Lectures on Brane World Scenarios

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Fine-tuning (naturalness) problem in the Standard Model

Standard model: gauge theory $SU(3)_c \times SU(2)_L \times U(1)_Y$

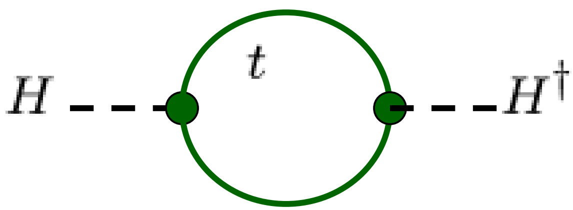
quarks & leptons & Higgs

Non-zero Higgs VEV determines the vacuum

$$SU(2)_L \times U(1)_Y \to U(1)_{em}$$

W, Z bosons get masses of order 100 GeV

Serious problem: SM vacuum is not stable under quantum corrections

$\Delta m_H^2 =$ 

$\sim -\Lambda_{new}^2$

$$\langle H \rangle \sim m_H^2 = (m_H^{tree})^2 + \Delta m_H^2$$
$$\sim (m_H^{tree})^2 - \Lambda_{new}^2$$

If new physics scale >> electroweak scale

For example $\Lambda_{new} = M_{Planck} \sim 10^{19}\text{GeV}$

$$\begin{aligned} \langle H \rangle &= (m_H^{tree})^2 - \Lambda_{new}^2 \\ &\rightarrow \mathcal{O}(M_W^2) = \mathcal{O}((10^{19}\text{GeV})^2) - \mathcal{O}((10^{19}\text{GeV})^2) \end{aligned}$$

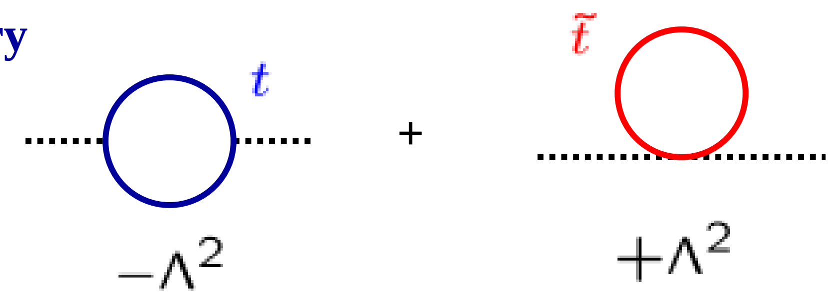
Fine-tuning is needed!

In order to solve this problem

I) New Physics without quadratic divergence in quantum corrections

→ Supersymmetry

$\Delta m_H^2 =$



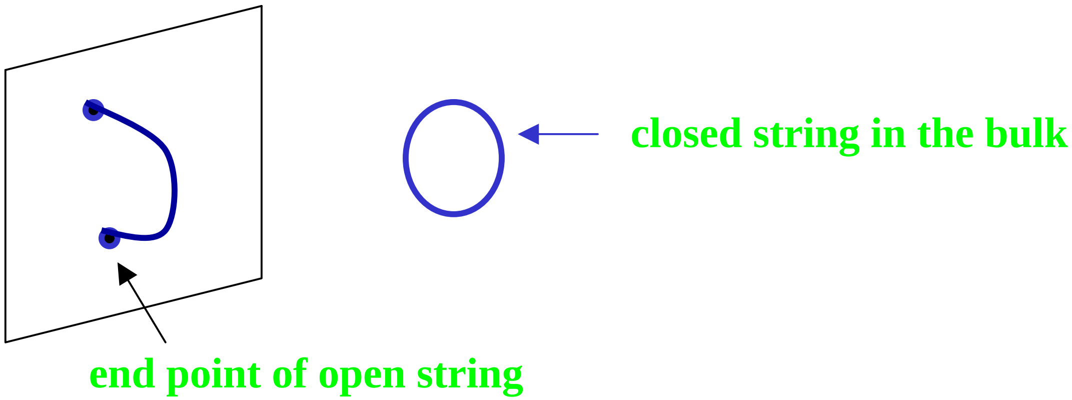
II) New Physics scale is close to the electorweak scale $\Lambda_{new} \sim \mathcal{O}(1\text{TeV})$

