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Contributed paper to the Linear Collider Workshop, March 2006, I.I.Sc Bangalore, India

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A Beam Test of Prototype TPCs using Micro-Pattern Gas Detectors at KEK

—– An Interpretation of the Results and Extrapolation to the ILC-TPC —–

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Abstract

We conducted a series of beam tests of prototype TPCs for the International Linear Collider (ILC) experiment, equipped with an MWPC, a MicroMEGAS, or GEMs as a readout device. The prototype operated successfully in a test beam at KEK under an axial magnetic field of up to 1 T. The analysis of data is now in progress and some of the preliminary results obtained with GEMs and MicroMEGAS are presented along with our interpretation. Also given is the extrapolation of the obtained spatial resolution to that of a large TPC expected for the central tracker of the ILC experiment.

keywords. TPC; MPGD; MicroMEGAS; GEM; ILC; Spatial Resolution.

1 Introduction

One of the major physics goals of the future linear collider experiment is to study properties of the Higgs boson, which is expected to be well within the reach of the center-of-mass energy of the machine [1] [2]. This goal demands unprecedented high performance of each detector component. For example, the central tracker is required to have a high momentum resolution, high two-track resolving power, and a high momentum resolution, for precise reconstruction of hard muons and each of charged particle tracks in dense jets.

A time projection chamber (TPC) is a strong candidate for the central tracker of the experiment since it can cover a large volume with a small material budget while maintaining a high tracking density (granularity). If micro-pattern gas detectors (MPGDs: micro-mesh gaseous structure (MicroMEGAS) [3], gas electron multiplier (GEM) [4] etc.) are employed for the detection devices of the TPC, instead of conventional multi-wire proportional chambers (MWPCs), one can expect a better spatial resolution at a lower gas gain, a higher granularity, and a smaller or negligible $E \times B$ effect at the entrance to the detection plane. Furthermore, the MPGDs have inherently smaller positiveion back flow rate than that of MWPCs. We therefore constructed a small prototype TPC with a replaceable readout device (MWPC, MicroMEGAS or triple GEM) and have conducted a series of beam tests at KEK in order to study the performance, especially its spatial resolution under an axial magnetic field.

We begin with brief descriptions of the prototype TPC and the experimental setup. Next, some preliminary results are presented along with our interpretation, in which special emphasis is placed on an analytic expression of the spatial resolution. Finally, the spatial resolution of the ILC-TPC is estimated from that measured with the prototype.

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2 Experimental setup

A photograph of the prototype is shown in Fig. 1. It consists of a field cage and an easily replaceable



Figure 1: Photograph of the prototype just before installation into the gas vessel.

gas amplification device attached to one end of the field cage. Gas amplified electrons are detected by a pad plane at ground potential placed right behind the amplification device. A drift electrode is attached to the other end of the field cage. The maximum drift length is about 260 mm.

The pad plane, with an effective area of $\sim 75 \times 75 \text{ mm}^2$, has 12 pad rows at a pitch of 6.3 mm, each consisting of 2×6 (1.27 × 6) mm² rectangular pads arranged at a pitch of 2.3 (1.27) mm when combined with MicroMEGAS (GEMs). Pad signals are fed to charge sensitive preamplifiers located on the outer surface of the bulkhead of the gas vessel behind the pad plane. The amplified signals are sent to shaper amplifiers with a shaping time of 500 ns in the counting room via coaxial cables, and then processed by 12.5 MHz digitizers.

The mesh of MicroMEGAS, made of 5- μ m thick copper, has 35 μ m^{\$\phi\$} holes spaced at intervals of 61 \$\mu\$m. The distance between the mesh and the pad plane is maintained to 50 \$\mu\$m m by kapton pillars arranged in-between. The typical gain is about 3650 at the mesh potential of -320 V. The triple GEM, CERN standard, has two 1.5-mm transfer gaps and a 1-mm induction gap. The transfer and induction fields are 2 kV/cm and 3 kV/cm, respectively. The typical total effective gain in a P5 (TDR) gas is about 3000 with 335 (340) V applied across each GEM foil.

The chamber gases are Ar-isobutane (5%) for MicroMEGAS, and a TDR gas (Ar-methane (5%)carbon dioxide (2%)) or Ar-Methane (5%) for GEMs, at atmospheric pressure and room temperature. The gas pressure and the ambient temperature are continuously monitored since they are not controlled actively. The drift-field strengths are 200, 220 and 100 V/cm, respectively for Ar-isobutane, TDR gas and Ar-methane.

The prototype TPC is placed in the uniform field region of a super conducting solenoid without return yoke, having bore diameter of 850 mm, effective length of 1000 mm, and the maximum field strength of 1.2 T. The prototype was then subjected to the beam, mostly 4 GeV/c pions, at the $\pi 2$ test beam facility of the KEK proton synchrotron.

