Lectures on Brane World Scenarios II

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Large Extra Dimension Scenario

Phenomenology

- $M_{4+\delta} \sim O(1\,\mathrm{TeV})$
- many graviton Kaluza-Klein modes

$$G_{\mu\nu}(x^\mu, y^1, y^2, .., y^n) = \sum_n g_{\mu\nu}^{(\vec{n})}(x^\mu)\,\chi^{(\vec{n})}(\vec{y})$$

$\chi^{(n)} \propto e^{i\frac{\vec{n}\cdot\vec{y}}{r}}$, $\left(m_{KK}^{(\vec{n})}\right)^2 = \frac{|\vec{n}|^2}{r^2}$ **If 6 dim. ➔** $\frac{1}{r} \sim 10^{-4}\mathrm{eV}$

Phenomenology of Extra-dimension Scenario

= Phenomenology of graviton Kaluza-Klein modes

Construction of effective action in 4D

1. 4D Lagrangian

4+ δ dim. Graviton → 4D graviton in 4D

KK gravitons

KK gravi-scalars

KK gravi-vectors

2. Feynman rules

Characteristic features: infinite tower of KK gravitons

universal couplings

4D reduction of 4+ δ dim. Einstein's Eqs.

$$\Box_{4+\delta} G_{MN} = -\frac{T_{MN}}{(M_{4+\delta})^{2+\delta}}$$

$$G_{MN} = \left[\begin{array}{c|c} G_{\mu\nu}^{(n)} & V_{\mu j}^{(n)} \\ \hline V_{\mu i}^{(n)} & H^{(n)}\delta_{ij} + S_{ij}^{(n)} \end{array}\right]$$

KK graviton $$(\Box + \hat{n}) G_{\mu\nu}^{(n)} = \frac{1}{\bar{M}_P}\left[-T_{\mu\nu} + \frac{1}{3}\left(\frac{\partial_\mu \partial_\nu}{\hat{n}^2}\right) T_\lambda^\lambda\right]$$

KK gravi-scalar $$(\Box + \hat{n}) H^{(n)} = \frac{\sqrt{\frac{3(\delta-1)}{\delta+2}}}{\bar{M}_P} T_\mu^\mu$$

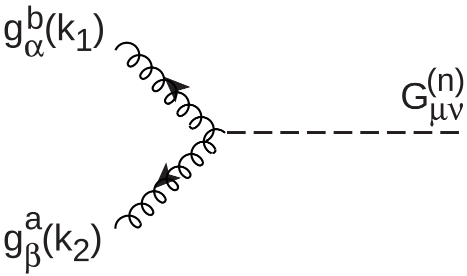
Vector $$(\Box + \hat{n}) V_{\mu j}^{(n)} = 0$$

Scalar $$(\Box + \hat{n}) S_{ij}^{(n)} = 0$$

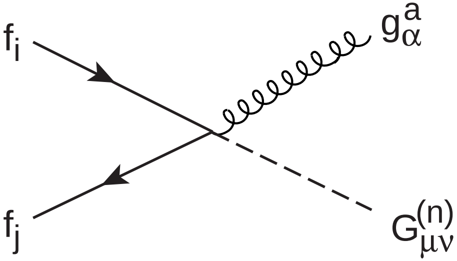
Feynman rules

$$T^{\mu\nu} = 2\frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} - g^{\mu\nu}\mathcal{L}$$

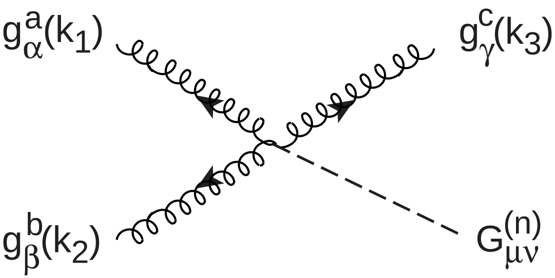
ex) QCD + KK graviton



$$-\frac{1}{\bar{M}_P}\left[\ \frac{i}{2}\eta_{\mu\nu}(k_{1\beta}k_{2\alpha} - (k_1k_2)\eta_{\alpha\beta}) + \eta_{\alpha\beta}k_{1\mu}k_{2\nu} + \eta_{\mu\alpha}(\eta_{\nu\beta}(k_1k_2) - k_{1\beta}k_{2\nu}) - \eta_{\mu\beta}k_{1\nu}k_{2\alpha}\right]$$



$$-\frac{i}{2\bar{M}_P}g_s t^a_{ji}[\gamma_\mu\eta_{\nu\alpha} + (\mu \leftrightarrow \nu)]$$



$$\frac{g_s}{\bar{M}_P} f^{abc}\Big[Y(k_1)_{\mu\nu\alpha\beta\gamma} + Y(k_2)_{\mu\nu\beta\gamma\alpha} + Y(k_3)_{\mu\nu\gamma\alpha\beta} + Y(k_1)_{\nu\mu\alpha\beta\gamma} + Y(k_2)_{\nu\mu\beta\gamma\alpha} + Y(k_3)_{\nu\mu\gamma\alpha\beta}\Big]$$

$$Y(k)_{\mu\nu\alpha\beta\gamma} = k_\mu(\eta_{\nu\beta}\eta_{\alpha\gamma} - \eta_{\nu\gamma}\eta_{\alpha\beta}) + [k_\beta(\eta_{\mu\alpha}\eta_{\nu\gamma} - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\gamma}) + (\beta \leftrightarrow \gamma)]$$

Detection of Extra-dimension @ colliders

→ **detection of KK graviton**

direct → KK graviton emission

indirect →KK graviton mediated process

First detection of spin 2 particle !