→ Brane World Scenario

Brane World Scenario

String Theory : classical solution **(Polchinski '95)**

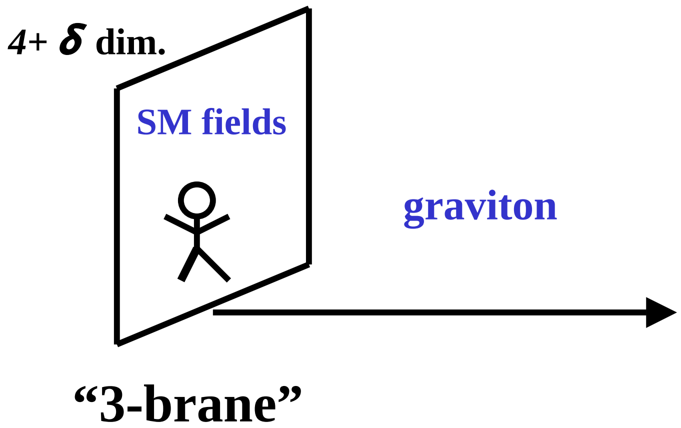
D-brane: D+1 dim. object embedded in higher dim. space time



Open string → fermion, boson, gauge on D-brane

Closed string → graviton in the bulk

Phenomenological model



δ (compact) extra-dimensions

We are confined on “3-brane”
Graviton lives in the bulk

Beyond the standard model → Brane World Scenario
4+ δ dimensions

New property
“geometry”

Typical Scenario: **Large (flat) Extra Dimensions** (Arkani-Hamed-Dimopoulos-Dvali, ’98)
Warped (small) Extra Dimensions (Randall-Sundrum, ’99)

Large Extra Dimension Scenario

Solution to fine-tuning problem without SUSY

Low scale gravity model in 4+ δ dimensions $\Lambda_{new} = M_{4+\delta} \sim \mathcal{O}(1\text{TeV})$

$$S_{4+\delta} = \left(M_{4+\delta}\right)^{2+\delta} \int d^4x d^\delta y \sqrt{-g_{4+\delta}} R_{4+\delta}$$
$$= M_{Planck}^2 \int d^4x \sqrt{-g_4} R_4$$

$$M_{Planck}^2 = M_{4+\delta}^{2+\delta} V_\delta$$

V δ **: volume of δ extra-dim.**

$V_\delta = (2\pi R)^\delta$ (compactified on T^δ)