3 Preliminary results

In this section we show some preliminary results of the analysis up to now, only for the data taken with an axial magnetic field of 1 T and with tracks normal to the pad rows. The results of analytic evaluations are used or presented here without comments. Readers are therefore advised to read Appendix and the slides available on-line [5] as well, where the analytic method is briefly summarized and illustrated.

The observed pad responses for different drift distances (z) are shown in Fig. 2 (a) while the widths of distributions are plotted as a function of drift distance in Fig. 2 (b). The measured spatial



Figure 2: (a) Pad responses for different drift distances. (b) Pad-response width squared (σ_{PR}^2) vs. drift distance (z). The width of pad response is parametrized as $\sigma_{PR}^2 = \sigma_{PR0}^2 + D^2 \cdot z$, with D being the diffusion constant.

resolution against drift distance is shown in Fig. 3 (a) and (b), respectively for the MicroMEGAS and triple GEM readout, along with the result of the analytic calculation. In the calculation the pad response function (PRF) was assumed to be δ function for the MicroMEGAS and a Gaussian for the Gems *.



Figure 3: (a) Spatial resolution vs. z obtained with MicroMEGAS. Gas: Ar-isobutane (5%). (b) Spatial resolution vs. z obtained with GEMs. Gas: Ar-methane (5%).

The obtained behavior of the pad response, and the spatial resolution at long drift distances [†] are compared with expectations in table 1. The comparisons show

^{*}PRF is the avalanche charge spread on the pad plane for a *single* drift electron and should not be confused with the pad response. In the case of MicroMEGAS it is much smaller than the pad pitch (2.3 mm) and is, therefore, neglected. The width (standard deviation) of the Gaussian PRF for the triple GEM has been determined from the intercept of the pad-response width squared vs. z (Fig. 2 (b)): $\sigma_{PR}^2 = \sigma_{PR0}^2 + D^2 \cdot z$ with $\sigma_{PR0}^2 = w^2/12 + \sigma_{PRF}^2$, where the pad pitch w = 1.27 mm and $\sigma_{PR0} \sim 511 \,\mu$ m, yielding $\sim 356 \,\mu$ m for σ_{PRF} . The value of σ_{PR0} thus obtained is consistent with a simple estimation taking into account only the diffusion in the transfer and induction gaps.

[†]When PRF is δ function the asymptotic behavior of the spatial resolution at long distances (diffusion dominant

- 1. σ_{PR0} is in reasonable agreement with the expectation $(\sqrt{w^2/12 + \sigma_{PRF}^2})$ if the contribution of σ_{PRF} is taken in to account (in the case of GEMs);
- 2. σ_{X0} is in good agreement with the expectation $(w/\sqrt{12 \cdot N_{eff}})$ for the MicroMEGAS, and better than this for the GEMs because of the significant charge spread in the transfer and induction gaps;
- 3. The values of diffusion constant (D) are comparable to those given by the simulation (MAG-BOLTZ [6]);
- 4. N_{eff} (16 ~ 22) is significantly smaller than the average number of drift electrons per pad row (~71) [7].

Table 1. Asymptotic behavior at long drift distances under B = 1 T.

()				
Detection device	MicroMEGAS	GEM		
Gas	Ar-isobutane (5%)	TDR	Ar-methane (5%)	
$\sigma_{PR0}~(\mu{ m m})$	758 ± 91	432 ± 3	511 ± 2	
$w/\sqrt{12} \; (\mu \mathrm{m})$	664	367		
$D \ (\mu m / \sqrt{cm})$	194 ± 18	213 ± 1	168 ± 1	
D [MAGBOLTZ]	193	200	166	

(a) Pad response

(b) Spatial resolution

Detection device	MicroMEGAS	GEM	
Gas	Ar-isobutane (5%)	TDR	Ar-methane (5%)
$\sigma_{X0} \; (\mu { m m})$	161 ± 54	44 ± 10	42 ± 17
$w/\sqrt{12 \cdot N_{eff}} \; (\mu \mathrm{m})$	166 ± 42	86 ± 3	78 ± 4
$D/\sqrt{N_{eff}} \ (\mu m/\sqrt{cm})$	48 ± 12	47 ± 1	35 ± 2
N_{eff}	16 ± 8	18 ± 1	22 ± 2

4 Expected spatial resolution of the ILC-TPC

Calculated spatial resolutions of the ILC-TPC at B = 4 T are shown in Fig. 4 for tracks perpendicular to the pad row. In the calculations the values of diffusion constants (D) given by MAGBOLTZ were used. The figure tells us that under a strong magnetic field it is important to reduce the pad-pitch dominant region (at small drift distances) in the ILC-TPC by enhancing the charge sharing among the readout pads, in order to maintain a good resolution over the entire sensitive volume.

