Kalman Filter Library

Kalman Filter Library Features

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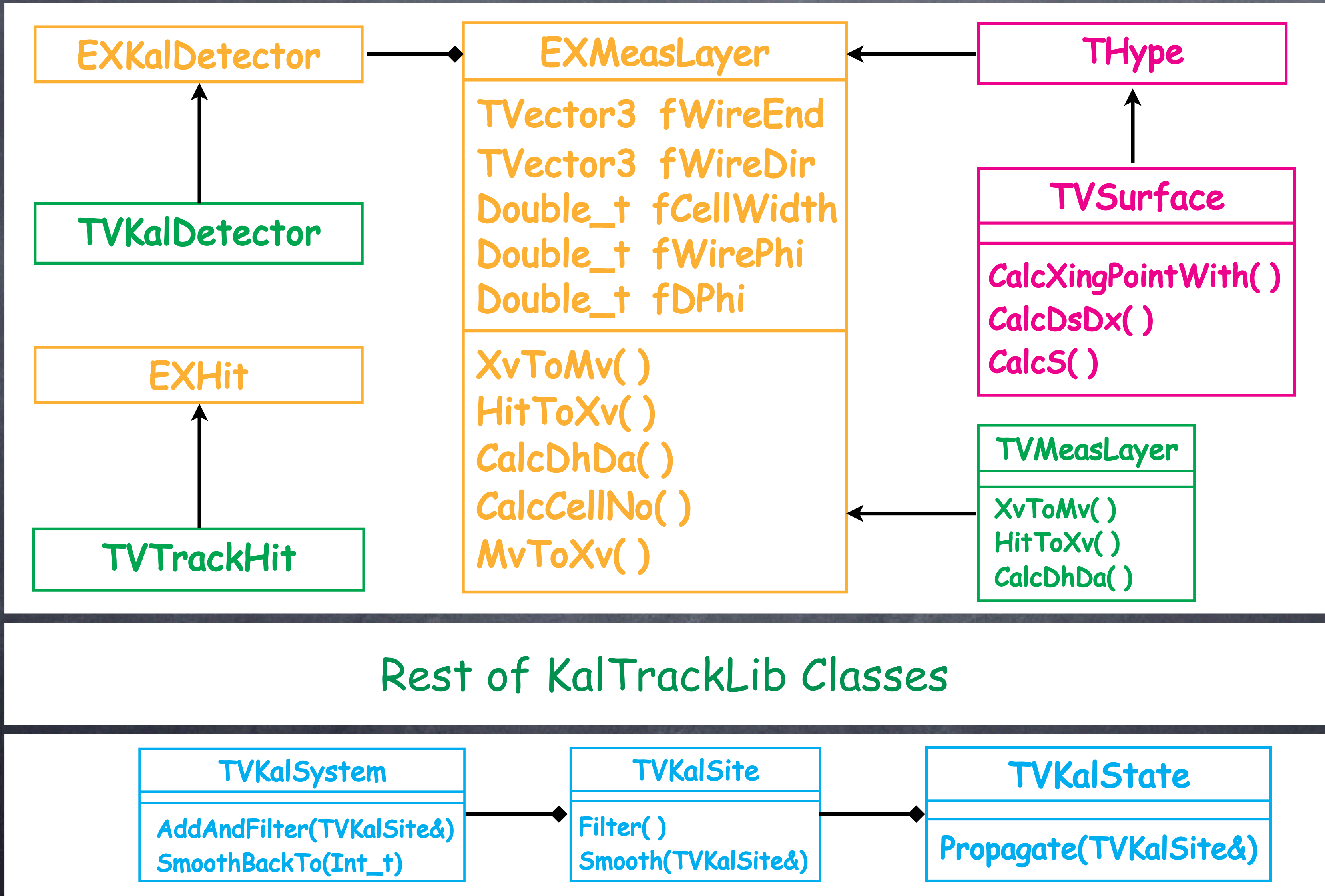
- KalLib: general base classes that implement algorithm
  - TVKalSystem, TVKalSite, TVKalState
- KalTrackLib: that implements pure virtuals of KalLib for track fitting purpose
- GeomLib: geometry classes that provide
  - track models (helix, straight line, ...)
  - surfaces (cylinder, hyperboloid, flat plane, ...)
  -
- Minimum number of user-implemented classes
  - MeasLayer : measurement layer
  - KalDetector : an array containing MeasLayers
    - You can put different kinds of MeasLayers
  - Hit : coordinate vector as defined by the MeasLayer
- Track model can change site to site which allows B-field variation along a particle trajectory

