Lectures on Brane World Scenarios III

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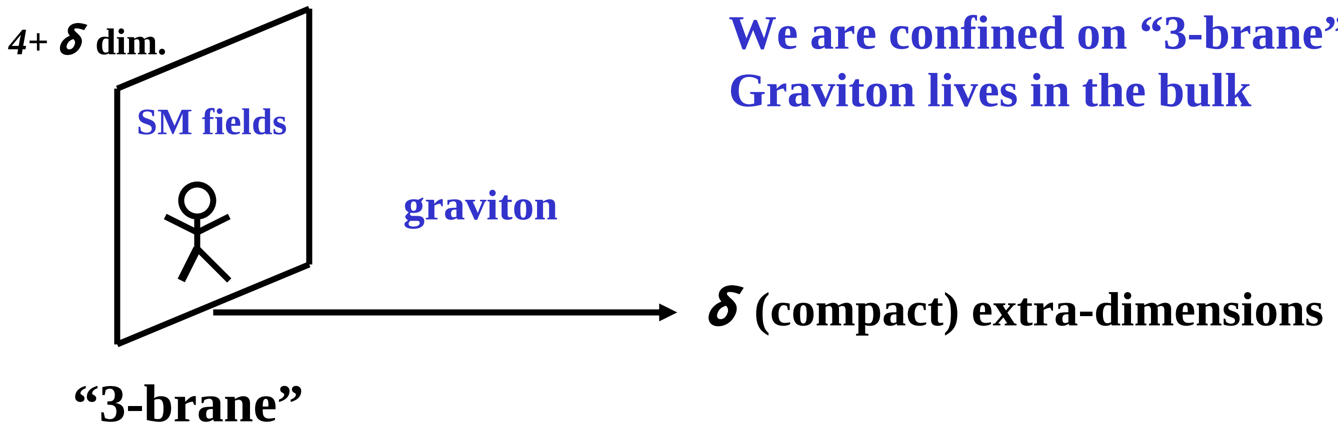
Brane World Scenarios

Phenomenological model



We are confined on "3-brane"
Graviton lives in the bulk

Beyond the standard model → Brane World Scenario
4+ δ dimensions

New property
"geometry"

Typical Scenario: Large (flat) Extra Dimensions (Arkani-Hamed-Dimopoulos-Dvali, '98)

Warped (small) Extra Dimensions (Randall-Sundrum, '99)

Today's topic

Models with Large Extra Dimensions

Conceptual problem

Is it really solution to gauge hierarchy problem?

$$M_{Planck} \sim 10^{19}\ \text{GeV} \gg M_{weak} \sim 100\ \text{GeV}$$

$$\searrow \quad M_{4+\delta} = \mathcal{O}(1\ \text{TeV}) \sim M_{weak}$$

Compactified on T^{δ}

$$M_{Planck}^2 = M_{4+\delta}^{2+\delta} \times R^{\delta}$$

$$\rightarrow \quad \boxed{\frac{1}{R} = 10^{-\frac{32}{\delta}}\ \text{TeV}}$$

δ	$1/R$	
1	10^{-20} eV	excluded
2	10^{-4} eV	
3	20 eV	
5	0.4 MeV	
10	0.6 GeV	

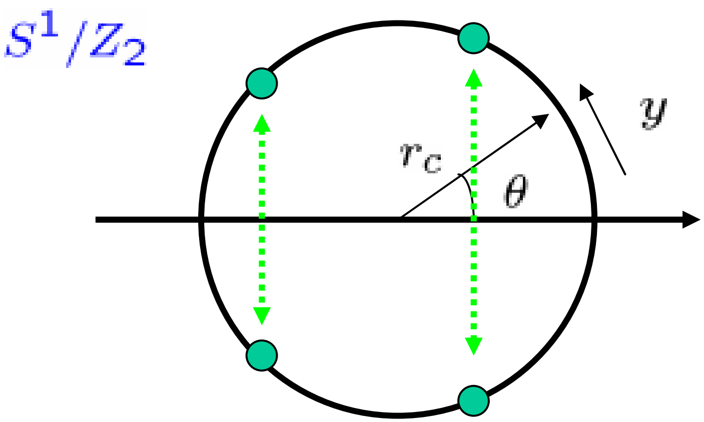
hierarchy! $\quad \delta = \mathcal{O}(1) \rightarrow M_{4+\delta} \gg \frac{1}{R}$