There are several possibilities to realize effective charge sharing:

- zigzag (chevron) pads.
- a smaller pad pitch with a larger number of readout channels.
- defocussing of electrons after gas amplification (natural dispersion in the transfer and induction gaps of GEMs, *stochastic* PRF).

asymptotic region) is described by $\sigma_X^2 \equiv \sigma_{X0}^2 + D_X^2 \cdot z \sim 1./N_{eff} \cdot (w^2/12 + D^2 \cdot z)$, where N_{eff} is the effective number of electrons and D is the diffusion constant (see Appendix).



Figure 4: Expected spatial resolutions of the ILC-TPC obtained with MicroMEGAS or GEMs. Gas: Ar-methane (5%), B = 4 T ($D = 50 \ \mu m/\sqrt{cm}$), and $N_{eff} = 22$.

- Use of resistive anode technique with a moderate number of readout channels (applicable to both GEMs and MicroMEGAS, *static* PRF) [8].
- pixel readout (Digital TPC) [9].

5 Summary

To summarize, the prototype TPC equipped with a MicroMEGAS or GEMs operated stably during the beam tests. The tests provided us with valuable information on its performance under axial magnetic fields of up to 1 T:

- The obtained spatial resolution is understood in terms of pad pitch, diffusion constant, PRF, and the effective number of electrons.
- The expected resolution can be estimated by a numerical calculation (NOT a Monte-Carlo) for given geometry, gas mixture and PRF if the relevant parameters are known.
- The calculation is based on a simple formula, easy to code and fast, though it is applicable only to tracks perpendicular to the pad row.
- In the case of MicroMEGAS, the spatial resolution as a function of drift distance is well described by the analytic formula, assuming δ function for PRF.
- In the case of GEMs, the spatial resolution as a function of drift distance is satisfactorily described by the analytic formula, assuming a Gaussian for PRF with the width determined from the intercept of the pad-response width squared as a function of drift distance.
- It is important to make the pad pitch small, *physically or effectively*, in order to reduce both the overall offset term (σ_{X0}) and the resolution degradation due to finite pad pitch.
- The spatial resolution required from the ILC-TPC (100 ~ 200 μ m for the maximum drift distance of ~ 2.5 m) is now within the reach for tracks normal to the pad row.

Appendix: An analytic estimation of pad response and spatial resolution

One way to estimate the spatial resolution of a TPC is to write a realistic Monte-Carlo simulation code. This technique is applicable to any situation, and has been developed by several groups. On the other hand, an analytic approach is applicable only to a restricted case where incident particles are normal to the pad row. However, the resultant formula is rather simple and is sometimes enlightening as shown below. Though a numerical calculation is needed to evaluate the formula, the demanded CPU time is much less than a Monte-Carlo simulation. In addition, the analytic calculation can be used to check the reliability of a Monte-Carlo simulation program, which is usually long and complicated. This appendix is devoted to briefly summarize our analytic approach, based on the following assumptions:

- 1. Particle tracks are normal to the pad row;
- 2. Track coordinate is determined by the charge centroid method;
- 3. Contribution of ambient electronic noise is negligible;
- 4. Displacement of arriving drift electrons due to $E \times B$ effect near the entrance to the detection device is negligible;
- 5. Displacement of arriving electrons due to the finite granularity of amplification elements of the detection device (line intervals in MicroMEGAS or a hole pitch in GEM) is negligible.

A.1 Pad response

Let us calculate here the width of pad response with respect to the true coordinate assuming that the "pad response function (PRF) [‡]" is δ function.

$$\left\langle (x^{\#} - \tilde{x})^2 \right\rangle = \sum_{N=1}^{\infty} P(N) \cdot \frac{1}{w} \cdot \int_{-w/2}^{+w/2} d\tilde{x} \left(\prod_{k=1}^N \int P_x(x_k) dx_k \int P_q(q_k) dq_k \right) \sum_{i=1}^N \frac{q_i}{\sum_{j=1}^N q_j} \cdot (x_i^{\#} - \tilde{x})^2 ,$$

where P(N) is the probability density function (PDF) of total number of drift electrons (N), w is the pad pitch, $P_x(x_k)$ is the PDF of k-th electron's arrival position (x_k) , $P_q(q_k)$ is the PDF of k-th electron's signal charge (q_k) , $x_i^{\#}$ is the central coordinate of the pad on which *i*-th electron arrives $(= j \cdot w, \text{ with } j \text{ being the corresponding pad number})$, and \tilde{x} is the original position (true coordinate) of electrons. $P_x(x)$, accounting for diffusion, is denoted later by $P_x(x; \tilde{x}, \sigma_d)$, where $\tilde{x}(\sigma_d)$ is the mean (width) of a Gaussian distribution:

$$P_x(x) \equiv P_x(x; \tilde{x}, \sigma_d) \equiv \frac{1}{\sqrt{2\pi\sigma_d}} \cdot \exp\left(-\frac{(x-\tilde{x})^2}{2\sigma_d^2}\right)$$

Figs. 5 and 6 illustrate the situation and give some of the definitions of relevant variables.

