Lectures on Brane World Scenarios III

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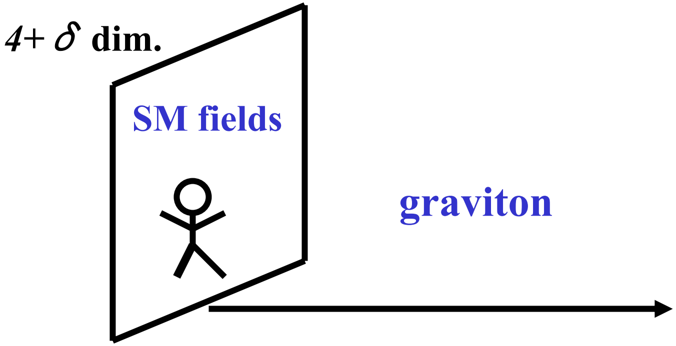
Brane World Scenarios

<u>Phenomenological model</u>



δ (compact) extra-dimensions

"3-brane"

We are confined on "3-brane"
Graviton lives in the bulk

<u>Beyond the standard model</u> → Brane World Scenario
$4+\delta$ dimensions

New property
"geometry"

Typical Scenario: Large (flat) Extra Dimensions (Arkani-Hamed-Dimopoulos-Dvali, '98)

Warped (small) Extra Dimensions (Randall-Sundrum, '99)

Today's topic

Models with Large Extra Dimensions

Conceptual problem

Is it really solution to gauge hierarchy problem?

$$M_{Planck} \sim 10^{19} \text{ GeV} \gg M_{weak} \sim 100 \text{ GeV}$$

$$M_{4+\delta} = \mathcal{O}(1 \text{ TeV}) \sim M_{weak}$$

Compactified on T^{δ}

$$M_{Planck}^2 = M_{4+\delta}^{2+\delta} \times R^{\delta}$$

$$\rightarrow \boxed{\frac{1}{R} = 10^{-\frac{32}{\delta}} \text{ TeV}}$$

δ	$1/R$	
1	10^{-20} eV	excluded
2	10^{-4} eV	
3	20 eV	
5	0.4 MeV	
10	0.6 GeV	

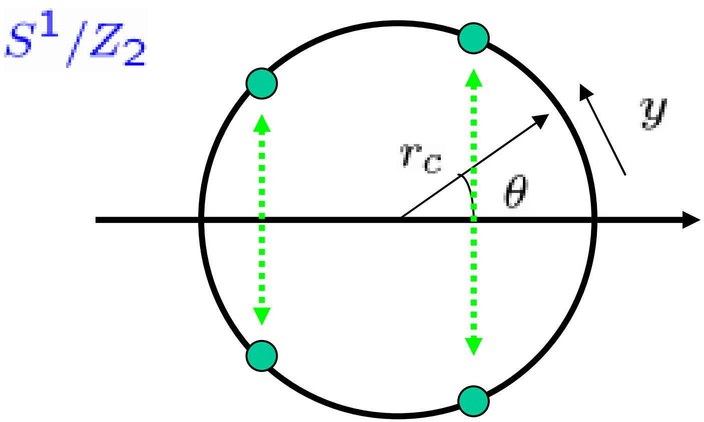
hierarchy! $\delta = \mathcal{O}(1) \rightarrow M_{4+\delta} \gg \frac{1}{R}$