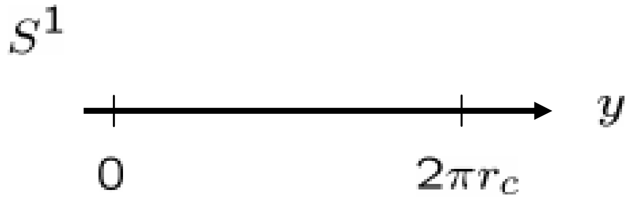
Warped (small) Extra Dimension Scenario

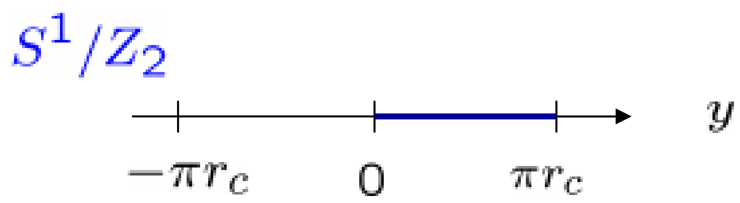
Randall-Sundrum, PRL 83 (1999) 3370

Setup: 5-dimimensional theory

5th dimension is compactified on S^1/Z_2



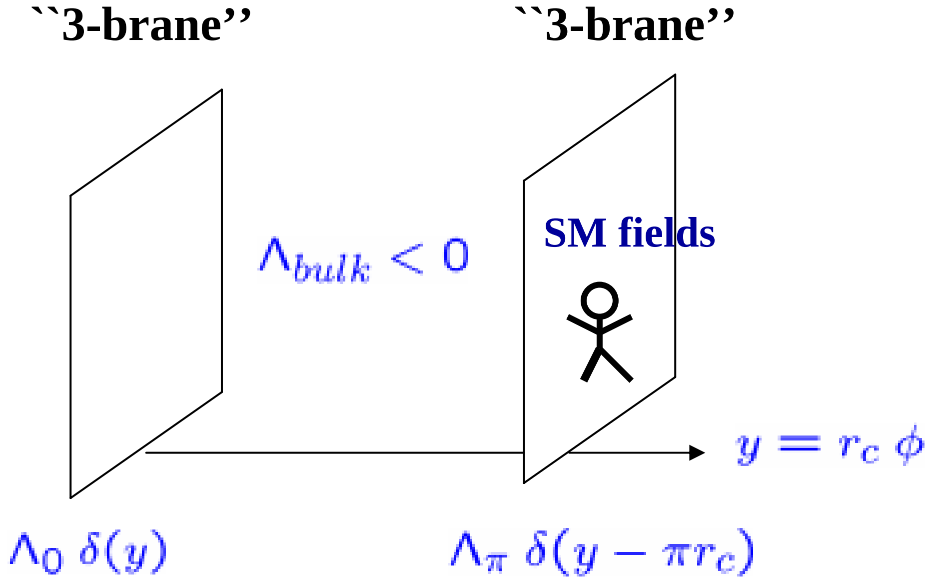




Identification $y \leftrightarrow -y$

$$y \to y + 2\pi r_c$$

slice of AdS_5



$$S_{total} = S_{bulk} + S_0 + S_\pi$$

$$\left\{ \begin{aligned} S_G &= \int d^4x \int_0^\pi d\phi \sqrt{-G}\left(4M_5^3 R_5 - 2\Lambda_{bulk}\right) \\ S_0 &= \int d^4x \sqrt{-g_0}\,(\mathcal{L}_0 - \Lambda_0) \\ S_\pi &= \int d^4x \sqrt{-g_\pi}\,(\mathcal{L}_\pi - \Lambda_\pi) \end{aligned} \right.$$

$$\begin{aligned} g_{0\mu\nu} &= G_{\mu\nu}(x^\mu, \phi = 0) \\ g_{\pi\mu\nu} &= G_{\mu\nu}(x^\mu, \phi = \pi) \end{aligned}$$

Solving Einstein's equations with cosmological constants { in bulk / on branes }

Metric ansatz

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2$$

4 dimensional Poincare invariance

$$\sqrt{-G}\left(R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R\right) = -\frac{1}{4M^3}\Lambda_{bulk}\sqrt{-G}G_{\mu\nu} + \Lambda_0\sqrt{-g_0}g_{0\mu\nu}\delta(\phi) + \Lambda_\pi\sqrt{-g_\pi}g_{\pi\mu\nu}\delta(\phi-\pi)$$

$$\left(R_{55} - \frac{1}{2}G_{55}R\right) = -\frac{1}{4M^3}\Lambda_{bulk}G_{55}$$

Others = 0

$$\sqrt{-G}\left(R_{\mu\nu}-\frac{1}{2}G_{\mu\nu}R\right) = -\frac{1}{4M^3}\Lambda_{bulk}\sqrt{-G}G_{\mu\nu} + \Lambda_0\sqrt{-g_0}g_{0\mu\nu}\delta(\phi)+\Lambda_\pi\sqrt{-g_\pi}g_{\pi\mu\nu}\delta(\phi-\pi)$$

$$\rightarrow \quad \frac{3}{r_c}\frac{d^2\sigma}{d\phi^2} = \frac{\Lambda_0}{4M^3 r_c}\delta(\phi)+\frac{\Lambda_\pi}{4M^3 r_c}\delta(\phi-\pi)$$

$$\left(R_{55}-\frac{1}{2}G_{55}R\right) = -\frac{1}{4M^3}\Lambda_{bulk}G_{55}$$

$$\rightarrow \quad \frac{6}{r_c^2}\left(\frac{d\sigma}{d\phi}\right)^2 = -\frac{\Lambda_{bulk}}{4M^3}$$