[‡]In the case of conventional MWPC readout, PRF is defined as the charge distribution on the pad plane caused by a single drift electron arriving at a sense wire. Therefore it is *static* and is determined electro-statically. On the other hand, in the case of MicroMEGAS or GEMs the charge distribution for a single drift electron is caused mainly by avalanche spread due to diffusion or by diffusion in the transfer and induction gaps. Therefore it is essentially *stochastic*. In the analytic approach discussed here, however, PRF is treated as if it were *static*, assuming a large avalanche multiplication factor.



Figure 5: Principle of the track coordinate measurement.



Figure 6: Illustration of the relevant parameters.

The calculation proceeds straightforwardly as follows:

$$\begin{split} \left\langle (x^{\#} - \hat{x})^{2} \right\rangle &= \sum_{N=1}^{\infty} P(N) \cdot \sum_{i=1}^{N} \left(\prod_{k=1}^{N} \int P_{q}(q_{k}) dq_{k} \right) \sum_{i=1}^{N} \frac{q_{i}}{\sum_{j=1}^{N} q_{j}} \\ &\times \frac{1}{w} \cdot \int_{-w/2}^{+w/2} d\hat{x} \left(\prod_{k=1}^{N} \int_{\infty}^{\infty} P_{x}(x_{k}) dx_{k} \right) \cdot (x_{i}^{\#} - \hat{x})^{2} \\ &= \sum_{N=1}^{\infty} P(N) \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \frac{1}{w} \cdot \int_{-w/2}^{+w/2} d\hat{x} \left(\prod_{k=1}^{N} \int_{-\infty}^{\infty} P_{x}(x_{k}) dx_{k} \right) \cdot (x_{i}^{\#} - \hat{x})^{2} \\ &= \sum_{N=1}^{\infty} P(N) \cdot \frac{1}{w} \cdot \int_{-w/2}^{+w/2} d\hat{x} \int_{-\infty}^{\infty} P_{x}(x) \cdot (x^{\#} - \hat{x})^{2} dx \\ &= \frac{1}{w} \cdot \int_{-w/2}^{+w/2} d\hat{x} \int_{-\infty}^{\infty} P_{x}(x) \cdot (x^{\#} - \hat{x})^{2} dx \\ &= \frac{1}{w} \cdot \int_{-w/2}^{+w/2} d\hat{x} \sum_{j=-\infty}^{\infty} \int_{jw-w/2}^{jw+w/2} P_{x}(x;\hat{x},\sigma_{d}) \cdot (jw - \hat{x})^{2} dx \\ &= \frac{1}{w} \cdot \sum_{j=-\infty}^{\infty} \int_{-w/2}^{jw+w/2} d\hat{x} (jw - \hat{x})^{2} \int_{jw-w/2}^{jw+w/2} P_{x}(x;jw - t,\sigma_{d}) dx \\ &= \frac{1}{w} \cdot \sum_{j=-\infty}^{\infty} \int_{jw-w/2}^{jw+w/2} dt \cdot t^{2} \int_{-w/2}^{+w/2} P_{x}(x; -t,\sigma_{d}) dx \\ &= \frac{1}{w} \cdot \int_{-w/2}^{\infty} dt \cdot t^{2} \int_{-w/2}^{+w/2} P_{x}(x; -t,\sigma_{d}) dx \\ &= \frac{1}{w} \cdot \int_{-w/2}^{-w/2} dx \int_{-\infty}^{\infty} t^{2} \cdot P_{x}(x; -t,\sigma_{d}) du \\ &= \frac{1}{w} \cdot \int_{-w/2}^{+w/2} dx \int_{-\infty}^{\infty} (u - x)^{2} \cdot P_{x}(u; 0, \sigma_{d}) du \quad \text{with } u \equiv x + t \\ &= \frac{1}{w} \cdot \int_{-w/2}^{+w/2} (\sigma_{k}^{2} + x^{2}) dx \\ &= \sigma_{d}^{2} + \frac{w^{2}}{12} . \end{split}$$

The interpretation of the result is quite simple. The squared pad-response width is a quadratic sum of the widths, one due to diffusion and the other originated from the finite pad pitch. This can be readily generalized for the case where the width of PRF (σ_{PRF}) is finite:

$$\left\langle (x^{\#} - \tilde{x})^2 \right\rangle = \sigma_d^2 + \sigma_{PRF}^2 + \frac{w^2}{12} = \frac{w^2}{12} + \sigma_{PRF}^2 + D^2 \cdot z ,$$
 (2)

where D is the diffusion constant and z is the drift distance. Therefore if the square of the width of pad response is plotted against z one gets a straight line with a slope of D^2 and an intercept of $w^2/12 + \sigma_{PRF}^2$.