KK graviton emission process @ hadron colliders

Emitted KK graviton → non-interacting & stable particle

→ missing energy event

For emissions of KK graviton with mass (m)

$$\frac{d\sigma_m}{dt}(q\bar{q} \to gG) = \frac{\alpha_s}{36} \frac{1}{s\bar{M}_P^2} F_1(t/s, m^2/s)$$

$$\frac{d\sigma_m}{dt}(qg \to qG) = \frac{\alpha_s}{96} \frac{1}{s\bar{M}_P^2} F_2(t/s, m^2/s)$$

$$\frac{d\sigma_m}{dt}(gg \to gG) = \frac{3\alpha_s}{16} \frac{1}{s\bar{M}_P^2} F_3(t/s, m^2/s)$$

$$\begin{aligned}
F_1(x,y) &= \frac{1}{x(y-1-x)}[-4x(1+x)(1+2x+2x^2) \\
&+ y(1+6x+18x^2+16x^3) - 6y^2x(1+2x) + y^3(1+4x)] \\
F_2(x,y) &= -(y-1-x)F_1\left(\frac{x}{y-1-x}, \frac{y}{y-1-x}\right) \\
F_3(x,y) &= \frac{1}{x(y-1-x)} 1 + 2x + 3x^2 + 2x^3 + x^4 \\
&- 2y(1+x^3) + 3y^2(1+x^2) - 2y^3(1+x) + y^4
\end{aligned}$$

$$\sum_n \to \int dn \to R^\delta \int d^\delta m$$

$\bar{M}_P^2$

$\propto$

$$\frac{d\sigma}{dt} = S_{\delta-1} \left(\frac{\bar{M}_P^2}{M_{4+\delta}^{2+\delta}} \right) \int_0^{\sqrt{s}} m^{\delta-1} dm \times \left(\frac{d\sigma_m}{dt} \right)$$

$$S_{\delta-1} = \frac{2\pi^{\delta/2}}{\Gamma(\delta/2)}$$ **: surface of a unit-radius sphere in δ dimensions**

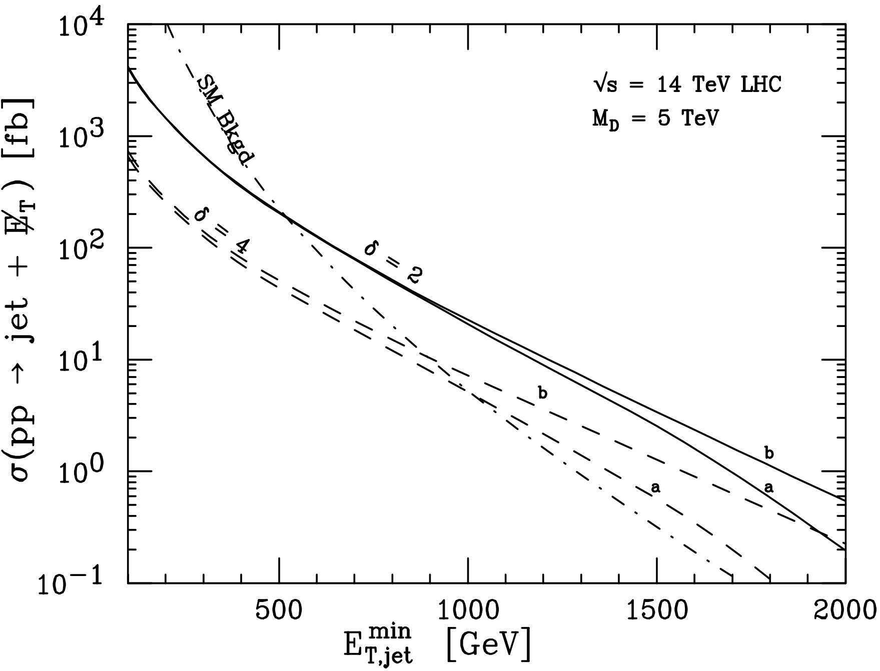
<u>4D Planck scale is cancelled out</u>

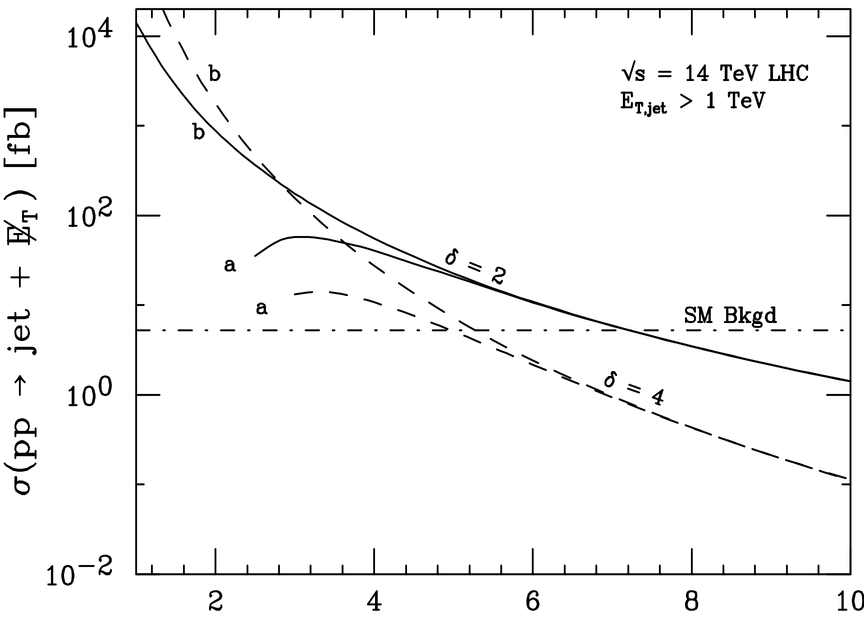
Cross sections depend on $\sigma \propto \left(\frac{\sqrt{s}}{M_{4+\delta}} \right)^{2+\delta}$