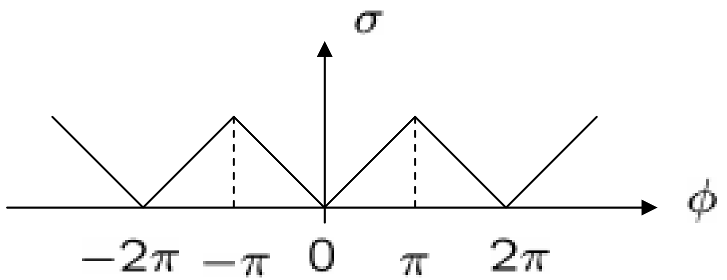
IF

$$\begin{aligned}\Lambda_{bulk} &= -24M^3k^2 \\ \Lambda_0 &= -\Lambda_\pi = 24M^3k\end{aligned}$$

satisfied

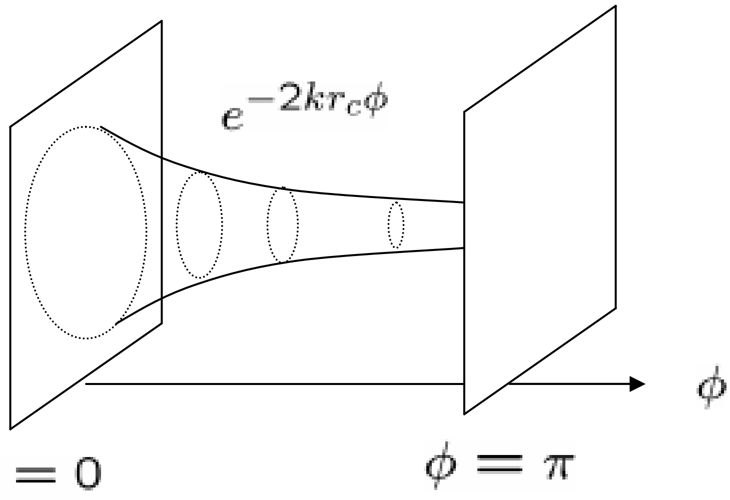
→ **Solution consistent with the orbifold symmetry** $\phi \leftrightarrow -\phi$

$$\sigma(\phi) = kr_c|\phi|$$



4-dim. Effective Planck scale

Solution: $$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2$$



$$V_5 \;=\; 2r_c \int_0^\pi d\phi\, e^{-2kr_c|y|} = \frac{1}{k}\left(1 - e^{-2kr_c|y|}\right)$$

$$\rightarrow \; M_4^2 = \frac{M^3}{k}\left(1 - e^{-2kr_c\pi}\right)$$

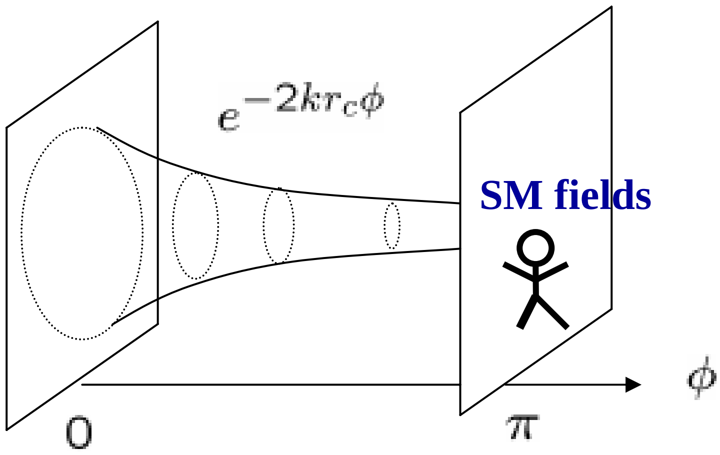
strongly warped

$$\rightarrow \; \frac{M^3}{k} \quad \text{for } kr_c\pi \gg 1$$

$$M^3(2\pi r_c) \quad \text{for } k \rightarrow 0$$

flat limit

Alternative solution to hierarchy problem!