In fact, we use the width of pad response with respect to the charge centroid ($\equiv \bar{x}$), instead of the unknown (precise) true coordinate (\tilde{x}), in the present paper. Therefore Eq. (1) needs a slight

modification accordingly as briefly shown below for the case where PRF is δ function ($\sigma_{PRF} = 0$). In the calculation, signal charge fluctuation represented by $P_q(q)$ is not included explicitly since it does not affect the final result. From now on we avoid to explicitly show the integrals weighted by PDFs and use instead average symbols denoted by $\langle \cdots \rangle$ in order to save space.

$$\left\langle (x^{\#} - \bar{x})^{2} \right\rangle = \frac{1}{N} \cdot \left\langle \sum_{i=1}^{N} (x_{i}^{\#} - \bar{x})^{2} \right\rangle$$

$$= \frac{1}{N} \cdot \left\langle \sum_{i=1}^{N} \left((x_{i}^{\#} - \tilde{x}) - (\bar{x} - \tilde{x}) \right)^{2} \right\rangle$$

$$= \frac{1}{N} \cdot \left\langle \sum_{i=1}^{N} \left((x_{i}^{\#} - \tilde{x})^{2} + (\bar{x} - \tilde{x})^{2} - 2 \cdot (x_{i}^{\#} - \tilde{x}) \cdot (\bar{x} - \tilde{x}) \right) \right\rangle$$

$$= \left\langle (x^{\#} - \tilde{x})^{2} \right\rangle + \left\langle (\bar{x} - \tilde{x})^{2} \right\rangle - \frac{2}{N} \cdot \left\langle (\bar{x} - \tilde{x}) \cdot \sum_{i=1}^{N} (x_{i}^{\#} - \tilde{x}) \right\rangle$$

$$= \left\langle (x^{\#} - \tilde{x})^{2} \right\rangle + \left\langle (\bar{x} - \tilde{x})^{2} \right\rangle - 2 \cdot \left\langle (\bar{x} - \tilde{x})^{2} \right\rangle$$

$$= \left\langle (x^{\#} - \tilde{x})^{2} \right\rangle - \left\langle (\bar{x} - \tilde{x})^{2} \right\rangle .$$

$$(3)$$

The first term is what we have calculated above (Eq. (1)) while the second term is nothing but the spatial resolution (squared) obtained with the charge centroid method, which is to be evaluated in the next section. The contribution of second term is small except at small drift distances.

A.2 Spatial resolution

Let us consider first the spatial resolution to be obtained with infinitesimal pad pitch and σ_{PRF} (PRF: δ function) since the calculation is very simple in this case [7]. In the following, the measured track coordinate is assumed to be determined by the centroid of charges collected by the readout pads:

$$X \equiv \frac{\sum_{i=1}^{N} q_i \cdot x_i}{\sum_{i=1}^{N} q_i}$$

where q_i , x_i are the signal charge and the arrival position, respectively, of *i*-th electron. In the calculation below and in the rest of this appendix, the symbol $< \dots >_x (q)$ stands for the average taken over the variables x(q) with the corresponding PDFs. The subscript x or q may be omitted when the meaning of average is clear itself. Then

$$\begin{aligned} \sigma_X^2 &= \left\langle (X - \tilde{x})^2 \right\rangle \\ &= \left\langle \left(\frac{\sum_{i=1}^N q_i \cdot (x_i - \tilde{x})}{\sum_{i=1}^N q_i} \right)^2 \right\rangle \\ &= \left\langle \frac{1}{(\sum_i q_i)^2} \cdot \left(\sum_i q_i^2 \cdot (x_i - \tilde{x})^2 + \sum_{i \neq j} q_i \cdot q_j \cdot (x_i - \tilde{x}) \cdot (x_j - \tilde{x}) \right) \right\rangle \\ &= \left\langle \frac{1}{(\sum_i q_i)^2} \cdot \left(\left\langle (x - \tilde{x})^2 \right\rangle_x \cdot \sum_i q_i^2 + \langle x - \tilde{x} \rangle_x^2 \cdot \sum_{i \neq j} q_i \cdot q_j \right) \right\rangle_q \\ &= \left\langle \frac{\sum_i q_i^2}{(\sum_i q_i)^2} \right\rangle_q \cdot \left\langle (x - \tilde{x})^2 \right\rangle_x \\ &= \left\langle \frac{\sum_i q_i^2}{(\sum_i q_i)^2} \right\rangle_q \cdot \sigma_d^2 \end{aligned}$$