For $M_{4+\delta} \sim 1\text{TeV}$

δ	r	
1	10^{13} cm	excluded
2	10^{-1} mm	allowed
3	10^{-6} mm	allowed

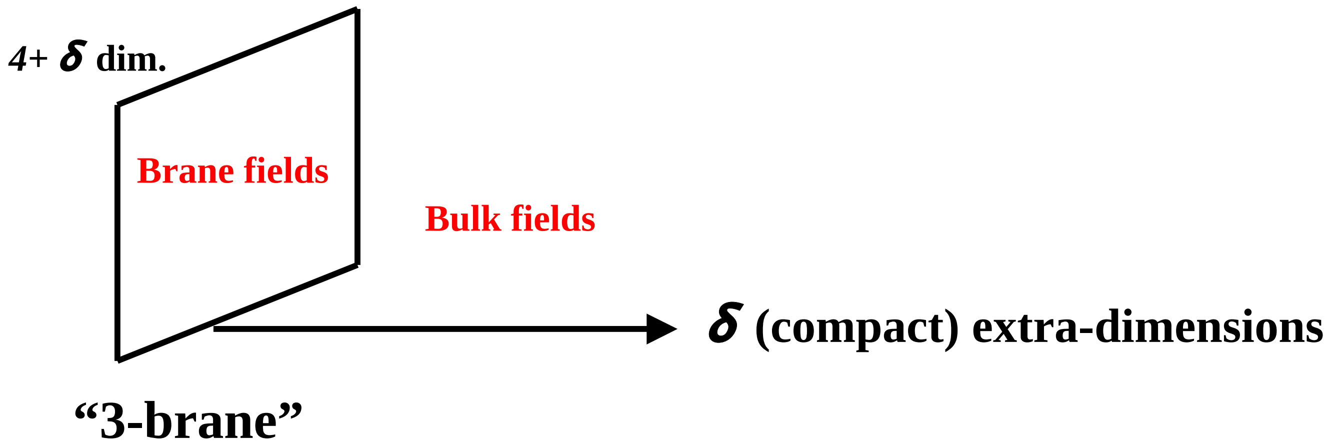
← **r < 218 μ*m***

Hoyle et al., PRL 86 (2001) 1418

General picture of Brane World Scenario

Setup: 4+ δ dimensional model ← compact δ extra-dimensions

some 3-branes



4 dimensional description

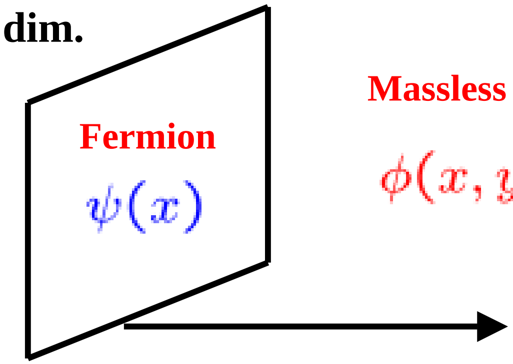
brane fields

infinite tower of Kaluza-Klein modes of bulk fields

characteristic couplings between brane and bulk fields

Toy model

4+ $\boldsymbol{\delta}$ **dim.**



Massless Bulk boson

$\phi(x, y_1, y_2, .., y_\delta)$

"3-brane"

$\boldsymbol{\delta}$ **(compact) extra-dimensions on the manifold** T^δ $\leftarrow$ $\boldsymbol{\delta}$ **dimensional box**

$V_\delta = \Pi_{i=1}^{\delta}(2\pi R_i)$

$$
\begin{aligned}
\mathcal{L}_{brane} &= \bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) \\
&- g\bar{\psi}(x)\psi(x)\phi(x, y_1 = 0, y_2 = 0, .., y_\delta = 0)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{bulk} &= (\partial_M \phi)^\dagger \left(\partial^M \phi\right) \\
&= (\partial_\mu \phi)^\dagger (\partial^\mu \phi) + \sum_{i=1}^{\delta} (\partial_{y_i}\phi)^\dagger (\partial^{y_i}\phi)
\end{aligned}
$$

Equation of motion

$$\partial_M \partial^M \phi = (\partial_\mu \partial^\mu - \sum_{i=1}^{\delta} \partial_{y_i}^2)\phi = 0$$

Plane wave: $\phi = \tilde{\phi}(x) e^{ik_1 y_1} e^{ik_2 y_2} ... e^{ik_\delta y_\delta}$

Compact δ extra-dimensions on T^δ

→ $\phi(x, y_1, y_2, .., y_\delta) = \phi(x, y_1 + 2\pi R_1, .., y_\delta + 2\pi R_\delta)$

→ $k_i = \frac{n_i}{R_i}$; $n_i = 0, \pm 1, \pm 2, ..$

$$\partial_M \partial^M \phi \to \left(\partial_\mu \partial^\mu + \sum_{i=1}^{\delta} \left(\frac{n_i}{R_i} \right)^2 \right) \tilde{\phi}(x) = 0$$

Kaluza-Klein modes: $m^2_{n_1, n_2, .., n_\delta} = \sum_{i=1}^{\delta} \left(\frac{n_i}{R_i} \right)^2$