K.Fujii @ ACFA LCWS2003

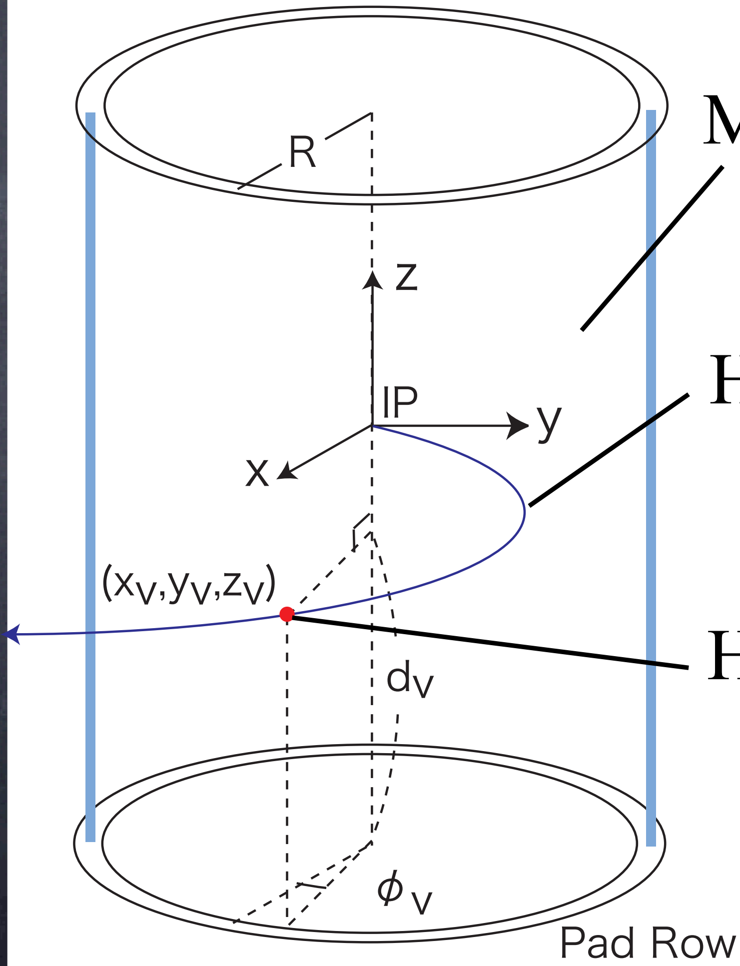
Kalman Filter Class Organization

CDC Example



Example of Detector Implementation

How to define TPC



Measurement Layer

$$S(x,y,z) = x^2 + y^2 - R^2 = 0$$

Helix

$$\left\{\begin{array}{lcl} x &=& x_0 + d_\rho \cos\phi_0 + \frac{\alpha}{\kappa}(\cos\phi_0 - \cos(\phi_o + \phi)) \\ y &=& y_0 + d_\rho \sin\phi_0 + \frac{\alpha}{\kappa}(\sin\phi_0 - \sin(\phi_o + \phi)) \\ z &=& z_0 + d_z - \frac{\alpha}{\kappa}\tan\lambda \cdot \phi \end{array}\right.$$

Hit

$$\begin{pmatrix} R \cdot \phi_v \\ d_v \end{pmatrix} = \begin{pmatrix} R \cdot \tan^{-1}\left(\frac{y_v}{x_v}\right) \\ L/2 \mp z_v \end{pmatrix}$$

EXMeasLayer for TPC

HitToXv

$$\begin{pmatrix} x_v \\ y_v \\ z_v \end{pmatrix} = \begin{pmatrix} R \cdot \cos\phi_v \\ R \cdot \sin\phi_v \\ \pm(L/2 - d_v) \end{pmatrix}$$

XvToMv

$$\begin{pmatrix} R \cdot \phi_v \\ d_v \end{pmatrix} = \begin{pmatrix} R \cdot \tan^{-1}\left(\frac{y_v}{x_v}\right) \\ L/2 \mp z_v \end{pmatrix}$$