 $\approx \frac{1}{N} \cdot \frac{\langle q^2 \rangle}{\langle q \rangle^2} \cdot \sigma_d^2 \text{, assuming } \sum_i q_i = \text{const} = N \cdot \langle q \rangle \text{, expecting a large } N.$

Averaging over N, we obtain

$$\sigma_X^2 \approx \sum_{i=1}^{\infty} P(N) \cdot \frac{1}{N} \cdot \frac{\langle q^2 \rangle}{\langle q \rangle^2} \cdot \sigma_d^2$$
$$\approx \left\langle \frac{1}{N} \right\rangle \cdot \frac{\langle q^2 \rangle}{\langle q \rangle^2} \cdot \sigma_d^2$$
$$\approx \frac{1}{N_{eff}} \cdot \sigma_d^2 , \qquad (4)$$

where N_{eff} is defined as

$$\frac{1}{N_{eff}} \equiv \left\langle \frac{1}{N} \right\rangle \cdot \frac{\langle q^2 \rangle}{\langle q \rangle^2} \equiv \left\langle \frac{1}{N} \right\rangle \cdot (1+K) ,$$

with K being the relative variance of avalanche fluctuation: $\sigma_q^2/\langle q \rangle^2$.

Next, let us assume a finite pad pitch (w) but still an infinitesimal PRF width (σ_{PRF}). In this case, the charge centroid is given by

$$X = \frac{\sum_{i=1}^{N} q_i \cdot x_i^{\#}}{\sum_{i=1}^{N} q_i} ,$$

where $x_i^{\#}$ $(= j \cdot w)$ is the central coordinate of the pad on which electron *i* arrives, and

$$\begin{split} \sigma_X^2 &= \left\langle (X - \tilde{x})^2 \right\rangle \\ &= \left\langle \left(\frac{\sum_{i=1}^N q_i \cdot (x_i^{\#} - \tilde{x})}{\sum_{i=1}^N q_i} \right)^2 \right\rangle \\ &= \left\langle \frac{1}{(\sum_i q_i)^2} \cdot \left(\sum_i q_i^2 \cdot (x_i^{\#} - \tilde{x})^2 + \sum_{i \neq j} q_i \cdot q_j \cdot (x_i^{\#} - \tilde{x}) \cdot (x_j^{\#} - \tilde{x}) \right) \right\rangle \\ &= \left\langle \frac{1}{(\sum_i q_i)^2} \cdot \left(\left\langle (x^{\#} - \tilde{x})^2 \right\rangle_x \cdot \sum_i q_i^2 + \langle x^{\#} - \tilde{x} \rangle_x^2 \cdot \left(\sum_{i,j} q_i \cdot q_j - \sum_i q_i^2 \right) \right) \right\rangle_q \\ &= \left\langle x^{\#} - \tilde{x} \right\rangle^2 + \left\langle \frac{\sum_i q_i^2}{(\sum_i q_i)^2} \right\rangle \cdot \left(\left\langle (x^{\#} - \tilde{x})^2 \right\rangle - \langle x^{\#} - \tilde{x} \rangle^2 \right) \\ &\approx \left\langle x^{\#} - \tilde{x} \right\rangle^2 + \frac{1}{N} \cdot \frac{\langle q^2 \rangle}{\langle q \rangle^2} \cdot \left(\langle (x^{\#})^2 \rangle - \langle x^{\#} \rangle^2 \right) . \end{split}$$

Averaging over N, and substituting $j \cdot w$ for $x^{\#}$, we obtain

$$\sigma_X^2 \approx \langle x^{\#} - \tilde{x} \rangle^2 + \frac{1}{N_{eff}} \cdot (\langle (x^{\#})^2 \rangle - \langle x^{\#} \rangle^2)$$
$$\approx \left(\sum_{j=-\infty}^{\infty} jw \cdot P_x^{\#}(jw) - \tilde{x} \right)^2 + \frac{1}{N_{eff}} \cdot \left(\sum_{j=-\infty}^{\infty} j^2 w^2 \cdot P_x^{\#}(jw) - \left(\sum_{j=-\infty}^{\infty} jw \cdot P_x^{\#}(jw) \right)^2 \right) ,$$

where $P_x^{\#}(jw) \equiv \int_{jw-w/2}^{jw+w/2} P_x(x)dx$, with $P_x(x) \equiv \frac{1}{\sqrt{2\pi\sigma_d}} \exp\left(-\frac{(x-\tilde{x})^2}{2\sigma_d^2}\right)$. (5)

The first term in the final expression originates from the bias due to the charge centroid method combined with the finite pad pitch. This term is independent of N and rapidly decreases with increasing z because of diffusion [5]. On the other hand, the second term is the square of observed charge spread relative to the charge centroid (Eq. (3): $\sim \sigma_d^2 + w^2/12$) divided by N_{eff} .