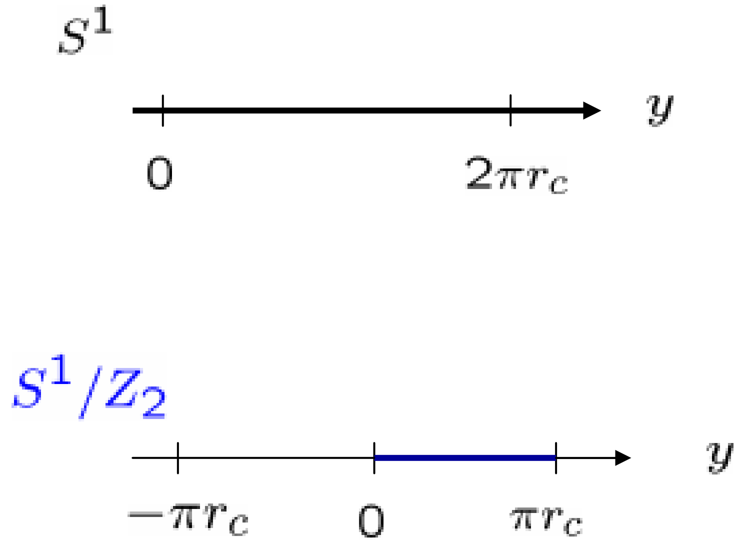
Warped (small) Extra Dimension Scenario

Randall-Sundrum, PRL 83 (1999) 3370

Setup: 5-dimimensional theory

5th dimension is compactified on S^1/Z_2

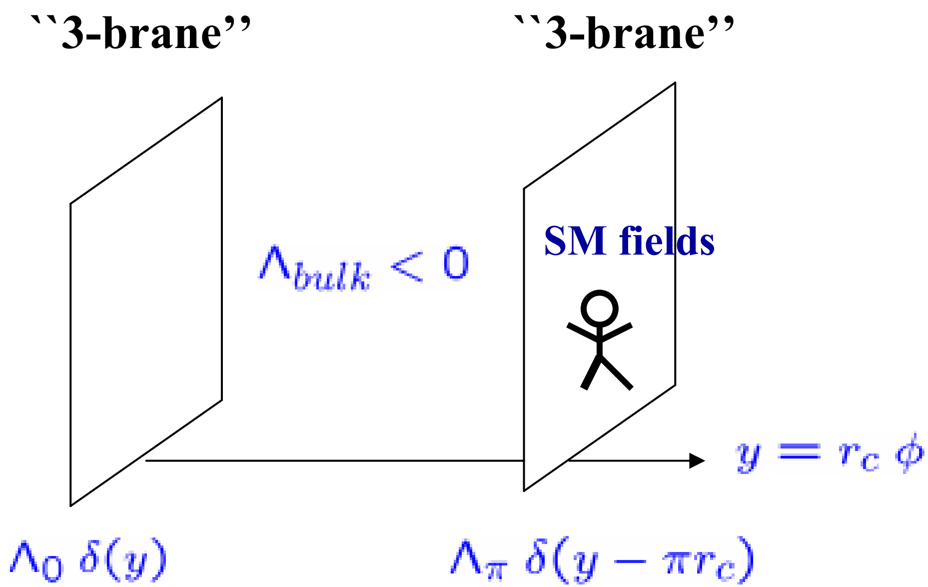




Identification $y \leftrightarrow -y$

$$y \rightarrow y + 2\pi r_c$$

slice of AdS_5



$$S_{total} = S_{bulk} + S_0 + S_\pi$$

$$\begin{cases} S_G = \int d^4x \int_0^\pi d\phi \sqrt{-G}\left(4M_5^3 R_5 - 2\Lambda_{bulk}\right) \\ S_0 = \int d^4x \sqrt{-g_0}\,(\mathcal{L}_0 - \Lambda_0) \\ S_\pi = \int d^4x \sqrt{-g_\pi}\,(\mathcal{L}_\pi - \Lambda_\pi) \end{cases}$$

$$g_{0\mu\nu} = G_{\mu\nu}(x^\mu, \phi = 0)$$
$$g_{\pi\mu\nu} = G_{\mu\nu}(x^\mu, \phi = \pi)$$

Solving Einstein's equations with cosmological constants $\begin{cases} \text{in bulk} \\ \text{on branes} \end{cases}$

Metric ansatz

$$\boxed{ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2}$$

4 dimensional Poincare invariance

$$\begin{cases} \begin{aligned} \sqrt{-G}\left(R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R\right) &= -\frac{1}{4M^3}\Lambda_{bulk}\sqrt{-G}G_{\mu\nu} \\ &+ \Lambda_0\sqrt{-g_0}g_{0\mu\nu}\delta(\phi) + \Lambda_\pi\sqrt{-g_\pi}g_{\pi\mu\nu}\delta(\phi-\pi) \end{aligned} \\[2em] \left(R_{55} - \frac{1}{2}G_{55}R\right) = -\frac{1}{4M^3}\Lambda_{bulk}G_{55} \\[2em] \text{Others} = 0 \end{cases}$$

$$\sqrt{-G}\left(R_{\mu\nu}-\frac{1}{2}G_{\mu\nu}R\right) = -\frac{1}{4M^3}\Lambda_{bulk}\sqrt{-G}G_{\mu\nu} + \Lambda_0\sqrt{-g_0}g_{0\mu\nu}\delta(\phi)+\Lambda_\pi\sqrt{-g_\pi}g_{\pi\mu\nu}\delta(\phi-\pi)$$

$$\rightarrow \quad \frac{3}{r_c}\frac{d^2\sigma}{d\phi^2} = \frac{\Lambda_0}{4M^3 r_c}\,\delta(\phi)+\frac{\Lambda_\pi}{4M^3 r_c}\,\delta(\phi-\pi)$$

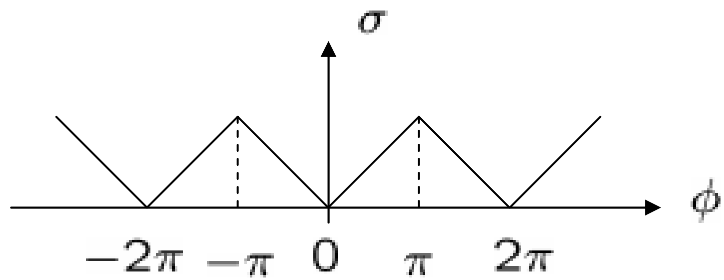
$$\left(R_{55}-\frac{1}{2}G_{55}R\right) = -\frac{1}{4M^3}\Lambda_{bulk}G_{55}$$

$$\rightarrow \quad \frac{6}{r_c^2}\left(\frac{d\sigma}{d\phi}\right)^2 = -\frac{\Lambda_{bulk}}{4M^3}$$