SM Higgs lives on the visible brane

$$S_{Higgs} = \int d^4x\sqrt{-g_\pi}\left[g_\pi^{\mu\nu}(D_\mu H)^\dagger(D^\mu H) - \lambda(H^\dagger H - M^2)^2\right]$$

$$\sqrt{-g_\pi}g_\pi^{\mu\nu} = e^{-2kr_c\pi}\eta^{\mu\nu} \Longrightarrow H \to e^{kr_c\pi}H$$

rescale

$$\to \int d^4x\left[\eta^{\mu\nu}(D_\mu H)^\dagger(D^\mu H) - \lambda(H^\dagger H - v^2)^2\right] \qquad v = M \times e^{-kr_c\pi}$$

Even if M ~M4, v can be the weak scale with $kr_c \sim 12$

Mild hierarchy

Phenomenology: graviton KK mode physics

KK mode decomposition

$$G_{MN} = e^{-2k|y|}\eta_{\mu\nu} + h_{\mu\nu}(x) \times \psi(y)$$

Mode equation

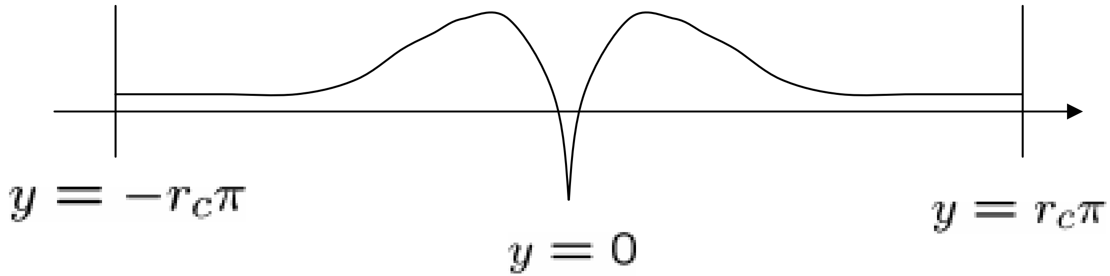
$$\left[-\frac{1}{2}\partial_z^2 + V(z)\right]\hat{\psi}(Z) = m^2\hat{\psi}(Z)$$

$$z = sgn(y) \times \frac{e^{k|y|/2} - 1}{k}$$

$$\hat{\psi}(z) = \psi(z)e^{k|y|/2}$$

$$V(z) = -\frac{3k}{2}\delta(z) + \frac{15k^2}{8(k|z|+1)^2}$$

(volcano potential)



KK mode configuration

$$f^{(n)}(\phi) = e^{kr_c|\phi|}\psi^{(n)}$$

$$f^{(0)}(\phi) \sim e^{-kr_c|\phi|}$$

localize around hidden brane

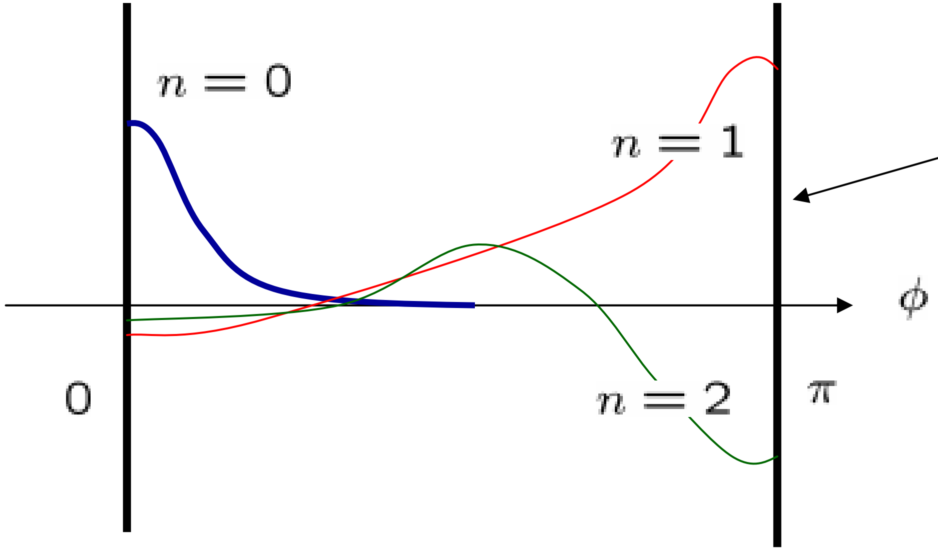
$$f^{(n)}(\phi) \sim e^{kr_c|\phi|} J_2(m_{(n)}/\kappa\, e^{kr_c|\phi|})$$

localize around visible brane

- $M \sim M_4 \gg M_W$
- **graviton KK mode mass** $m_{KK}^{(n)} \sim x_n \kappa e^{-kr_c\pi} \sim x_n\, \mathcal{O}(M_W)$

$$J_1(x_n) = 0 \quad \begin{aligned} x_1 &\sim 3.83 \\ x_2 &\sim 7.02 \\ x_3 &\sim 10.17 \\ x_4 &\sim 13.32 \end{aligned}$$

KK mode configuration



We live here

$$\mathcal{L}_{int} = -\frac{1}{\bar{M}_4} G^{(0)}_{\mu\nu} T^{\mu\nu} - \frac{1}{\bar{M}_4 e^{-\kappa r_c \pi}} \sum_{n=1} G^{(n)}_{\mu\nu} T^{\mu\nu}$$