CalcDhDa

Meas.Vector Derivative w.r.t. Track Parameter Vector

$$\begin{pmatrix} \frac{\partial (R\cdot\phi_v)}{\partial \mathrm{a}} \\ \frac{\partial d_v}{\partial \mathrm{a}} \end{pmatrix} = \begin{pmatrix} -\frac{y_v}{R}\left(\frac{\partial x_v}{\partial \mathrm{a}}\right) + \frac{x_v}{R}\left(\frac{\partial y_v}{\partial \mathrm{a}}\right) \\ \mp \frac{\partial z_v}{\partial \mathrm{a}} \end{pmatrix}$$

$$\frac{\partial \mathrm{x}(\phi(\mathrm{a}),\mathrm{a})}{\partial \mathrm{a}} = \frac{\partial \mathrm{x}}{\partial \phi} \cdot \frac{\partial \phi}{\partial \mathrm{a}} + \frac{\partial \mathrm{x}}{\partial \mathrm{a}}$$

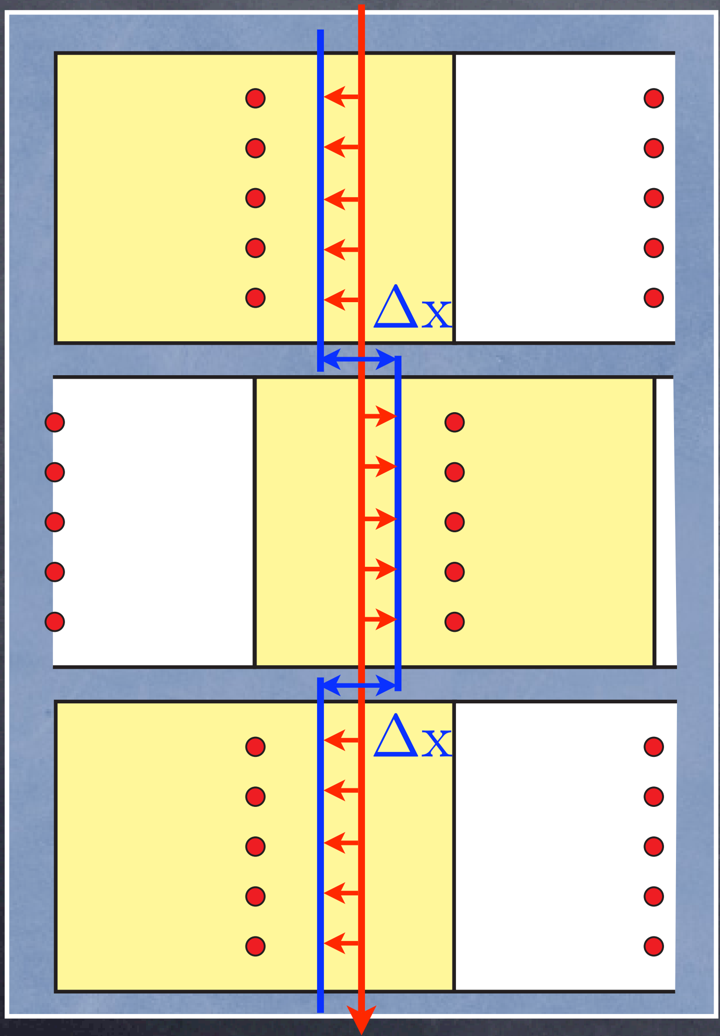
$$\frac{\partial \phi}{\partial \mathrm{a}} = -\frac{1}{\left(\frac{\partial S}{\partial \mathrm{X}} \cdot \frac{\partial \mathrm{X}}{\partial \phi}\right)} \frac{\partial S}{\partial \mathrm{x}} \cdot \frac{\partial \mathrm{x}}{\partial \mathrm{a}}$$

Kalman Filter Library now being imported into Satellites

Time Stamping

Application of Kalman Filter Library

In the Case of JLC-CDC



Staggered Cells

Wrong T0 breaks a track!

$$\Delta \mathrm{x} = 2\ v_{\mathrm{drift}} \times \Delta T_0$$

Naively we expect

$$\sigma_{\Delta T_0} \simeq \frac{\sigma_{\mathrm{xy}}}{v_{\mathrm{drift}}\sqrt{n}}$$