IF $\boxed{\begin{aligned}\Lambda_{bulk} &= -24M^3k^2\\ \Lambda_0 &= -\Lambda_\pi = 24M^3k\end{aligned}}$ **satisfied**

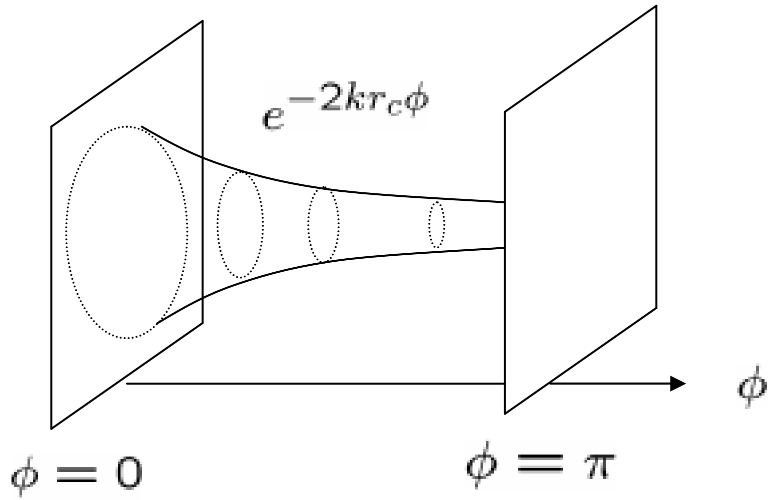
→ **Solution consistent with the orbifold symmetry** $\phi \leftrightarrow -\phi$

$$\sigma(\phi) = kr_c|\phi|$$



4-dim. Effective Planck scale

Solution: $$\boxed{ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2}$$



$$V_5 = 2r_c \int_0^\pi d\phi\, e^{-2kr_c|y|} = \frac{1}{k}\left(1 - e^{-2kr_c|y|}\right)$$

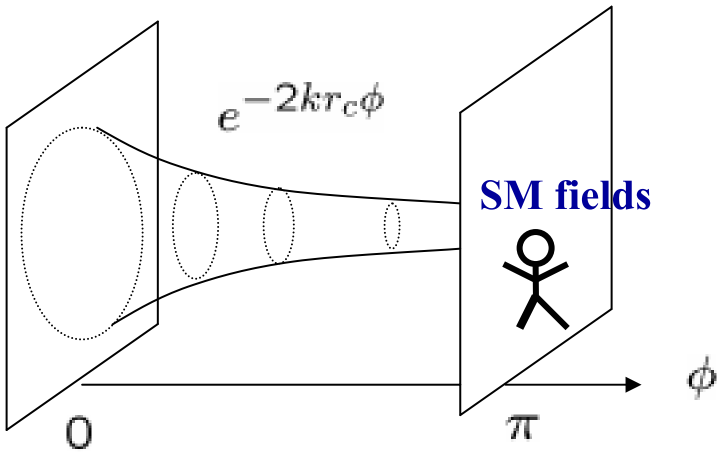
$$\to \quad M_4^2 = \frac{M^3}{k}\left(1 - e^{-2kr_c\pi}\right)$$

strongly warped

$$\to \quad \begin{cases} \dfrac{M^3}{k} & \text{for } kr_c\pi \gg 1 \\ M^3(2\pi r_c) & \text{for } k \to 0 \end{cases}$$

flat limit

Alternative solution to hierarchy problem!



SM Higgs lives on the visible brane

$$S_{Higgs} = \int d^4x\sqrt{-g_\pi}\left[g_\pi^{\mu\nu}(D_\mu H)^\dagger(D_\nu H) - \lambda(H^\dagger H - M^2)^2\right]$$

$$\sqrt{-g_\pi}g_\pi^{\mu\nu} = e^{-2kr_c\pi}\eta^{\mu\nu} \Longrightarrow H \to e^{kr_c\pi}H$$ **rescale**

$$\to \int d^4x\left[\eta^{\mu\nu}(D_\mu H)^\dagger(D_\nu H) - \lambda(H^\dagger H - v^2)^2\right] \qquad v = M \times e^{-kr_c\pi}$$