Example: **5(=4+1) dim. model** **(compact 5-th dim. on S^1)**

$$\mathcal{L}_{brane} = \bar{\psi}(x) i\gamma^{\mu}\partial_{\mu}\psi(x) - g\bar{\psi}(x)\psi(x)\phi(x, y=0)$$

$$\mathcal{L}_{bulk} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) - \frac{1}{2}(\partial_{y}\phi)^2$$

KK mode expansion:

$$\phi(x, y) = \phi_{odd}(x, y) + \phi_{even}(x, y)$$

$$\phi_{odd}(x, y) = \sum_{n=1} \phi_{odd}^{(n)}(x) \sin\left(\frac{n}{R} y\right)$$

$$\phi_{even}(x, y) = \phi_{even}^{(0)}(x) + \sum_{n=1} \phi_{even}^{(n)}(x) \cos\left(\frac{n}{R} y\right)$$

4 dimensional description

$$\begin{aligned}
\mathcal{L} &= \int_0^{2\pi R} dy \mathcal{L}_{bulk} \\
&= \frac{1}{2}(\pi R) \sum_{n=1} \left[(\partial_\mu \phi(x)^{(n)}_{odd})^2 - \left(\frac{n}{R}\right)^2 (\phi(x)^{(n)}_{odd})^2 \right] \\
&+ \frac{1}{2}(2\pi R)(\partial_\mu \phi(x)^{(0)}_{even})^2 \\
&+ \frac{1}{2}(\pi R) \sum_{n=1} \left[(\partial_\mu \phi(x)^{(n)}_{even})^2 - \left(\frac{n}{R}\right)^2 (\phi(x)^{(n)}_{even})^2 \right]
\end{aligned}$$

$$\mathcal{L}_{brane} = \bar{\psi} i \gamma^\mu \partial_\mu \psi - g \bar{\psi} \psi \left(\phi^{(0)}_{even} + \sum_{n=1} \phi^{(n)}_{even} \right)$$

After rescaling

$$\mathcal{L} = \frac{1}{2}\sum_{n=1}\left[(\partial_\mu\phi(x)_{odd}^{(n)})^2 - \left(\frac{n}{R}\right)^2(\phi(x)_{odd}^{(n)})^2\right] + \frac{1}{2}(\partial_\mu\phi(x)_{even}^{(0)})^2 + \frac{1}{2}\sum_{n=1}\left[(\partial_\mu\phi(x)_{even}^{(n)})^2 - \left(\frac{n}{R}\right)^2(\phi(x)_{even}^{(n)})^2\right]$$

$$\mathcal{L}_{brane} = \bar{\psi}i\gamma^\mu\partial_\mu\psi - \bar{\psi}\psi\left(\frac{g}{\sqrt{2\pi R}}\phi_{even}^{(0)} + \sum_{n=1}\frac{g}{\sqrt{\pi R}}\phi_{even}^{(n)}\right)$$

Characteristic features: KK mode mass n/R

universal coupling

Large Extra Dimension Scenario

Phenomenology

- $M_{4+\delta} \sim O(1\,\mathrm{TeV})$
- many graviton Kaluza-Klein modes

$$G_{\mu\nu}(x^\mu, y^1, y^2, .., y^n) = \sum_n g_{\mu\nu}^{(\vec{n})}(x^\mu)\, \chi^{(\vec{n})}(\vec{y})$$

$$\chi^{(n)} \propto e^{i\frac{\vec{n}\cdot\vec{y}}{r}} \;,\quad \left(m_{KK}^{(\vec{n})}\right)^2 = \frac{|\vec{n}|^2}{r^2}$$

If 6 dim. ➔ $\frac{1}{r} \sim 10^{-4}\mathrm{eV}$

Phenomenology of Extra-dimension Scenario

= Phenomenology of graviton Kaluza-Klein modes

Construction of effective action in 4D

1. 4D Lagrangian

4+ δ dim. Graviton → 4D graviton in 4D

KK gravitons

KK gravi-scalars

KK gravi-vectors

2. Feynman rules

4D reduction of 4+ δ dim. Einstein's Eqs.