pp → jet + nothing @ LHC $\sqrt{s} = 14\,\text{TeV}$

SM background = jet + Z ; Z → two neutrinos

$M_{4+\delta} = 5\,\text{TeV}$ $E_{T\,\text{jet}} > 1\,\text{TeV}$





$M_{4+\delta}[\text{TeV}]$

a: integrating the cross section over $\hat{s} < M_{4+\delta}^2$

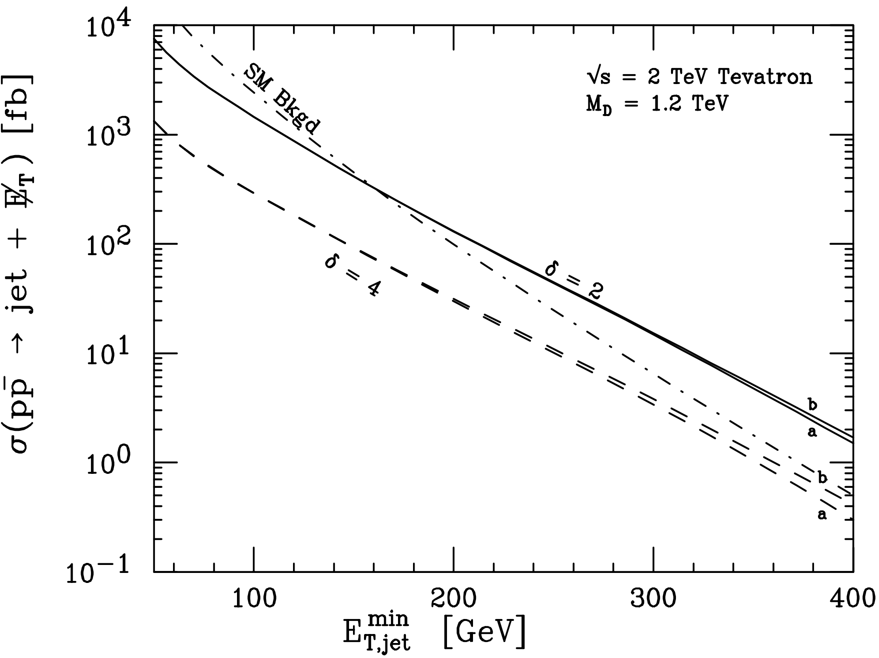
b: all

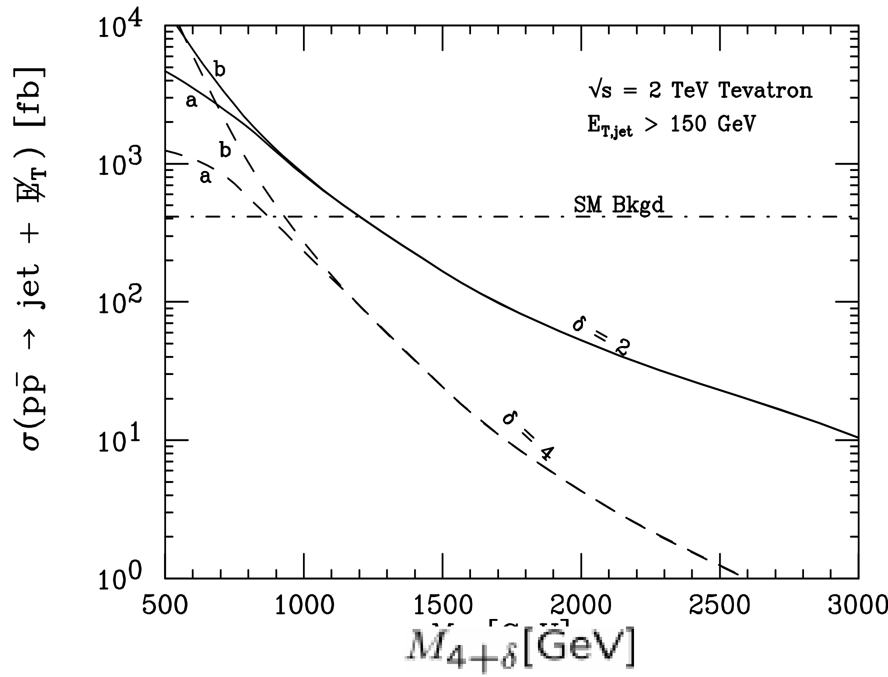
Giudice et al., NPB544 (1999) 3

p pbar → jet + nothing @ Tevatron $\sqrt{s} = 2\text{TeV}$

SM background = jet + Z ; Z → two neutrinos

$M_{4+\delta} = 1.2\text{TeV}$ $E_{T\,\text{jet}} > 150\text{GeV}$





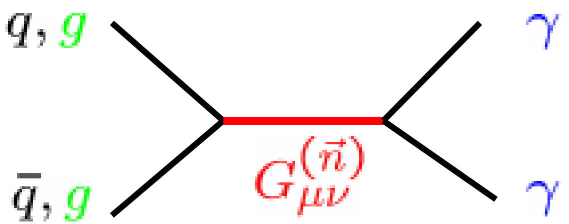
a: integrating the cross section over $\hat{s} < M_{4+\delta}^2$