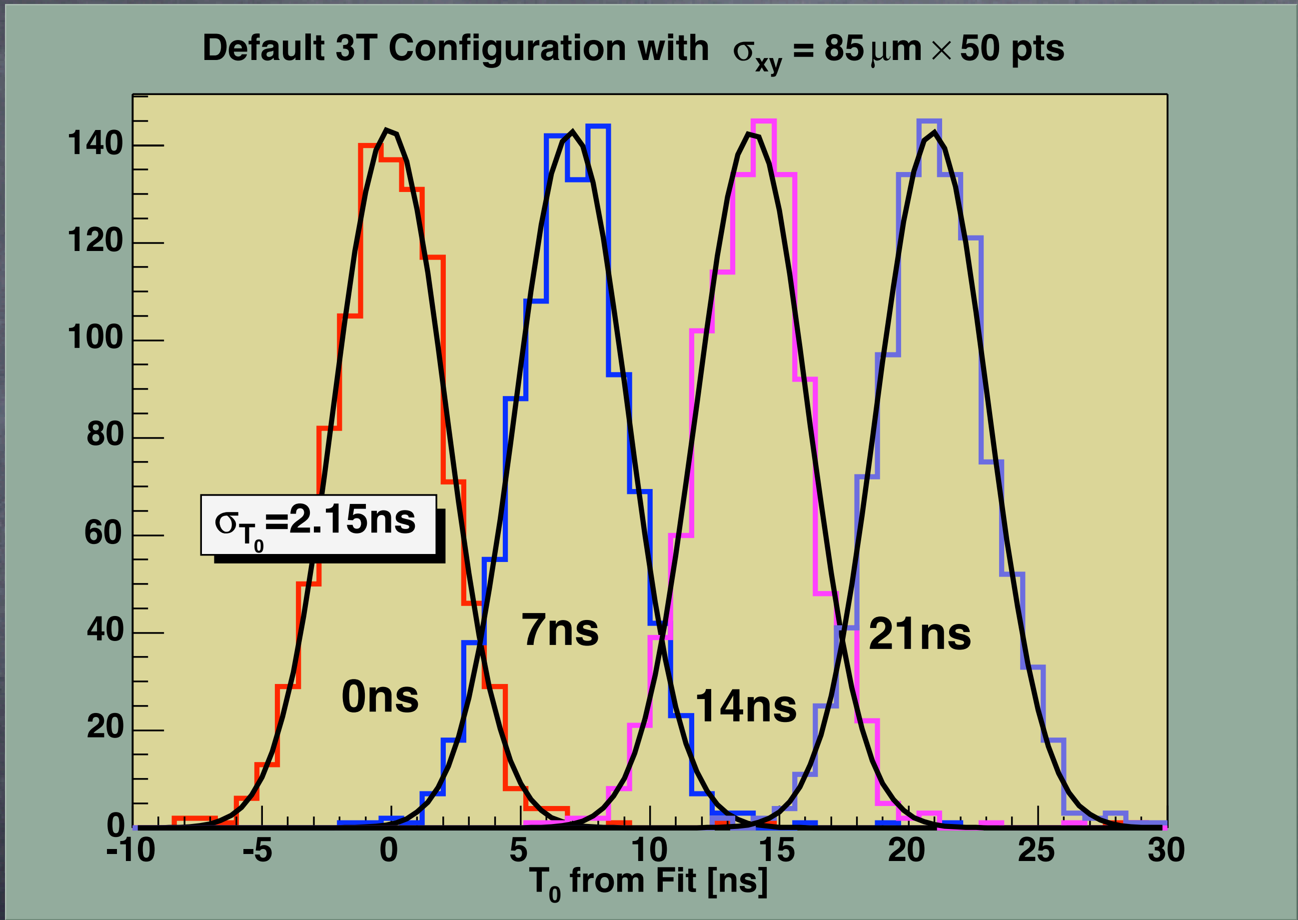
$$\sigma_{\mathrm{xy}} = 85\ \mu\mathrm{m}$$