$$\Box_{4+\delta} G_{MN} = -\frac{T_{MN}}{(M_{4+\delta})^{2+\delta}}$$

$$G_{MN} = \left[\begin{array}{c|c} G^{(n)}_{\mu\nu} & V^{(n)}_{\mu j} \\ \hline V^{(n)}_{\mu i} & H^{(n)}\delta_{ij} + S^{(n)}_{ij} \end{array}\right]$$

KK graviton $$(\Box + \hat{n})G^{(n)}_{\mu\nu} = \frac{1}{\bar{M}_P}\left[-T_{\mu\nu} + \frac{1}{3}\left(\frac{\partial_\mu\partial_\nu}{\hat{n}^2}\right)T^\lambda_\lambda\right]$$

KK gravi-scalar $$(\Box + \hat{n})H^{(n)} = \frac{\sqrt{\frac{3(\delta-1)}{\delta+2}}}{\bar{M}_P}T^\mu_\mu$$

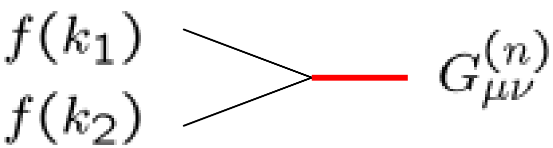
Vector $$(\Box + \hat{n})V^{(n)}_{\mu j} = 0$$

Scalar $$(\Box + \hat{n})S^{(n)}_{ij} = 0$$

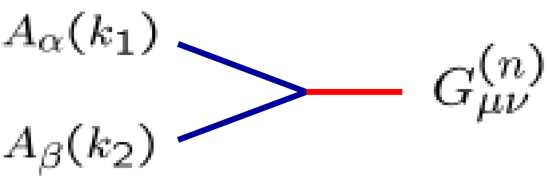
Feynman rules

$$T^{\mu\nu} = 2\frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} - g^{\mu\nu}\mathcal{L}$$

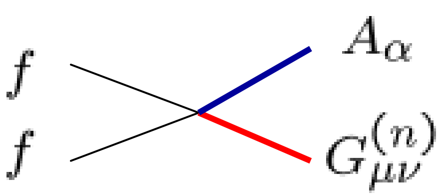
ex) QED + KK graviton



$$-\frac{i}{4\bar{M}_P}[(k_1+k_2)_\mu\gamma_\nu + (\nu \leftrightarrow \mu)]$$



$$\begin{aligned}-\frac{1}{\bar{M}_P}\quad [\quad & \frac{1}{2}\eta_{\mu\nu}(k_{1\beta}k_{2\alpha} - (k_1k_2)\eta_{\alpha\beta}) + \eta_{\alpha\beta}k_{1\mu}k_{2\nu} \\ + \quad & \eta_{\mu\alpha}(\eta_{\nu\beta}(k_1k_2) - k_{1\beta}k_{2\nu}) - \eta_{\mu\beta}k_{1\nu}k_{2\alpha} \\ + \quad & (\mu \leftrightarrow \nu)]\end{aligned}$$



$$-\frac{ieQ}{2\bar{M}_P}[\gamma_\mu\eta_{\nu\alpha} + (\mu \leftrightarrow \nu)]$$

$$\frac{iP_{\mu\nu\alpha\beta}}{k^2 - m^2}$$

$$\begin{aligned}P_{\mu\nu\alpha\beta} \;=\; & \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}) \\ - \; & \frac{1}{2m^2}(\eta_{\mu\alpha}k_\nu k_\beta + \eta_{\nu\beta}k_\mu k_\alpha + \eta_{\mu\beta}k_\nu k_\alpha + \eta_{\nu\alpha}k_\mu k_\beta) \\ + \; & \frac{1}{6}\left(\eta_{\mu\nu} + \frac{2}{m^2}k_\mu k_\nu\right)\left(\eta_{\alpha\beta} + \frac{2}{m^2}k_\alpha k_\beta\right)\end{aligned}$$