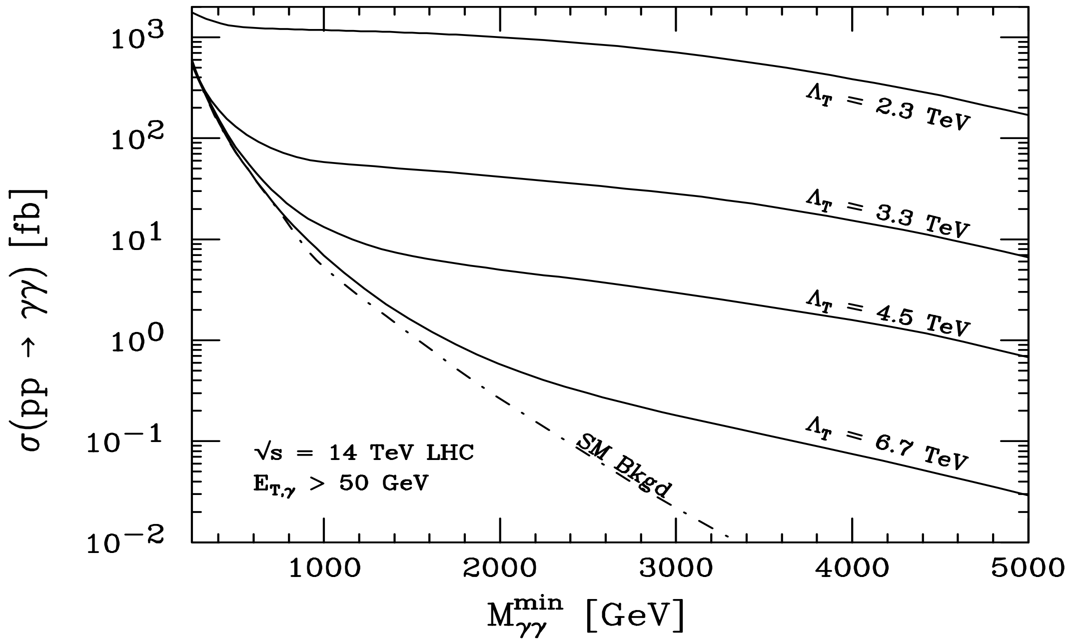
b: all

Giudice et al., NPB544 (1999) 3

KK graviton mediated processes @ LHC

EX) pp → γ γ





$\sqrt{s} = 14\text{TeV}$

$E_{T,\gamma} > 50\text{GeV}$

Giudice et al., NPB544 (1999) 3

$$\mathcal{M} = \frac{4\pi}{\Lambda_T^4} T_{\mu\nu}(p_1, p_2) T^{\mu\nu}(p_3, p_4)$$

Constraints from collider experiments

Lower bounds on $M_{4+\delta}$
Upper bounds on R
} from missing-energy exp.

Example) LEPII $e^+e^- \to \gamma + \text{nothing}$

Tevatron $p\bar{p} \to \gamma + \text{nothing}$

Mirabelli, Perelstein & Peskin,

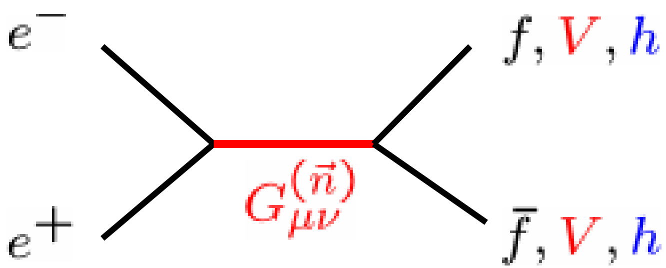
Phys. Rev. Lett. 82 (1999) 2236

	$R(\text{cm})/M_{4+\delta}(\text{GeV})$		
LEPII	$4.8 \times 10^{-2}/1200$	$1.9 \times 10^{-9}/730$	$6.8 \times 10^{-12}/530$
Tevatrn	$5.5 \times 10^{-2}/1140$	$1.4 \times 10^{-9}/860$	$4.1 \times 10^{-12}/780$
	$\delta = 2$	$\delta = 4$	$\delta = 6$

Updates by Cullen et at.,

Phys. Rev. D62 (2000) 055012

Virtual KK graviton mediated processes in a soft brane scenario



$$\sum_{\vec{n}} \frac{1}{s - \left(m_{KK}^{(\vec{n})}\right)^2} \to \infty \quad (\text{for } \delta \geq 2)$$

Need regularization!

Naïve: Cut Off $m_{KK}^{MAX} \sim M_{4+\delta}$

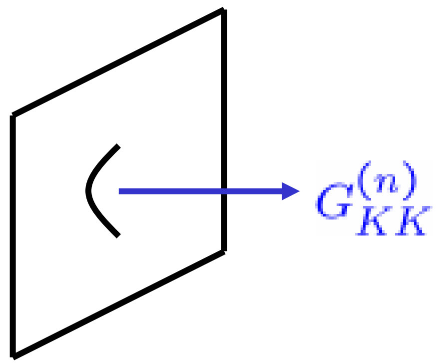
$$-\frac{8\pi}{M_P^2} \sum_{\vec{n}} \frac{1}{s - \left(m_{KK}^{(\vec{n})}\right)^2} \to \frac{4\pi\lambda}{M_S{}^4} \qquad \lambda = \pm 1$$

More physically justified regularization?

Introduction of finite brane tension (= brane with finite width)

Brane fluctuation

Bando et al., PRL 83 (1999) 3601



Recoil effect

hard breaking of momentum conservation

→ KK mode universal coupling

Restoration of momentum conservation for higher KK modes

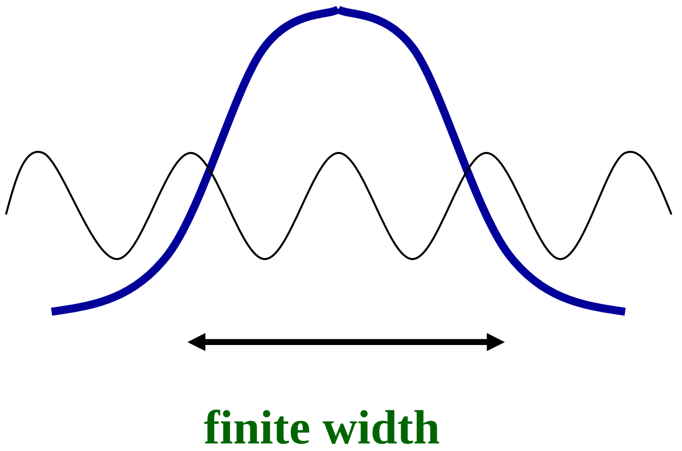
$$g \longrightarrow g \times e^{-\frac{m_{KK}^2}{2\Delta^2}}$$