Even if M ∼M4, v can be the weak scale with $kr_c \sim 12$

Mild hierarchy

Phenomenology: graviton KK mode physics

KK mode decomposition

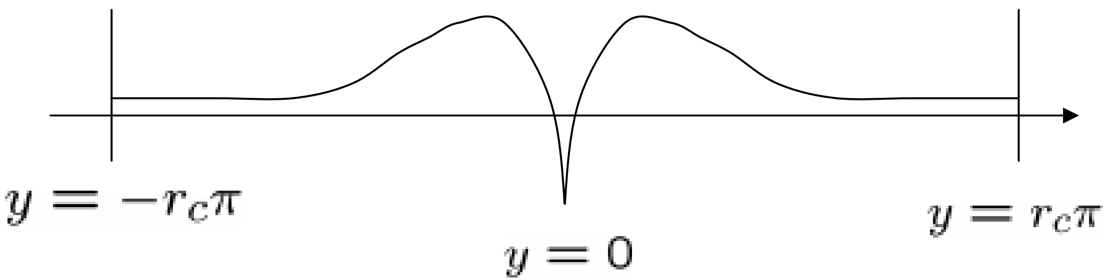
$$G_{MN} = e^{-2k|y|}\eta_{\mu\nu} + h_{\mu\nu}(x) \times \psi(y)$$

Mode equation

$$\left[-\frac{1}{2}\partial_z^2 + V(z)\right]\hat{\psi}(Z) = m^2\hat{\psi}(Z)$$

$$\begin{aligned} z &= sgn(y) \times \frac{e^{k|y|/2} - 1}{k} \\ \hat{\psi}(z) &= \psi(z)e^{k|y|/2} \end{aligned}$$

$$V(z) = -\frac{3k}{2}\delta(z) + \frac{15k^2}{8(k|z|+1)^2}$$ **(volcano potential)**



KK mode configuration

$$f^{(n)}(\phi) \;=\; e^{kr_c|\phi|}\psi^{(n)}$$

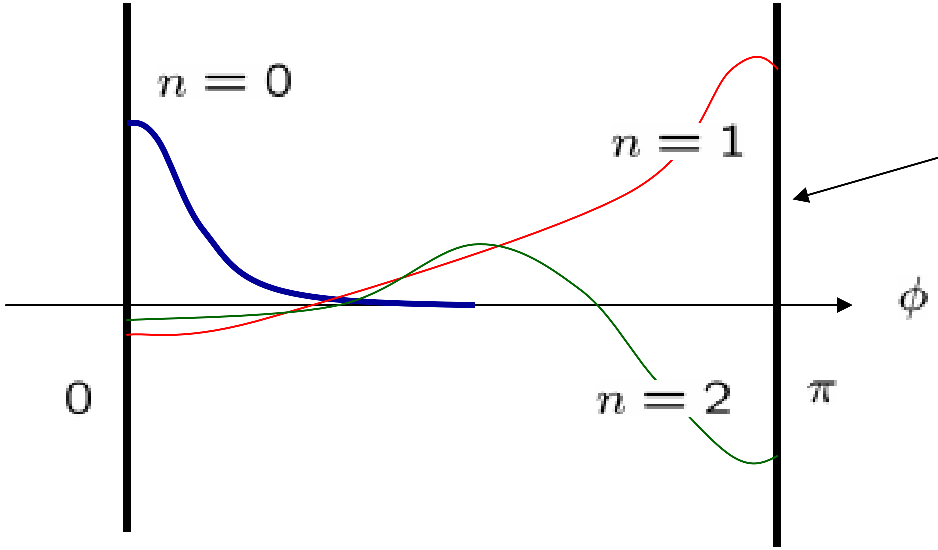
$$f^{(0)}(\phi) \;\sim\; e^{-kr_c|\phi|}$$
localize around hidden brane

$$f^{(n)}(\phi) \;\sim\; e^{kr_c|\phi|}\, J_2(m_{(n)}/\kappa\, e^{kr_c|\phi|})$$
localize around visible brane

- $M \sim M_4 \gg M_W$
- graviton KK mode mass $m_{KK}^{(n)} \sim x_n \kappa e^{-kr_c\pi} \sim x_n\, \mathcal{O}(M_W)$

$$J_1(x_n) = 0 \qquad \begin{aligned} x_1 &\sim 3.83 \\ x_2 &\sim 7.02 \\ x_3 &\sim 10.17 \\ x_4 &\sim 13.32 \end{aligned}$$

KK mode configuration



We live here

$$
\begin{aligned}
\mathcal{L}_{int} = & -\frac{1}{\bar{M}_4} G^{(0)}_{\mu\nu} T^{\mu\nu} \\
& - \frac{1}{\bar{M}_4 e^{-\kappa r_c \pi}} \sum_{n=1} G^{(n)}_{\mu\nu} T^{\mu\nu}
\end{aligned}
$$

