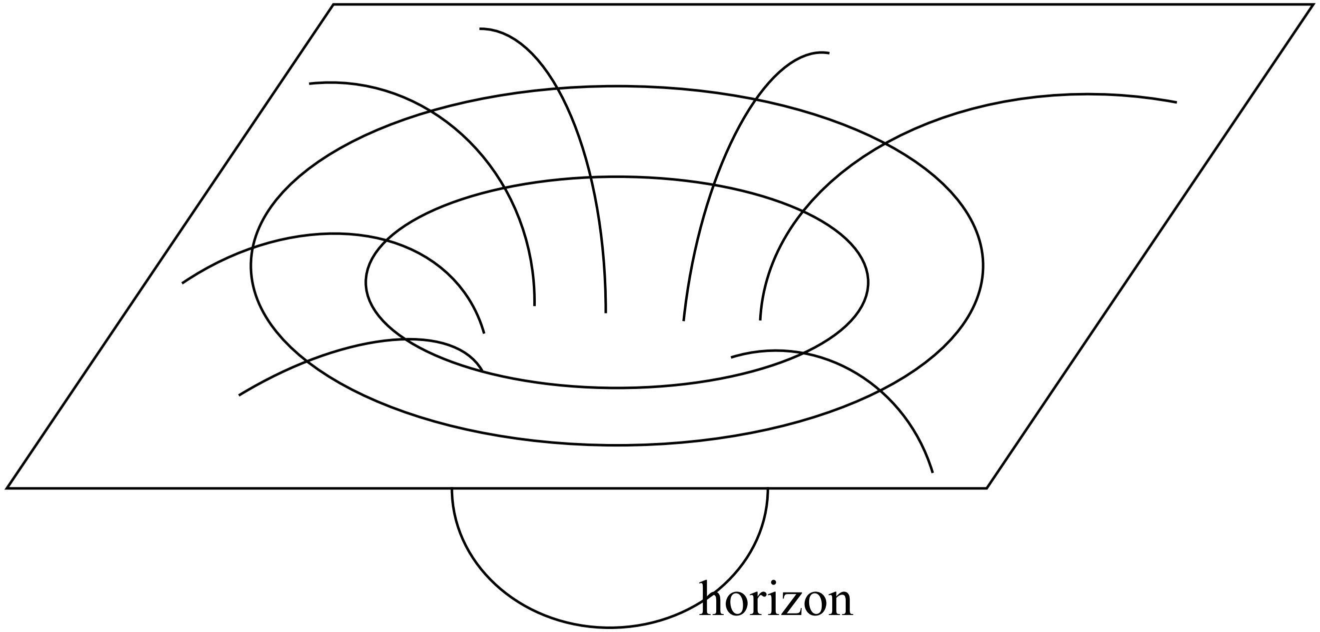
J_{M}^μ, $T_{\mathsf{M}}^{\mu\nu}$ **: matter current and stress-tensor**

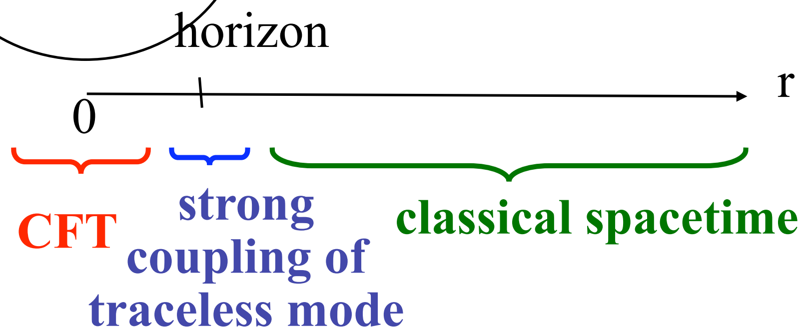
Black hole picture changes

No singularity

Horizon would disappear for tiny black hole obtained at final stage of evaporation

➡ solve information loss problem?





WMAP data

⇒ Determination of strong coupling effective action

⇒ Study of BH structure

7. Conclusions and Future projects

- Quantum gravity scenario of inflation was given.
- Sharp damping of the power spectrum at low multipoles was explained by dynamical scale of quantum gravity.
- Large blue spectrum at large angle was predicted.
- Tensor/scalar ratio is negligible, because of the conformal mode dominance.