Δ ~ brane tension

More intuitive picture: brane with finite width

Hisano& N. Okada,

Phys. Rev. D61 (2000) 106003



Toy model:

brane fermion + bulk scalar

ψ $\phi_{KK}^{(n)}$

Simple example: effective Yukawa coupling

$$Y_{eff} = Y_{univ} \int dy \, \psi(y) \, \phi_{KK}^{(n)} \, \psi(y)$$

$$\psi(y) \sim e^{-y^2/w^2} \qquad \rightarrow Y_{eff} \sim Y_{univ} \times \boxed{e^{-m_{KK}^2 w^2}}$$

$$\phi_{kk} \sim \cos(m_{KK} y)$$

Restoration of momentum conservation for higher KK modes

Effect on KK graviton emission processes

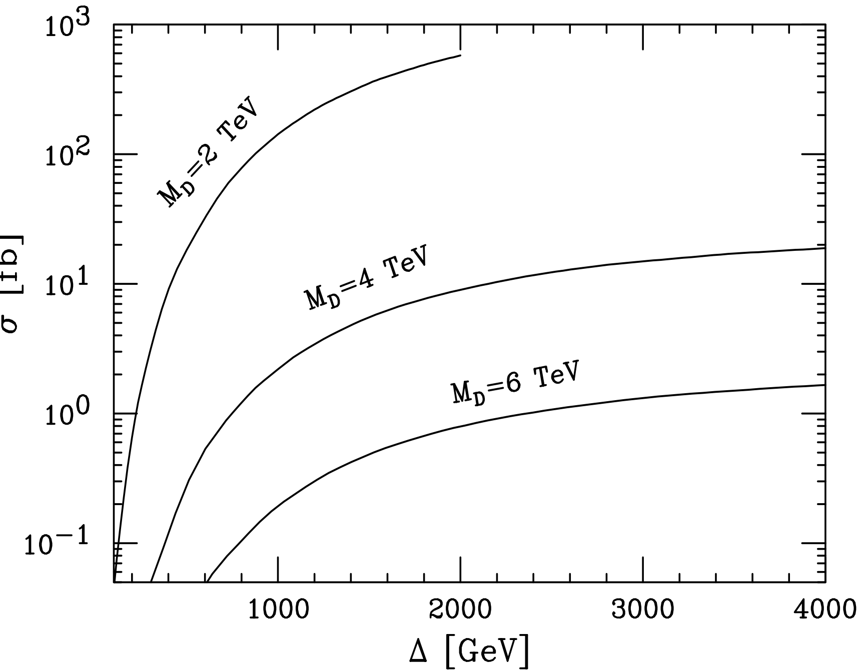
Murayama & Wells,

PRD 65 (2002) 056011

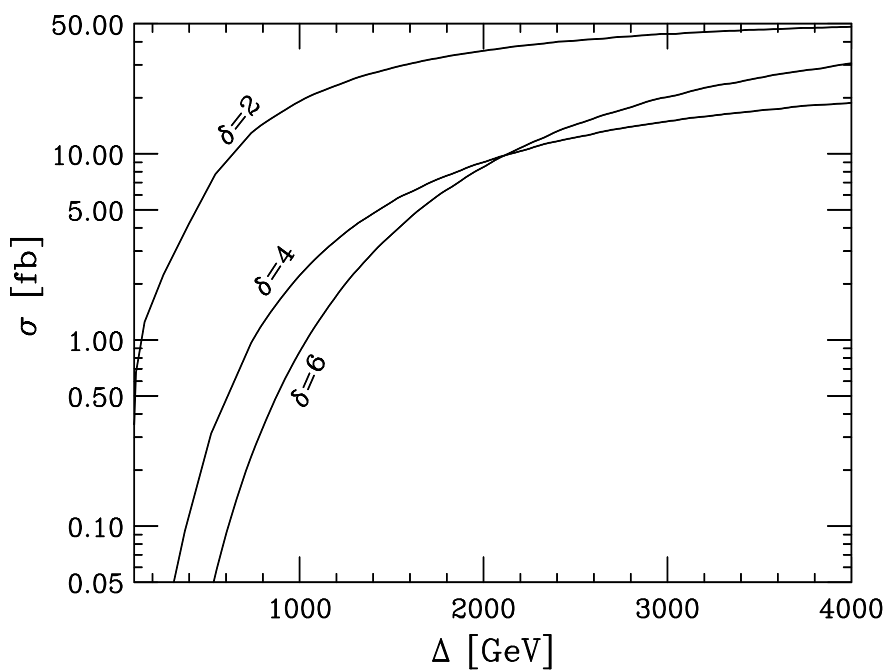
pp → jet + missing @ LHC

$$\mathcal{L} \sim -\frac{1}{\bar{M}_P} G^{(n)}_{\mu\nu} T^{\mu\nu} \times e^{-\frac{m^2_{(n)}}{2\Delta^2}}$$

$\delta = 4$



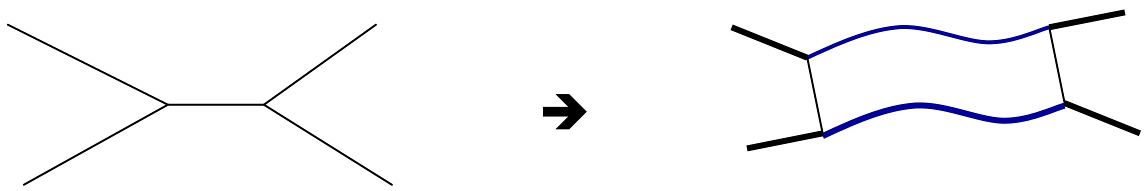
$E_{T,jet} > 1\text{TeV}$



$M_{4+\delta} = 4\text{TeV}$

TeV scale string theory

Evidence of TeV scale string theory? $M_{4+n} \sim M_{st} \sim 1\,\mathrm{TeV}$



String amplitude

(Perturbative) String Theory

Many states: string Regge excitations

$$M_n = \sqrt{n} M_{st} \; (n = 0, 1, 2, \ldots)$$
$$J = 0, 1/2, 1, 3/2, 2, 5/2 \ldots\ldots$$