← **Weak: small overlap**

← **Strong: large overlap**

$$m^{(n)}_{KK} \sim x_n \kappa e^{-k r_c \pi} \sim x_n \, \mathcal{O}(M_W)$$

Collider physics

KK graviton resonance production, if $\sqrt{s} > m_{KK}^{(1)}$

$$\left\{ \begin{array}{l} \mathcal{L}_{int}^{(1)} = -\dfrac{1}{\bar{M}_4 e^{-kr_c\pi}} G_{\mu\nu}^{(1)} T^{\mu\nu} \\ \\ \bar{M}_4 e^{-kr_c\pi} \sim \dfrac{m_{KK}^{(1)}}{3.83} \dfrac{\bar{M}_4}{k} \end{array} \right.$$

$$\left\{ \begin{array}{l} m_{KK}^{(n)} = x_n k e^{-kr_c\pi} \\ \\ J_1(x_n) = 0 \end{array} \right.$$

Model parameters → $\left\{ \begin{array}{l} \dfrac{k}{\bar{M}_4} \\ \\ m_{KK}^{(1)} \end{array} \right.$

KK graviton resonance production @ LC

Davoudiasl, Hewett & Rizzo,

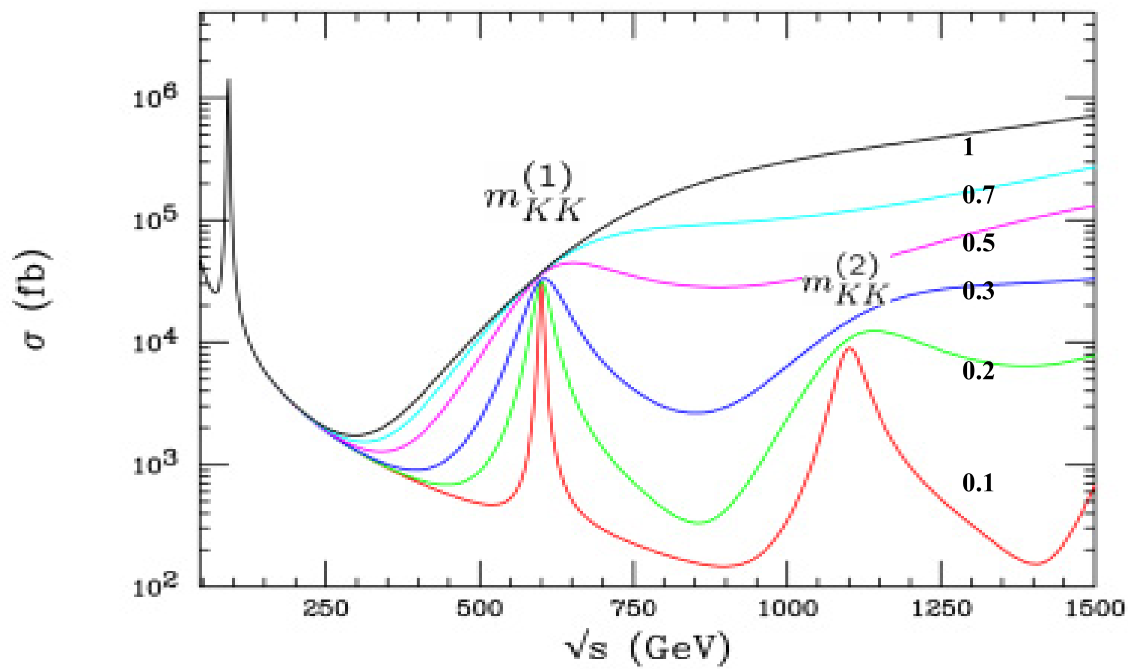
PRL 84 (2000) 2080

$$e^+e^- \to g_{KK}^{(1)} \to \mu^+\mu^-$$

$$\Gamma_1 = c \times m_{KK}^{(1)} \frac{k}{\bar{M}_4}$$

c: calculable numerical factor

$$m_{KK}^{(1)} = 600\text{GeV}$$



$$\frac{k}{M_4}$$

Width becomes large as k/M4 becomes large

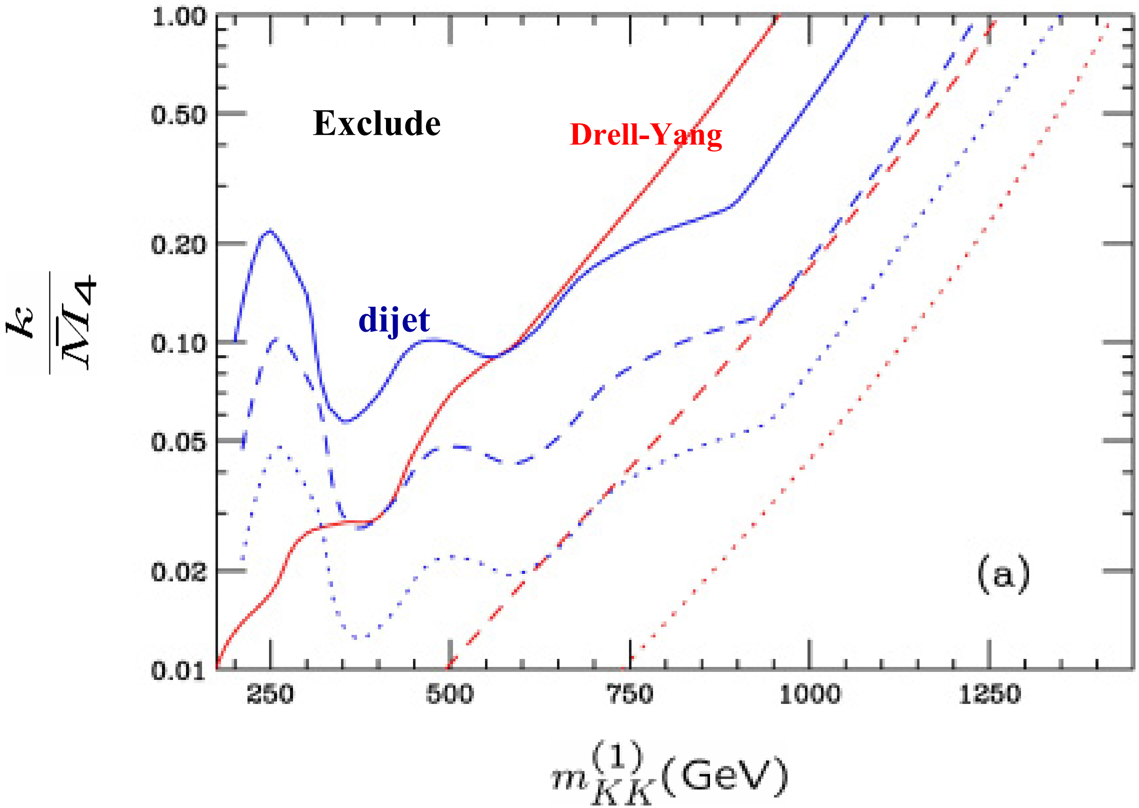
KK graviton resonance production @ hadron collider

Direct searches for the 1st KK graviton production @Tevatron, LHC

Drell-Yang events $\rightarrow \ell^+\ell^-$

Dijet events $\rightarrow q\bar{q},\ gg$

$$q\bar{q},\ gg \rightarrow G^{(1)}_{KK}$$



Davoudiasl et al., PRL 84 (2000) 2080

Tevatron Run I (solid line)