← **Weak: small overlap**

← **Strong: large overlap**

$$m^{(n)}_{KK} \sim x_n \kappa e^{-k r_c \pi} \sim x_n \, \mathcal{O}(M_W)$$

Collider physics

KK graviton resonance production, if $\sqrt{s} > m_{KK}^{(1)}$

$$\begin{cases} \mathcal{L}_{int}^{(1)} = -\dfrac{1}{\bar{M}_4 e^{-kr_c\pi}} G_{\mu\nu}^{(1)} T^{\mu\nu} \\ \bar{M}_4 e^{-kr_c\pi} \sim \dfrac{m_{KK}^{(1)}}{3.83} \dfrac{\bar{M}_4}{k} \end{cases}$$

$$\begin{cases} m_{KK}^{(n)} = x_n k e^{-kr_c\pi} \\ J_1(x_n) = 0 \end{cases}$$

Model parameters → $\begin{cases} \dfrac{k}{\bar{M}_4} \\ m_{KK}^{(1)} \end{cases}$

KK graviton resonance production @ LC

Davoudiasl, Hewett & Rizzo,

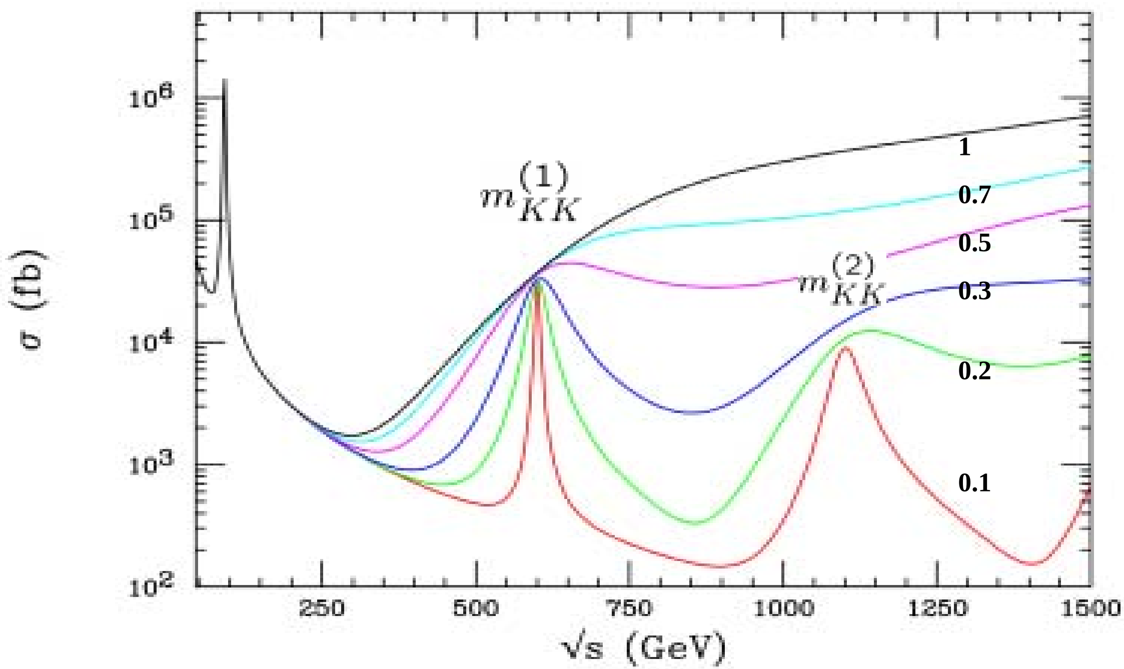
PRL 84 (2000) 2080

$$e^+e^- \to g_{KK}^{(1)} \to \mu^+\mu^-$$

$$\Gamma_1 = c \times m_{KK}^{(1)} \frac{k}{\bar{M}_4}$$

c: calculable numerical factor

$$m_{KK}^{(1)} = 600\text{GeV}$$



$$\frac{k}{M_4}$$

Width becomes large

as k/M4 becomes large

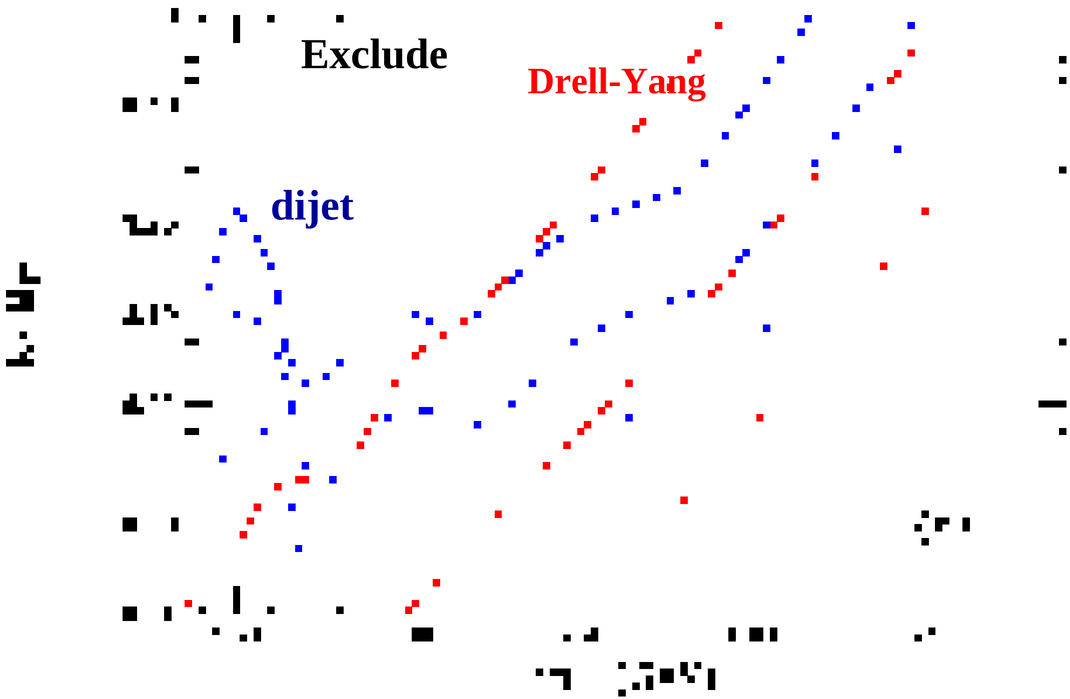
KK graviton resonance production @ hadron collider

Direct searches for the 1st KK graviton production @Tevatron, LHC

- **Drell-Yang events** $\to \ell^+\ell^-$
- **Dijet events** $\to q\bar{q},\ gg$