$$v_{\mathrm{drift}} = 0.7\ \mathrm{cm}/\mu\mathrm{s}$$

$$n = 50$$

$$\sigma_{\Delta T_0} \simeq 1.7\ \mathrm{ns}$$

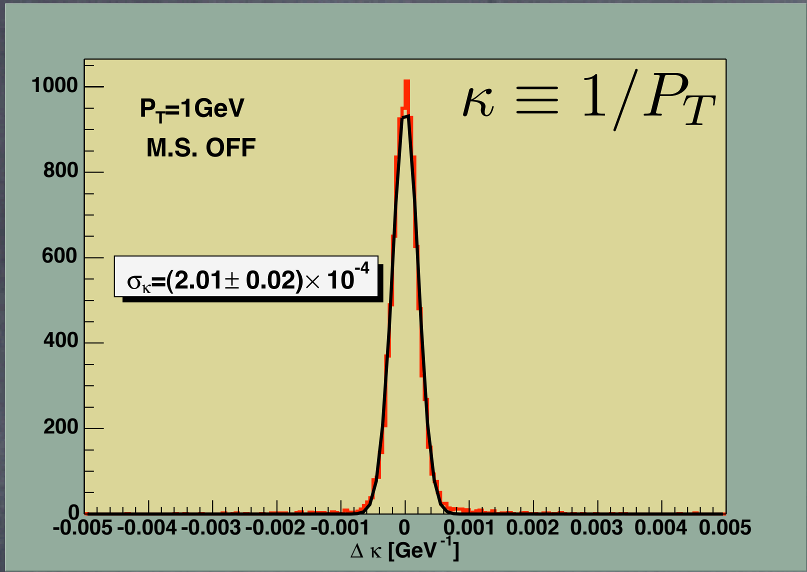
T0 from Helix Fit (axial+stereo, 100GeV)

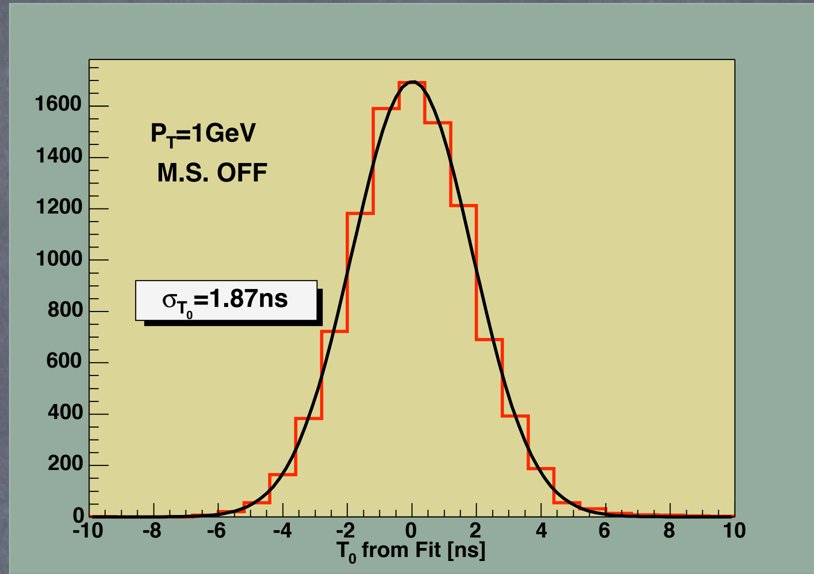


We can still determine T0 with ~2.2ns accuracy!

Multiple Scattering Effects (axial+stereo, 1GeV)