Tevatron Run II

(dashed line) $2 fb^{-1}$

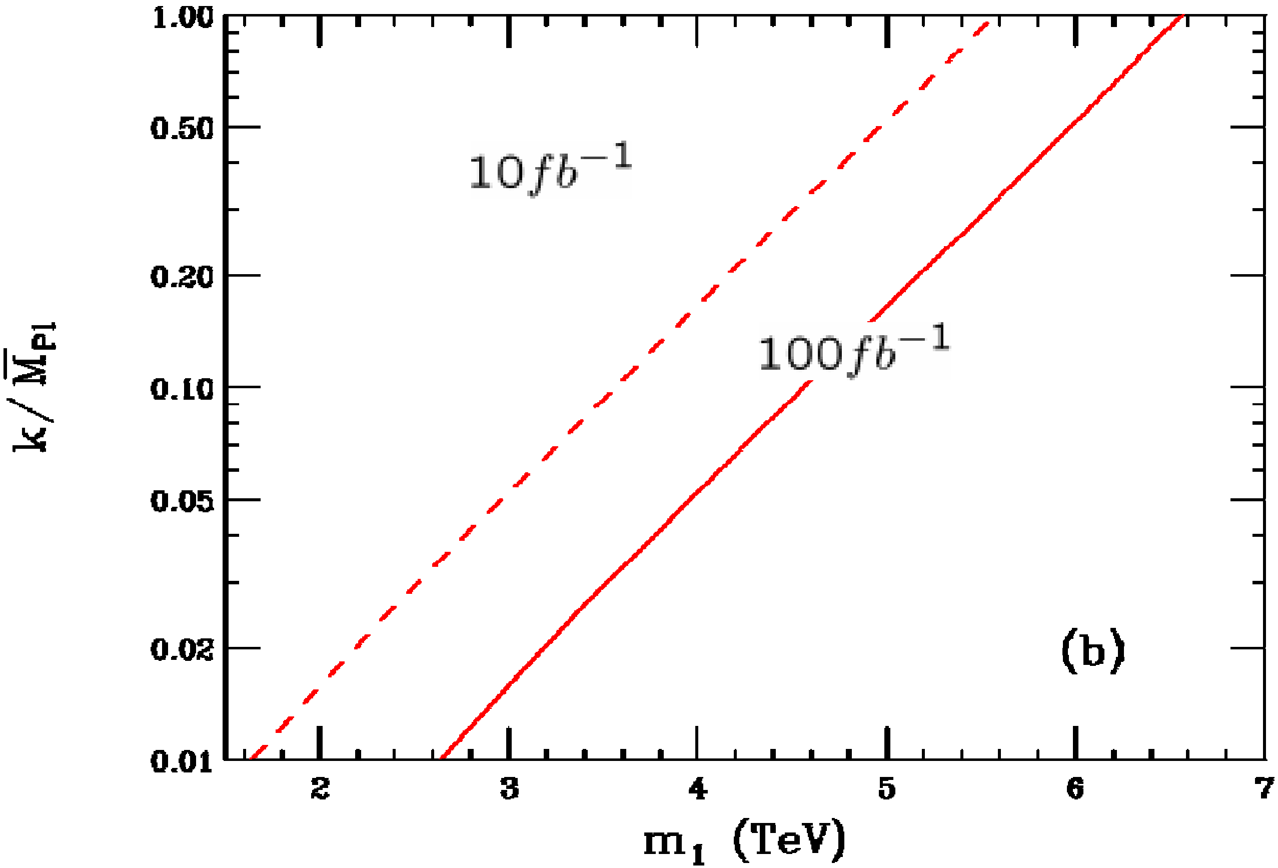
(dotted line) $30 fb^{-1}$

@ LHC

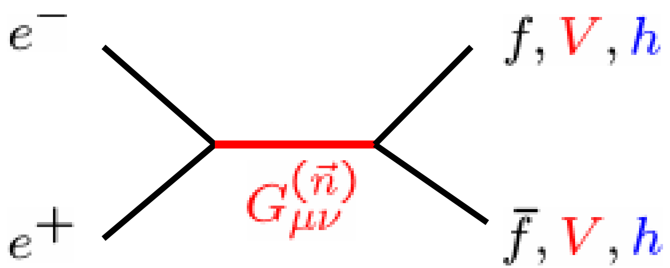
Davoudiasl et al.,

PRL 84 (2000) 2080

95% CL exclusion region



Virtual KK graviton mediated process



Models with Large Extra Dimensions

$$\sum_{\vec{n}} \frac{1}{s - \left(m_{KK}^{(\vec{n})}\right)^2} \to \infty \quad (\text{for } \delta \geq 2)$$

Need regularization

Naïve: Cut Off by $m_{KK}^{MAX} \sim M_{4+\delta}$

$$\boxed{\frac{4\pi\lambda}{M_S{}^4}} = -\frac{1}{M_P^2} \sum_{\vec{n}} \frac{1}{s - \left(m_{KK}^{(\vec{n})}\right)^2}; \qquad \lambda = \pm 1$$

$$\mathcal{M} = \frac{4\pi\lambda}{M_S^4} T_{\mu\nu}(p_1, p_2) T^{\mu\nu}(p_3, p_4)$$

Five dimensional case

$$\sum_{n=1}^{\infty} \frac{1}{s - \left(m_{KK}^{(n)}\right)^2}$$ → **converge!**

For $\sqrt{s} << m_{KK}^{(1)}$ → $$\sum_{n=1}^{\infty} \frac{1}{s - \left(m_{KK}^{(n)}\right)^2} \sim -\frac{e^{2kr_c\pi}}{k^2} \times \sum_{n=1}^{\infty} \frac{1}{x_n^2}$$

Correspondence: $$\frac{e^{2kr_c\pi}}{k^2} \sum_{n=1}^{\infty} \frac{1}{x_n^2} \Longleftrightarrow \frac{4\pi\lambda}{M_S^4}$$

→ **analysis in LED scenario can be interpreted**

Note: no brane fluctuation can be introduced in the orbifold case

Alternative to compactification

Randall-Sundrum, PRL 83 (1999) 4690

In order for models with extra dimensions to be realistic,

compactifications of extra dimensions is necessary

$M_4^2 = M_{4+\delta}^{2+\delta} R^\delta$ → **finite** M_4 **and** $M_{4+\delta}$

→ **finite *R***

But, general requirement is finiteness of V_δ

$M_4^2 = M_{4+\delta}^{2+\delta} V\delta$ → **finite** M_4 **and** $M_{4+\delta}$

→ **finite** V_δ

In <u>warped</u> extra dimension scenario

$$V_5 = 2r_c \int_0^{\pi} d\phi \, e^{-2kr_c|y|} = \frac{1}{k}\left(1 - e^{-2kr_c\pi}\right)$$