$$q\bar{q},\ gg \to G^{(1)}_{KK}$$

Davoudiasl et al., PRL 84 (2000) 2080



Tevatron Run I ———

Tevatron Run II

- - - - $2fb^{-1}$

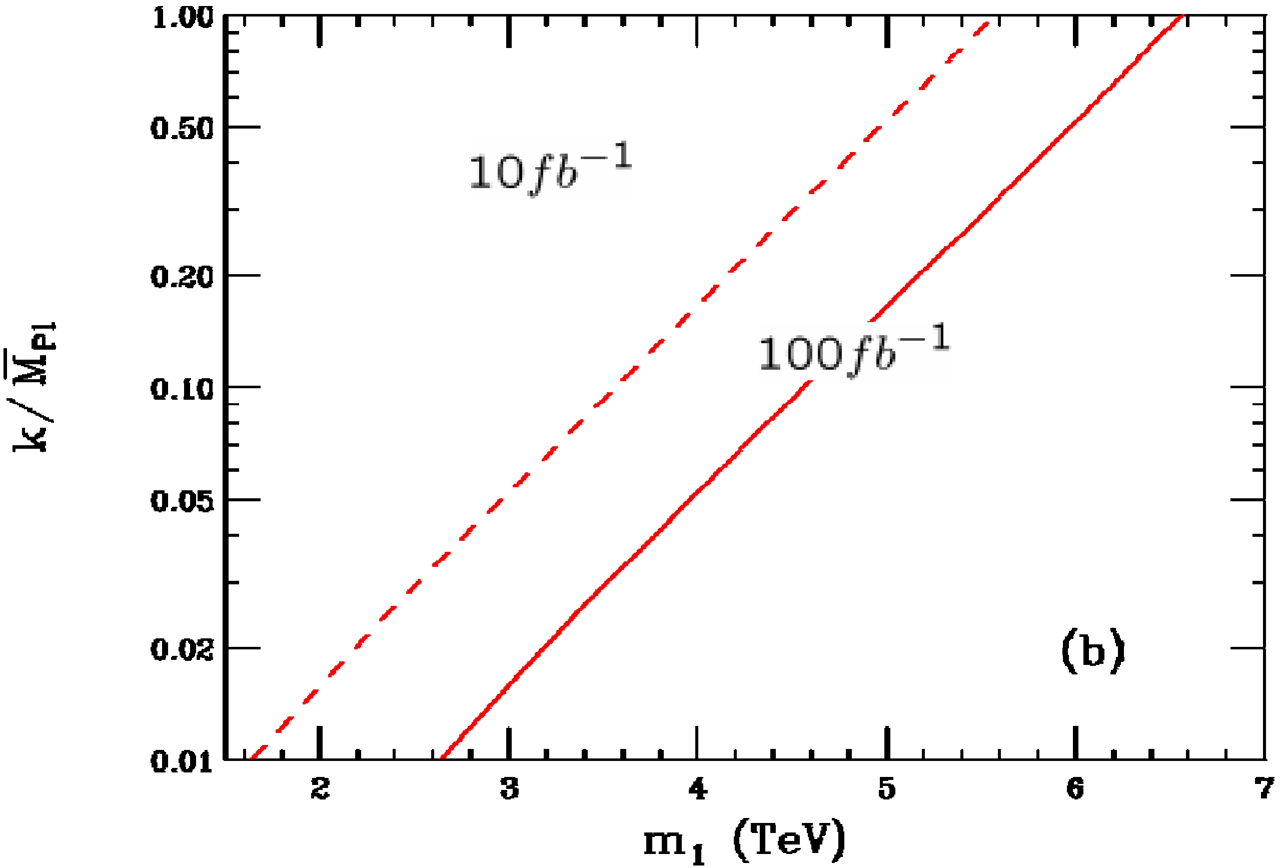
.... $30fb^{-1}$

@ LHC

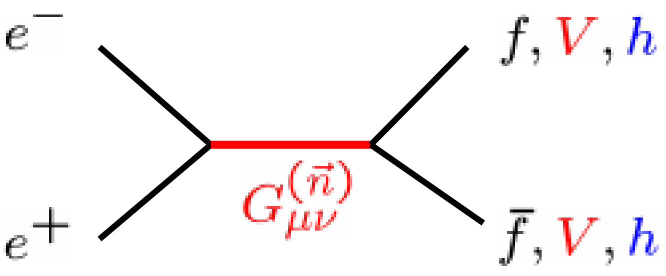
Davoudiasl et al.,

PRL 84 (2000) 2080

95% CL exclusion region



Virtual KK graviton mediated process



Models with Large Extra Dimensions

$$\sum_{\vec{n}} \frac{1}{s - \left(m_{KK}^{(\vec{n})}\right)^2} \to \infty \quad (\text{for } \delta \geq 2)$$

Need regularization

Naïve: Cut Off by $m_{KK}^{MAX} \sim M_{4+\delta}$

$$\boxed{\frac{4\pi\lambda}{{M_S}^4}} = -\frac{8\pi}{M_P^2} \sum_{\vec{n}} \frac{1}{s - \left(m_{KK}^{(\vec{n})}\right)^2}; \qquad \lambda = \pm 1$$

$$\mathcal{M} = \frac{4\pi\lambda}{M_S^4} T_{\mu\nu}(p_1, p_2) T^{\mu\nu}(p_3, p_4)$$

Five dimensional case

$$\sum_{n=1}^{\infty} \frac{1}{s - \left(m_{KK}^{(n)}\right)^2}$$ → **converge!**