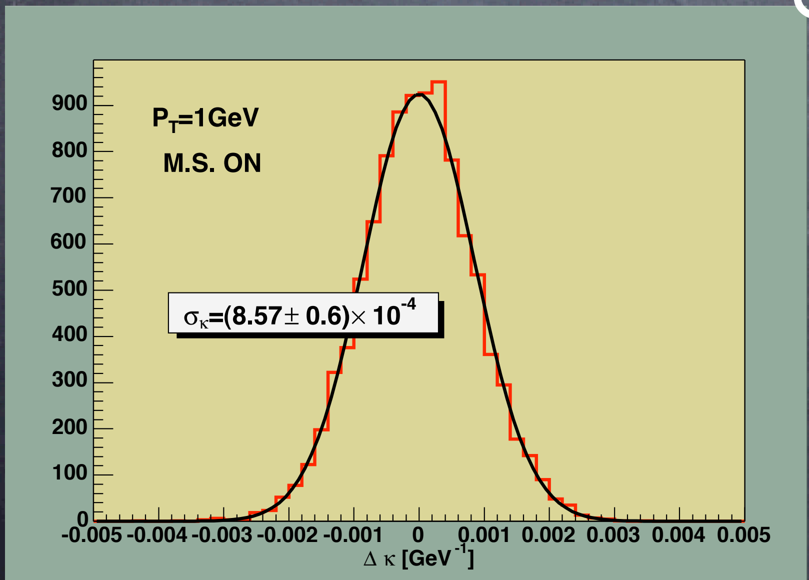
MS OFF

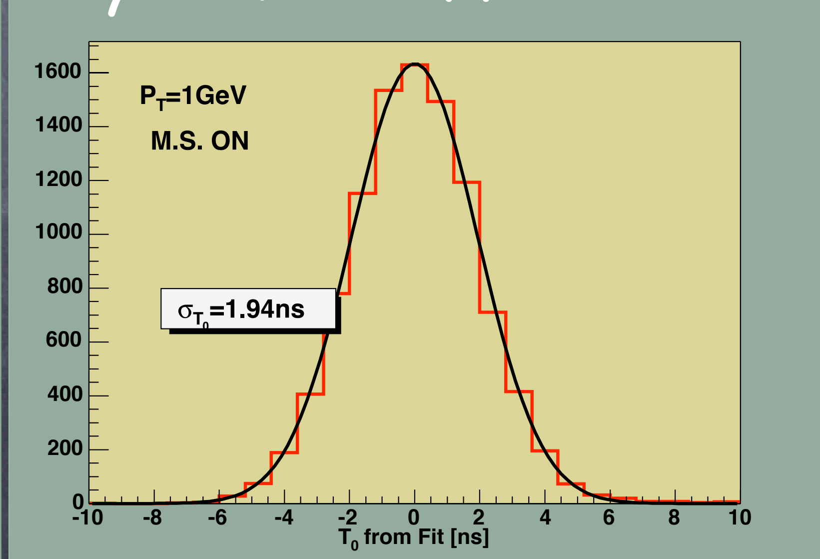




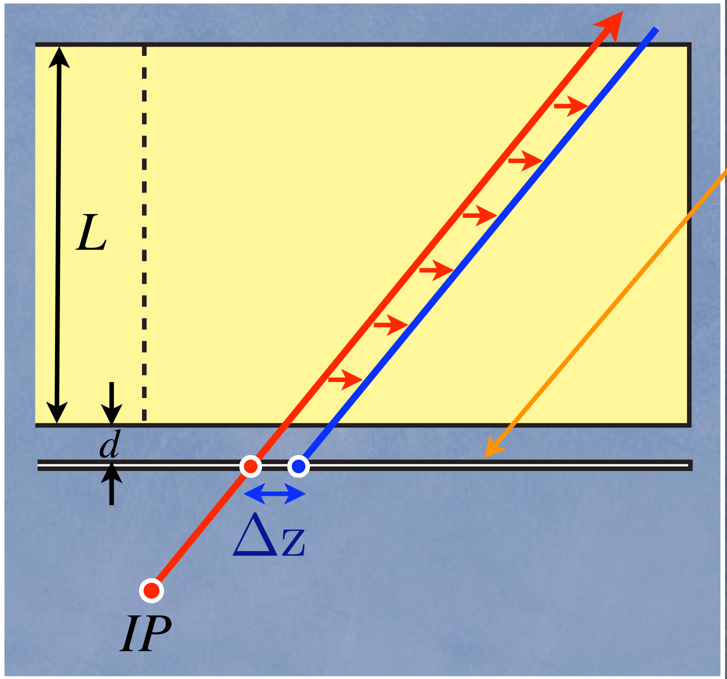
Only Small Effect on T0

MS ON





In the Case of TPC



External Z Detector (T0 Device)

Wrong T0 makes a Z-shift!

$$\Delta z = v_{\rm drift} \times \Delta T_0$$

Naively we expect

$$\sigma_{\Delta T_0} \simeq \frac{2\sigma_z}{v_{\rm drift}\sqrt{n}} \left[1 + 3\left(\frac{d}{L}\right) + 3\left(\frac{d}{L}\right)^2 \right]^{-\frac{1}{2}}$$