Finally let us assume a finite PRF width (σ_{PRF}). In this case, the charge centroid is given by

$$X = \frac{\sum_{i=1}^{N} \sum_{j} q_{ji} \cdot x_{j}^{*}}{\sum_{i=1}^{N} \sum_{j} q_{ji}} \equiv \frac{\sum_{i=1}^{N} Q_{i} \sum_{j} F_{j}(x_{i}) \cdot x_{j}^{*}}{\sum_{i=1}^{N} Q_{i}} ,$$

where (see Fig. 6)

 $\begin{array}{lll} q_{ji} &\equiv& Q_i \cdot F_j(x_i) \ : \ \text{signal charge on pad } j, \ \text{created by electron } i \ , \\ x_i &: \ \text{arrival position of electron } i \ \text{at the entrance to the detection device }, \\ x_j^* &\equiv& j \cdot w \quad : \ \text{central coordinate of pad } j \quad (j = \cdots, -2, -1, 0, +1, +2, \cdots) \ , \\ Q_i &\equiv& \sum_j q_{ji} \ : \ \text{total signal charge created by electron } i \ , \\ F_j(x_i) &\equiv& \frac{q_{ji}}{Q_i} \equiv \int_{jw-w/2}^{jw+w/2} f(\xi - x_i) d\xi \ , \\ f(\xi) &: \ (\text{normalized}) \ \text{PRF} \ . \end{array}$

And

$$\begin{split} \sigma_X^2 &= \left\langle (X - \tilde{x})^2 \right\rangle \\ &= \left\langle \left(\frac{\sum_{i=1}^N Q_i \sum_j F_j(x_i) \cdot x_j^*}{\sum_{i=1}^N Q_i} - \tilde{x} \right)^2 \right\rangle \\ &= \left\langle \left(\frac{\sum_{i=1}^N Q_i \sum_j F_j(x_i) \cdot (x_j^* - \tilde{x})}{\sum_{i=1}^N Q_i} \right)^2 \right\rangle \\ &= \left\langle \frac{1}{(\sum_i Q_i)^2} \cdot \left(\sum_i Q_i^2 \cdot \left(\sum_j F_j(x_i) \cdot (x_j^* - \tilde{x}) \sum_k F_k(x_i) \cdot (x_k^* - \tilde{x}) \right) \right) \right. \\ &+ \sum_{i \neq j} Q_i Q_j \cdot \left(\sum_k F_k(x_i) \cdot (x_k^* - \tilde{x}) \sum_l F_l(x_j) \cdot (x_l^* - \tilde{x}) \right) \right) \right\rangle \\ &= \left\langle \frac{1}{(\sum_i Q_i)^2} \cdot \left(\sum_i Q_i^2 \cdot \left((\sum_j F_j(x_i) \cdot x_j^* - \tilde{x}) (\sum_k F_k(x_i) \cdot x_k^* - \tilde{x}) \right) \right. \\ &+ \sum_{i \neq j} Q_i Q_j \cdot \left((\sum_k F_k(x_i) \cdot x_k^* - \tilde{x}) (\sum_l F_l(x_j) \cdot x_l^* - \tilde{x}) \right) \right) \right\rangle \\ &= \left\langle \frac{1}{(\sum_i Q_i)^2} \cdot \left(\sum_i Q_i^2 \cdot \left\langle (\sum_j F_j(x) \cdot x_j^* - \tilde{x}) (\sum_k F_k(x) \cdot x_k^* - \tilde{x}) \right\rangle_x \right. \\ &+ \left(\sum_{i,j} Q_i Q_j - \sum_i Q_i^2 \right) \cdot \left(\sum_k F_k(x) \cdot x_k^* - \tilde{x} \right)_x \right) \\ &= \left\langle \sum_j F_j(x) \cdot x_j^* - \tilde{x} \right\rangle_x^2 + \left\langle \frac{\sum_i Q_i^2}{(\sum_i Q_i)^2} \right\rangle \cdot \left(\left\langle \sum_{j,k} F_j(x) \cdot F_k(x) \right\rangle \cdot x_j^* \cdot x_k^* - \left\langle \sum_k F_k(x) \right\rangle \cdot x_k^* \right)^2 \right) \\ &= \left(\sum_j \langle F_j(x) \rangle \cdot x_j^* - \tilde{x} \right)^2 + \left\langle \frac{\sum_i Q_i^2}{(\sum_i Q_i)^2} \right\rangle \cdot \left(\sum_i Q_i (x) \cdot F_k(x) \right) \cdot x_j^* \cdot x_k^* - \left(\sum_k \langle F_k(x) \right\rangle \cdot x_k^* \right)^2 \right) \end{split}$$

$$\approx \left(\sum_{j} \langle F_{j}(x) \rangle \cdot x_{j}^{*} - \tilde{x} \right)^{2} + \frac{1}{N} \cdot \frac{\langle Q^{2} \rangle}{\langle Q \rangle^{2}} \cdot \left(\sum_{j,k} \langle F_{j}(x) \cdot F_{k}(x) \rangle \cdot x_{j}^{*} \cdot x_{k}^{*} - \left(\sum_{k} \langle F_{k}(x) \rangle \cdot x_{k}^{*} \right)^{2} \right).$$