For $\sqrt{s} << m_{KK}^{(1)}$ → $$\sum_{n=1}^{\infty} \frac{1}{s - \left(m_{KK}^{(n)}\right)^2} \sim -\frac{e^{2kr_c\pi}}{k^2} \times \sum_{n=1}^{\infty} \frac{1}{x_n^2}$$

Correspondence: $$\frac{e^{2kr_c\pi}}{k^2} \sum_{n=1}^{\infty} \frac{1}{x_n^2} \Longleftrightarrow \frac{4\pi\lambda}{M_S^4}$$

→ **analysis in LED scenario can be interpreted**

Note: no brane fluctuation can be introduced in the orbifold case

Alternative to compactification

Randall-Sundrum, PRL 83 (1999) 4690

In order for models with extra dimensions to be realistic,

compactifications of extra dimensions is necessary

$M_4^2 = M_{4+\delta}^{2+\delta} R^\delta$ → finite M_4 and $M_{4+\delta}$

→ finite ***R***

But, general requirement is finiteness of V_δ

$M_4^2 = M_{4+\delta}^{2+\delta} R^\delta$ → finite M_4 and $M_{4+\delta}$

→ finite V_δ

In <u>warped</u> extra dimension scenario

$$V_5 = 2r_c \int_0^\pi d\phi\, e^{-2kr_c|y|} = \frac{1}{k}\left(1 - e^{-2kr_c|y|}\right)$$
$$\rightarrow M_4^2 = \frac{M^3}{k}\left(1 - e^{-2kr_c\pi}\right)$$