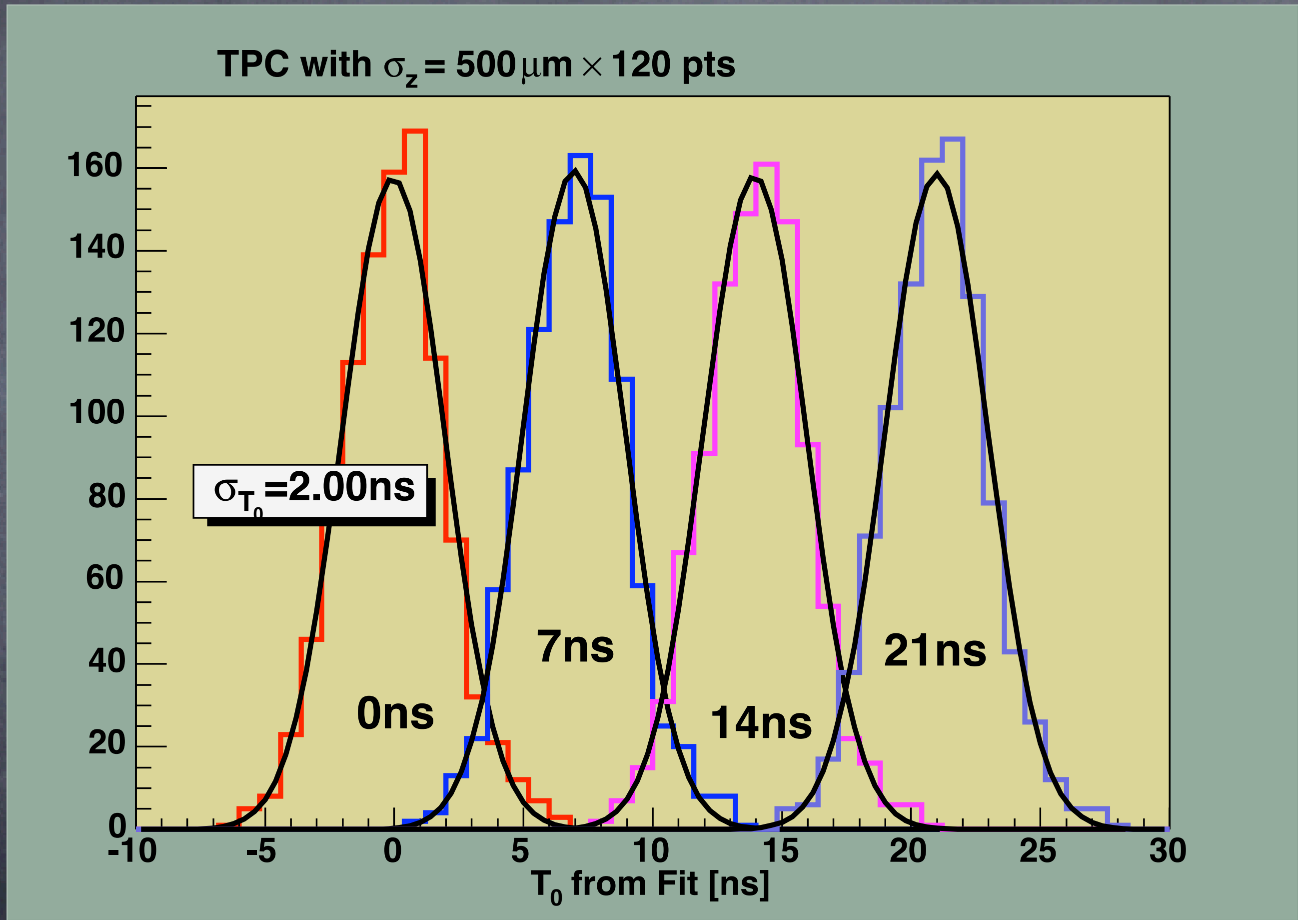
$$\simeq \frac{2\sigma_z}{v_{\rm drift}\sqrt{n}} \quad \text{if} \quad \left(\frac{d}{L}\right) \ll 1$$