A great advantage of this QG model is that it ignites the inflation naturally without any additional fields

Future projects

- **Full analysis of angular power spectrum**

  Need a model for strong α_G

- **Analysis of multi-point correlation/bi-spectrum**

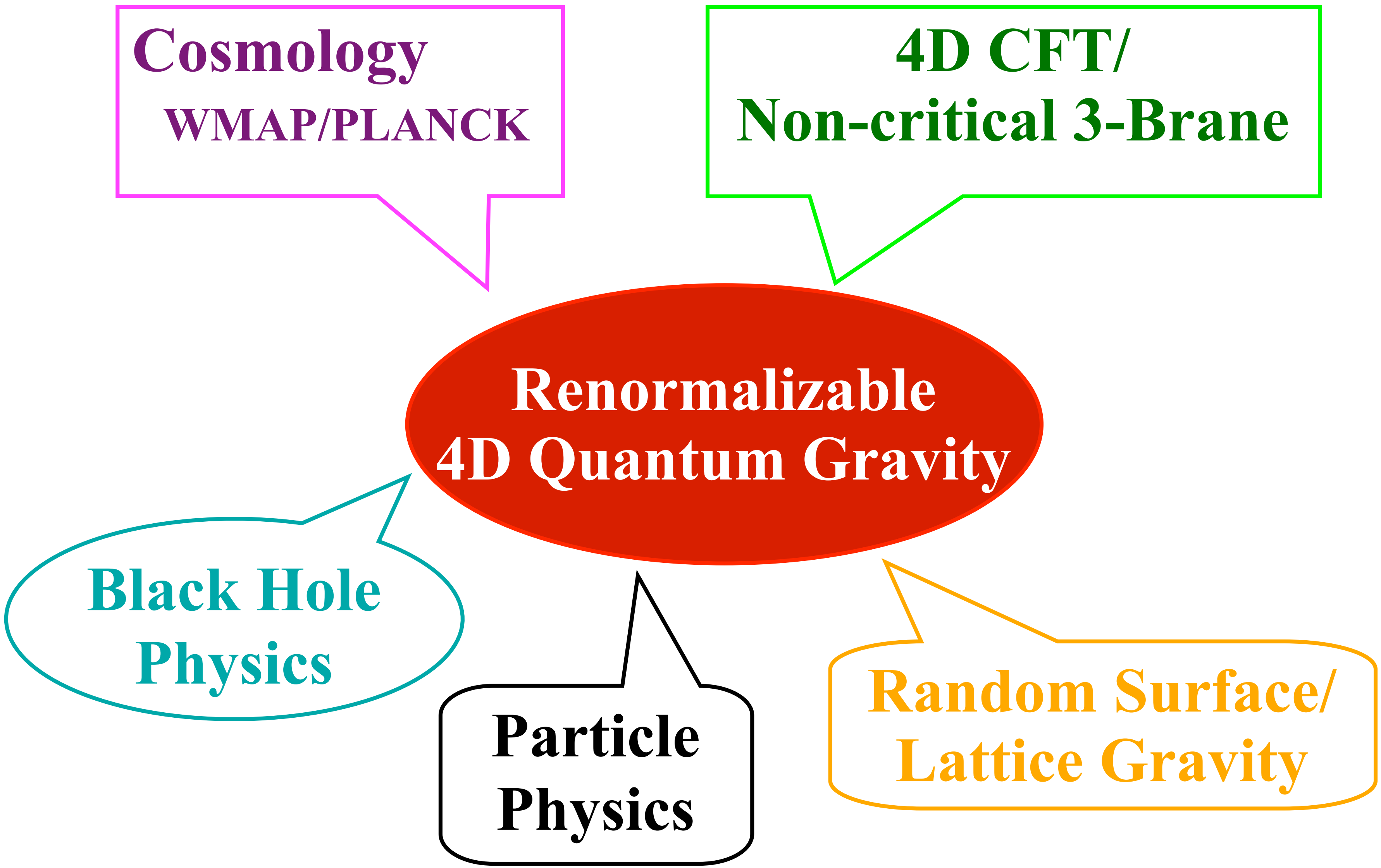
  $\longrightarrow$ **PLANCK mission (2007)**

Non-Gaussianity

WMAP: $-58 < f_{NL} < 134$ **By Komatsu et al**

Standard models of inflation : $f_{NL} \sim 10^{-1} - 10^{-2}$

Theoretical consistency $\longrightarrow$ **Experimental test**


Cosmology
WMAP/PLANCK
4D CFT/
Non-critical 3-Brane
Renormalizable
4D Quantum Gravity
Black Hole
Physics
Particle
Physics
Random Surface/
Lattice Gravity