Effective volume is finite even if $r_c \rightarrow \infty$

implies → **<u>Alternative compactification scenario</u>**

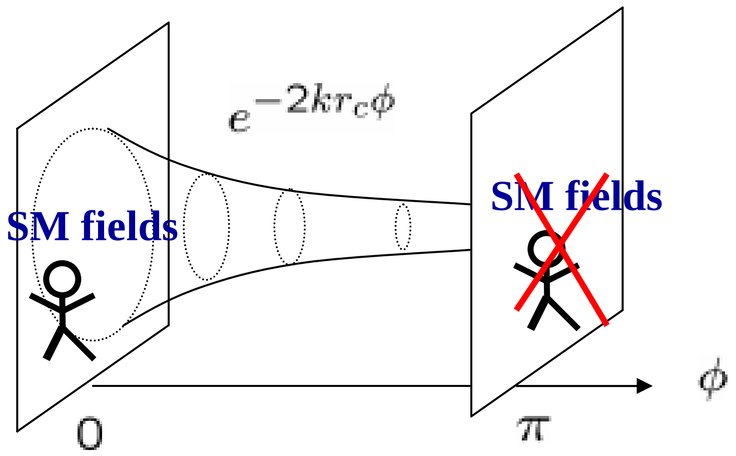
<u>What happen?</u>

$$r_c \rightarrow \infty \quad \begin{cases} m_{KK}^{(1)} \rightarrow 0 \\ m_{KK}^{(n+1)}/m_{KK}^{(n)} \rightarrow 1 \end{cases}$$

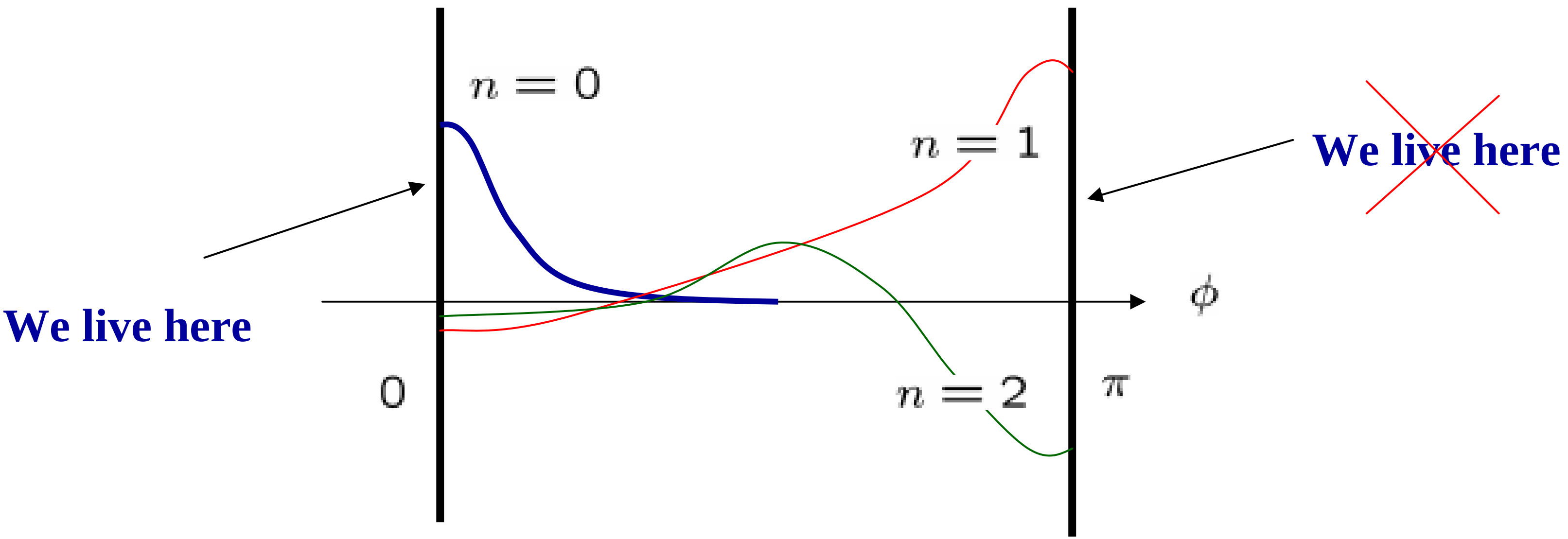
Continuous KK mode spectrum from 0 to infinity

→ Realistic model?

Changing the setup



KK mode configuration



$$\mathcal{L}_{int} = -\frac{1}{\bar{M}_4} G^{(0)}_{\mu\nu} T^{\mu\nu} - \frac{1}{\bar{M}_4 e^{+\kappa r_c \pi}} \sum_{n=1} G^{(n)}_{\mu\nu} T^{\mu\nu}$$

← **Strong: large overlap**

← **weak: small overlap**

Newton potential for continuum KK mode

$$V(r) \sim G_N \frac{m_1 m_2}{r} + \int_0^\infty \frac{mdm}{k^2} G_N \frac{m_1 m_2 e^{-mr}}{r} = G_N \frac{m_1 m_2}{r}\left(1 + \frac{1}{r^2 k^2}\right)$$

Gravity precision measurement → ***r k >1* for *r = 0.1 mm***

→ $k > 10^{-4}$ eV

$M \sim \left(\bar{M}_4^2 k\right)^{\frac{1}{3}}$ → $M > 10^{7.7}$ GeV

Fig. on page 15