Assuming that Z resolution of the external detector is negligible

$$\sigma_z = 500\ \mu\text{m}$$
$$v_{\rm drift} = 5\ \text{cm}/\mu\text{s}$$
$$n = 120$$

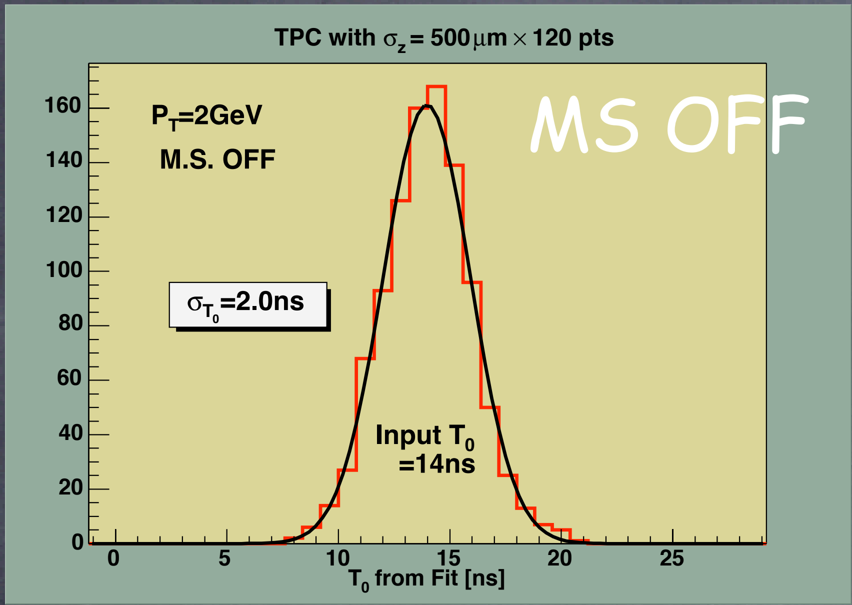
$$\Rightarrow \quad \sigma_{\Delta T_0} \simeq 2.0\ \text{ns}$$

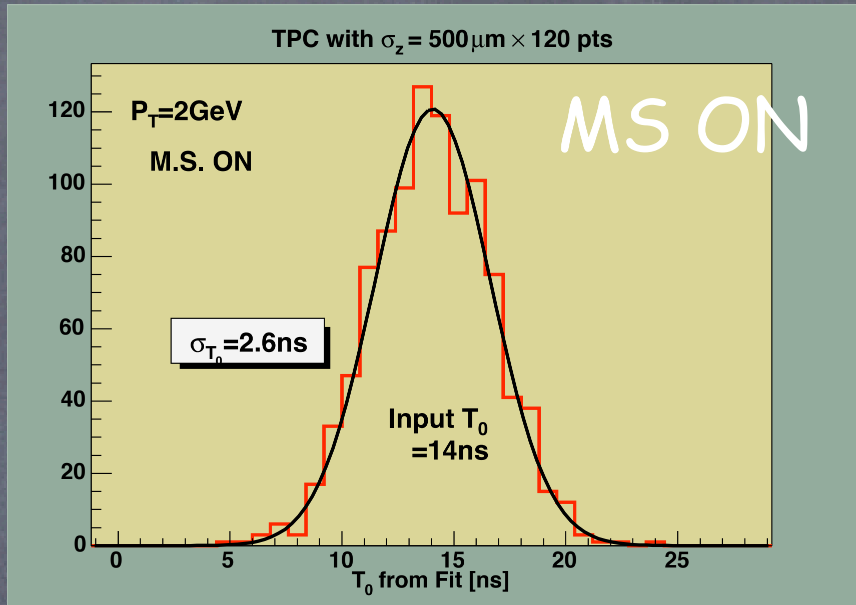
T0 from Helix Fit (d=5cm, 100GeV)



We can determine T0 with ~2.0ns accuracy as expected!

Multiple Scattering Effects (d=5cm,0.6%X0, 2GeV)





MS Effect more significant than for CDC

This is probably due to the fact that there is only a single break point to decide T0.

The material thickness between TPC and T0 detector does not matter as long as it stays just in front of the T0 detector.

0.6%X0 to 3.0%X0 --> 2% shift in T0 resolution