Averaging over N , and substituting $j\cdot w$ and $k\cdot w,$ respectively for $x_j^* \; and \; x_k^*$, we get

$$\sigma_X^2 \approx \left(\sum_j jw \cdot \langle F_j(x) \rangle - \tilde{x}\right)^2 + \frac{1}{N_{eff}} \cdot \left(\sum_{j,k} jkw^2 \cdot \langle F_j(x) \cdot F_k(x) \rangle - \left(\sum_j jw \cdot \langle F_j(x) \rangle\right)^2\right), \quad (6)$$

where

$$\begin{split} \langle F_j(x) \rangle &\equiv \int_{-\infty}^{\infty} P_x(x) \cdot F_j(x) \, dx \;, \\ \langle F_j(x) \cdot F_k(x) \rangle &\equiv \int_{-\infty}^{\infty} P_x(x) \cdot F_j(x) \cdot F_k(x) \, dx \;, \\ \text{with} \\ P_x(x) &\equiv \frac{1}{\sqrt{2\pi\sigma_d}} \exp\left(-\frac{(x-\tilde{x})^2}{2\sigma_d^2}\right) \;, \\ F_j(x) &\equiv \int_{jw-w/2}^{jw+w/2} f(\xi-x) d\xi \;, \\ f(\xi) : \; \text{PRF} \;. \end{split}$$

It should be pointed out here that σ_X^2 depends on the position of \tilde{x} relative to the corresponding pad center, and that the beam spot size is usually much larger than the pad pitch. Therefore unless the incident positions of incoming particles are measured precisely by an external tracker (e.g. by a set of silicon strip detectors) on an event-by-event basis, σ_X^2 obtained above (Eq. (5) or (6)) has to be averaged over \tilde{x} in a range, say, [-w/2, +w/2].

It is easy to show that Eq. (6) is a generalization of Eq. (4) and (5). Eq. (5) is expected to be a good approximation when σ_{PRF} is much smaller than the pad pitch w, i.e. in the case of MicroMEGAS. On the other hand, Eq. (6) has to be used for GEM readout since σ_{PRF} is several hundred microns and is not negligible as compared to w.

Evaluation of Eq. (5) or (6), including the average over \tilde{x} , can be done numerically using a short and simple program, with much shorter demanded CPU time than Monte-Carlo simulations. The results of the analytic calculation and a Monte-Carlo simulation are compared in Fig. 7 for the triple GEM readout. The Monte-Carlo simulation takes into account the primary ionization statistics, diffusion in the drift space, avalanche multiplication and its fluctuation in the GEM holes, and the diffusion in the transfer and induction gaps. The figure shows that they are almost identical, indicating the reliability of both the analytic approach and the Monte-Carlo simulation. A major advantage of Monte-Carlo simulation is that it can easily be generalized to be applicable to inclined tracks.

To summarize, the analytic calculation gives reliable evaluation of the spatial resolution of a TPC for tracks perpendicular to the pad row once the effective number of electrons (N_{eff}) , the diffusion constant (D), and the pad response function (PRF) are known. N_{eff} is determined from the primary ionization statistics (average density of primary ionizations and their cluster size distribution) and the relative variance of avalanche fluctuation for a single drift electron [7]. They are experimentally measurable or found in literature. The diffusion constant in the drift region is determined from the slope of the pad-response width squared as a function of drift distance (Eq. (2)). It may be estimated using the simulation by MAGBOLTZ. Finally, the width of pad response function is estimated from the intercept of the squared pad-response width plotted against drift distance (Eq. (2)). This can be estimated also by using the simulated value(s) of diffusion constant in the detection gap(s). The most reliable PRF would, however, be provided by a dedicated experiment using a single-electron source and finer readout pads.



Figure 7: Comparison between the analytic calculation and the Monte-Carlo simulation. In the calculation N_{eff} is assumed to be 21 and PRF is assumed to be a Gaussian with $\sigma = 363 \ \mu\text{m}$. The diffusion constant (D) is set to 166 $\mu\text{m}/\sqrt{\text{cm}}$ in both cases.

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