Phenomenology of TeV string theory

string Regge excitations: new contact terms $\sqrt{s} \leq M_{st}$

string resonances $\sqrt{s} \geq M_{st}$

Stringy correction to scattering amplitudes

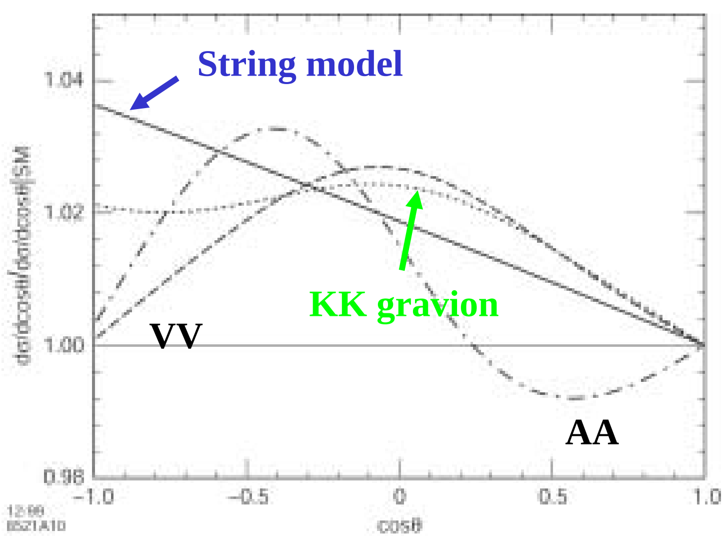
Cullen, Perelstein & Peskin,
PRD 62 (2000) 0550212

$$\frac{d\sigma}{d\cos\theta} = \left(\frac{d\sigma}{d\cos\theta}\right)_{SM} |S(s,t)|^2$$

String form factor: $$S(s,t) = \frac{\Gamma(1 - s/M_{st}^2)\Gamma(1 - t/M_{st}^2)}{\Gamma(1 - s/M_{st}^2 - t/M_{st}^2)}$$

$\sqrt{s} \le M_{st} \rightarrow$ **Contact interaction mediated SR excitations**

Example: Bhabha scattering @ 1TeV LC



$M_{st} = 3.1\text{TeV}$

$\Lambda_T = 6.2\text{TeV}$

VV contact with $\Lambda = 88\text{TeV}$

AA contact with $\Lambda = 62\text{TeV}$

$$\mathcal{L}_{KK} = i\frac{4}{\Lambda_T} T^{\mu\nu}T_{\mu\nu}$$

$$\mathcal{L}_{VV} = -\frac{4\pi}{2\Lambda^2}\overline{e_L}\gamma^\mu e_L \overline{e_L}\gamma_\mu e_L + \cdots$$

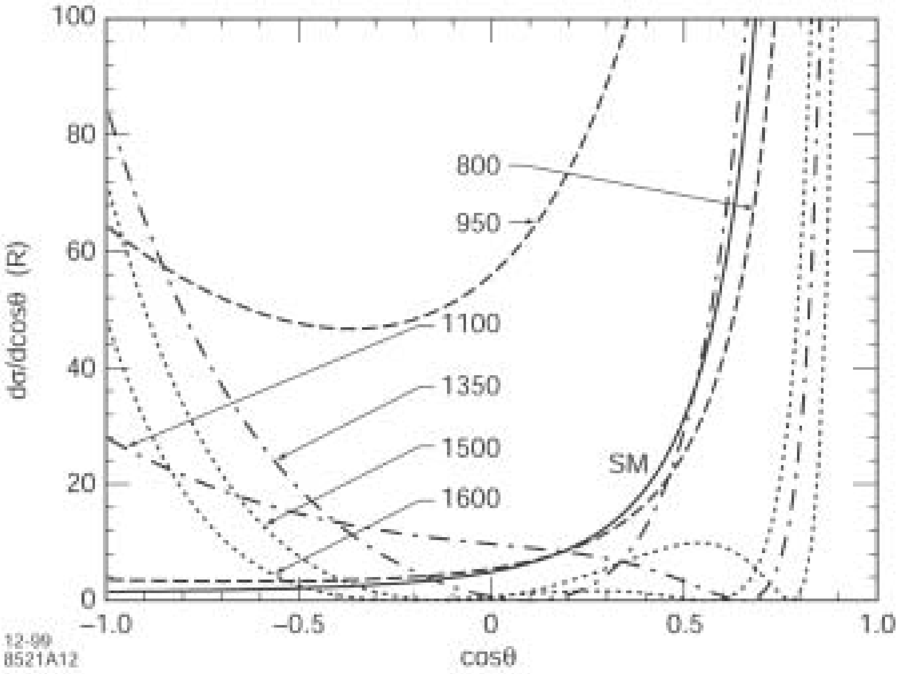
$s = M_n^2 = nM_{st}^2 \rightarrow$ **String resonance**