$$\to M_4^2 = \frac{M^3}{k}\left(1 - e^{-2kr_c\pi}\right)$$

Effective volume is finite even if $r_c \to \infty$

implies → **<u>Alternative compactification scenario</u>**

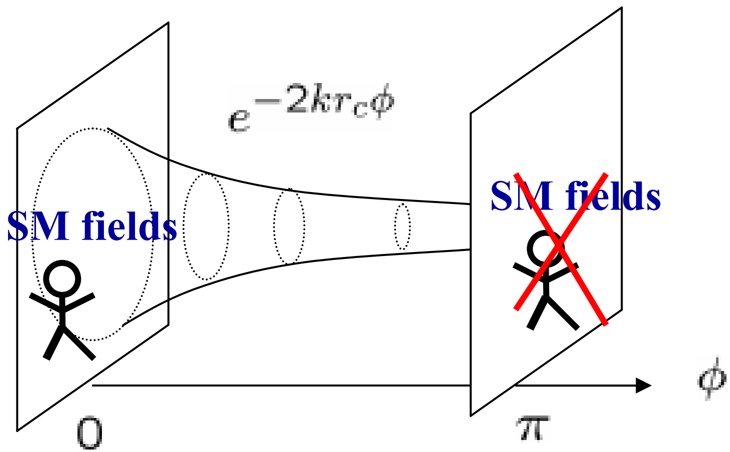
<u>What happen?</u>

$$r_c \to \infty \qquad \begin{cases} m_{KK}^{(1)} \to 0 \\ m_{KK}^{(n+1)}/m_{KK}^{(n)} \to 1 \end{cases}$$

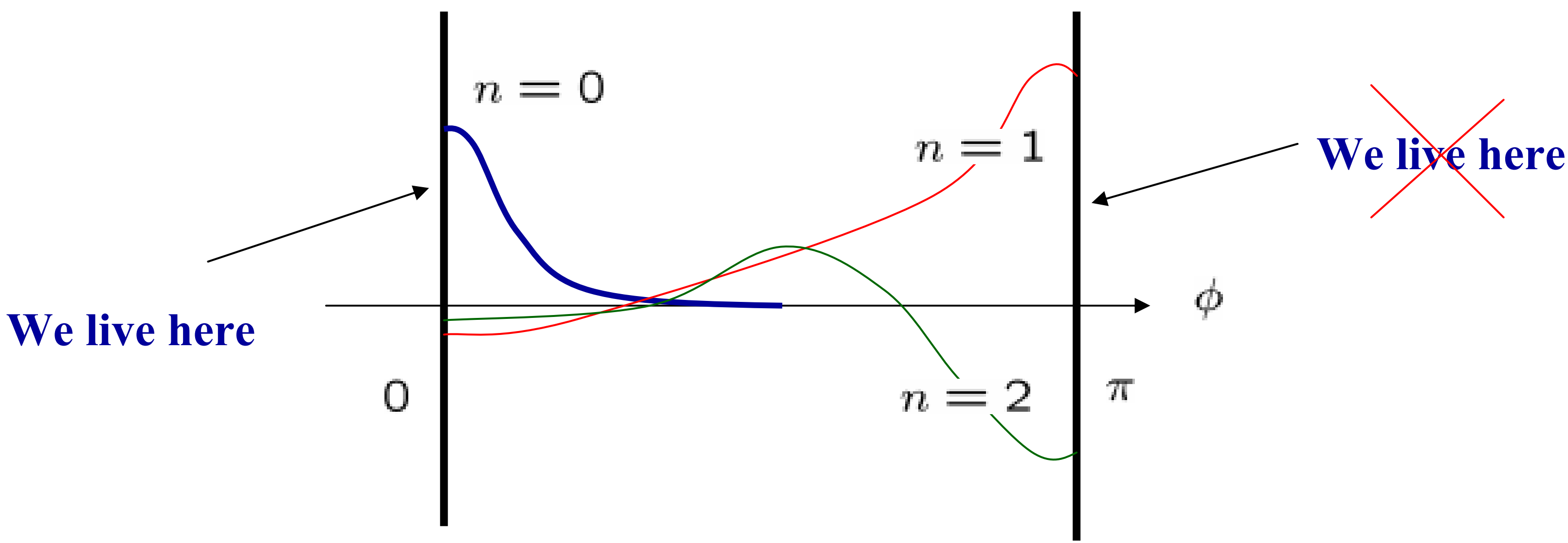
Continuous KK mode spectrum from 0 to infinity

→ Realistic model?

Changing the setup



KK mode configuration



$$\mathcal{L}_{int} = -\frac{1}{\bar{M}_4} G^{(0)}_{\mu\nu} T^{\mu\nu} - \frac{1}{\bar{M}_4 e^{+\kappa r_c \pi}} \sum_{n=1} G^{(n)}_{\mu\nu} T^{\mu\nu}$$

← **Strong: large overlap**

← **weak: small overlap**

Newton potential for continuum KK mode

$$V(r) \sim G_N \frac{m_1 m_2}{r} + \int_0^\infty \frac{m\,dm}{k^2} G_N \frac{m_1 m_2 e^{-mr}}{r} = G_N \frac{m_1 m_2}{r}\left(1 + \frac{1}{r^2 k^2}\right)$$

Gravity precision measurement → ***r k >1*** **for** ***r = 0.1 mm***

→ $k > 10^{-4}$ eV

$M \sim \left(\bar{M}_4^2 k\right)^{\frac{1}{3}}$ → $M > 10^{7.7}$ GeV