$nM_{st}^2 < s < (n+1)M_{st}^2$ **behavior of string form factor**

Bhabha scattering @ 1TeV LC

Cullen, Perelstein & Peskin, PRD 62 (2000) 0550212

In units of $\frac{4\pi\alpha^2}{3s}$



$M_{st} = 1\text{TeV}$

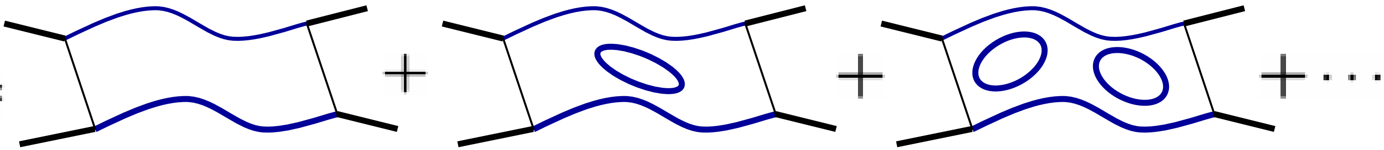
$s < M_{st}^2$

$M_{st}^2 < s < 2M_{st}^2$

$2M_{st}^2 < s < 3M_{st}^2$

Stringy effect: hard scattering limit

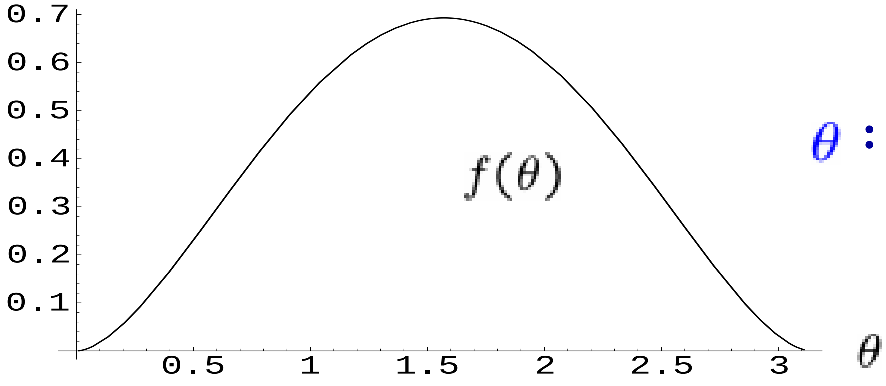
Mende & Ooguri, NPB 339 (1990) 641

$S(s,t,u) =$  $+ \cdots$

$$\sim e^{-\sqrt{\frac{s f(\theta)}{M_{st}^2}}}$$

$$f(\theta) = -\lambda \ln(\lambda) - (1-\lambda)\ln(1-\lambda)$$

$$\lambda = \sin^2(\theta/2)$$

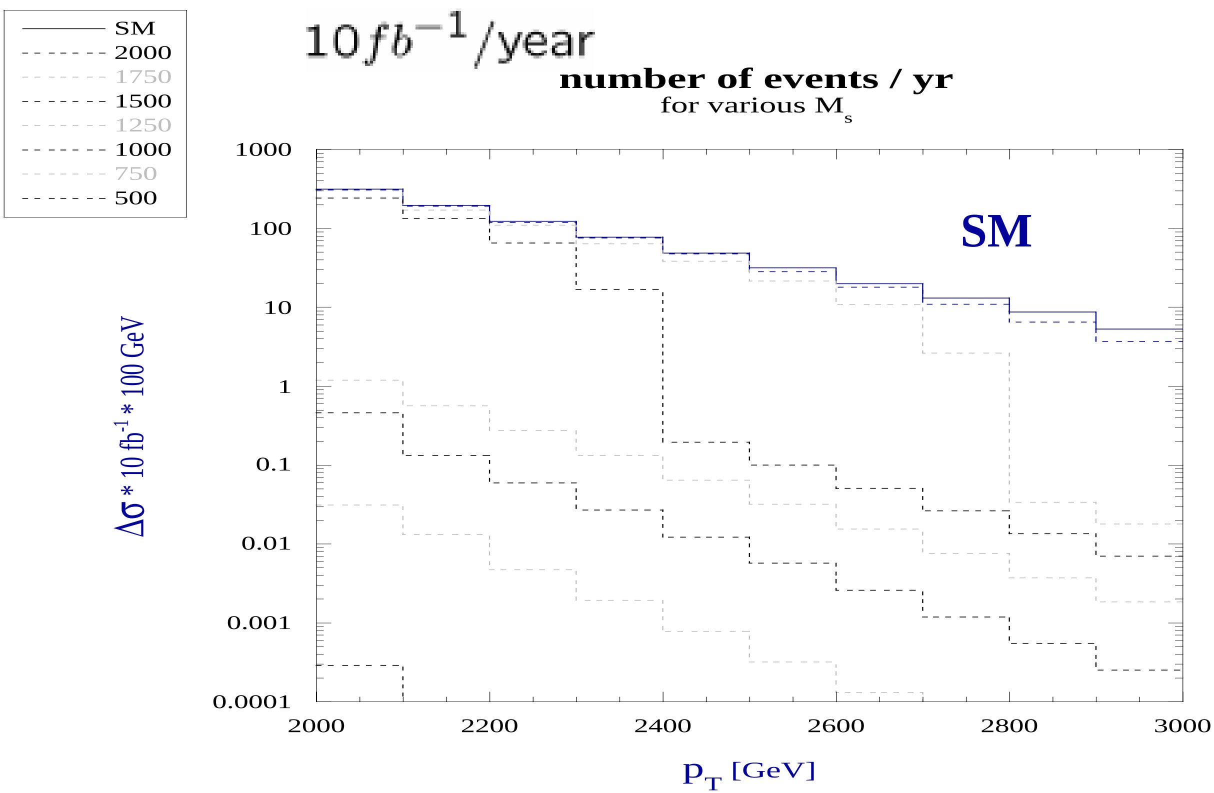


θ : scattering angle

Stringy behavior @ LHC, if M_{st} =O(1 TeV) $M_{st} \ll \sqrt{s_{LHC}}$

Large angle scattering → $\sigma \sim \sigma_{SM} \times e^{-\frac{s}{M_{st}^2}}$

Dramatic reduction in QCD jet production @ LHC



K. Oda & N. Okada,

PRD 66 (2002) 095005

Non-perturbative process?

$\sqrt{s} > M_{4+\delta}$ **TeV string, strong gravity**

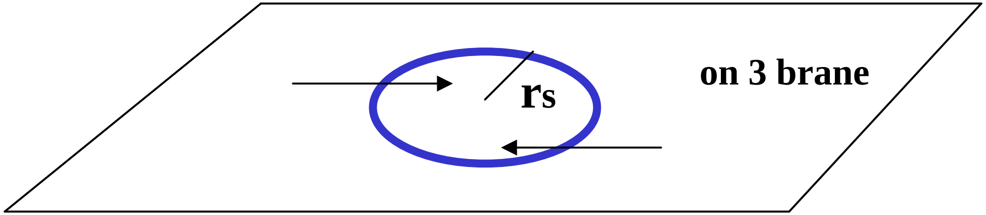
4+ δ dim. Black Hole formation?

Giddings-Thormas, PRD65 (2002) 056010
Dimopoulos-Lansberg, PRL87 (2001) 161602

4+ δ dim. BH solution

$$ds^2 = -R(r)dt^2 + R(r)^{-1}dr^2 + r^2 d\Omega^2_{\delta+2}$$

$$\begin{cases} R(r) = 1 - \left(\frac{r_s}{r}\right)^{\delta+1} \\ r_s = \frac{1}{\sqrt{\pi}M_{4+\delta}}\left(\frac{8\Gamma\left(\frac{\delta+2}{2}\right)}{\delta+2}\right)^{\frac{1}{\delta+1}}\left(\frac{M_{BH}}{M_{4+\delta}}\right)^{\frac{1}{\delta+1}} \end{cases}$$



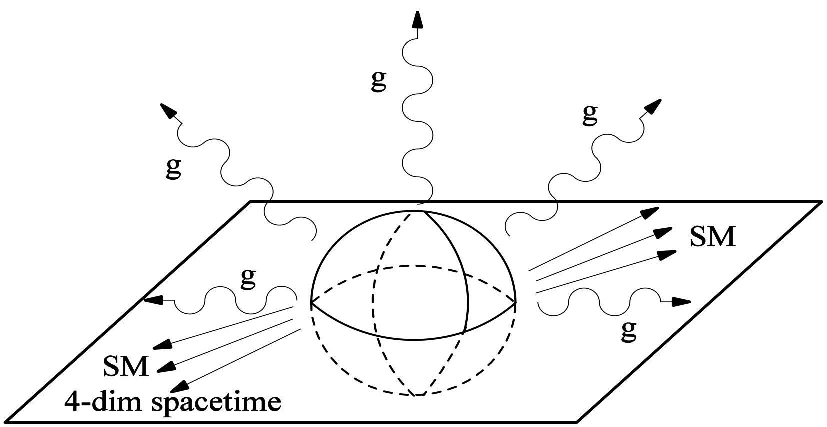
$$\sigma(M_{BH}) \sim \pi r_s^2$$

BH production @ LHC

$$\sigma_{pp\to BH}(x_m,\delta) \sim \sum_{ij}\int_{x_m}^{1} dx \int_{x}^{1} \frac{dy}{y} f_i(y,Q) f_j(x/y,Q) \times \sigma_{ij\to BH}(xs;\delta)$$

$$x_m = \frac{M_{BH}^{min}}{\sqrt{s}} \sim \frac{M_{4+\delta}}{\sqrt{s}}$$

BH decay → Evaporate by emitting Hawking Radiation



- particle blind
- E ~ T_H for each quanta
- emission rate → # of d.o.f

$$T_H = \frac{\delta+1}{4\pi r_s}$$

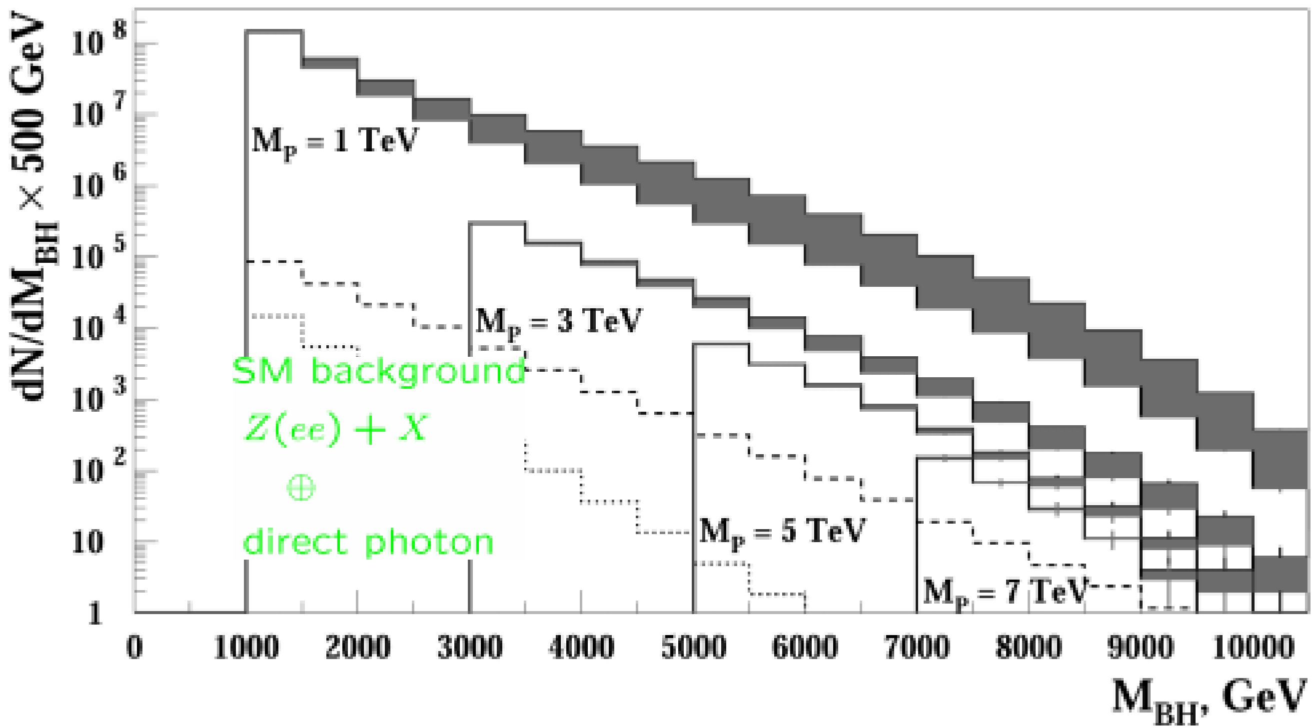
BH →
- quarks & gluons ~ 75 %
- charged leptons ~ 10 %
- neutrinos ~ 5 %
- γ, W, Z ~ 2 % each

of produced BH @ LHC ($100 fb^{-1}$ integrated luminosity)

electron/photon decay channels

Dimopoulos-Landsberg,

PRL 87 (2001) 161602



$\delta = 1 \sim 7$

LHC can be "BH Factory"

if $M_{BH}^{min} \sim \mathcal{O}(1\,\text{TeV})$

Far future …..

BH property has been known

Collider will be designed in order to

NOT produce BH