Detection of Extra-dimension @ future colliders

→ **detection of KK graviton**

direct → KK graviton emission

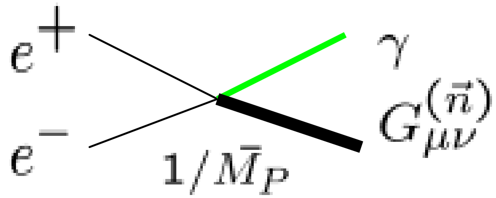
indirect → KK graviton mediated process

First detection of spin 2 particle !

KK graviton emission process

Emitted KK graviton → non-interacting & stable particle
→ missing energy event

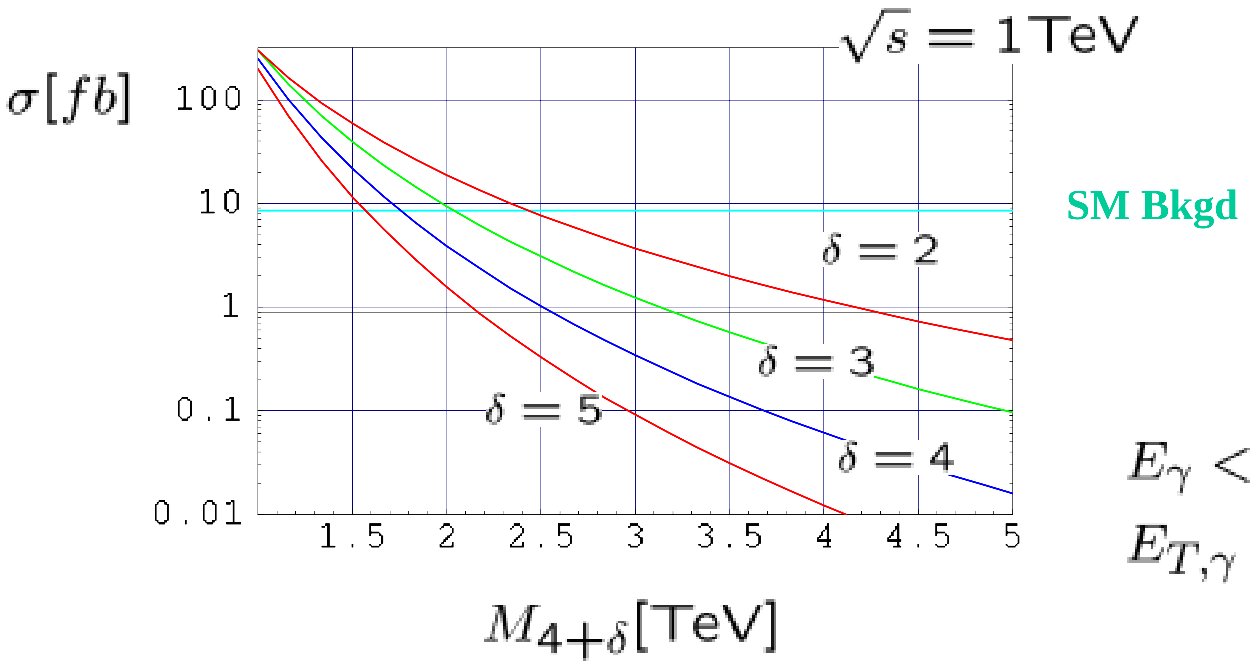
Example: $e^+e^- \to \gamma + \text{nothing}$



SM background: $e^+e^- \to \gamma\bar{\nu}\nu$

Each process is suppressed by $1/\bar{M}_P{}^2$
But × # of KK modes $\Longrightarrow \frac{1}{M_{4+\delta}^2}$

Result $e^+e^- \rightarrow \gamma + E_{\text{missing}}$

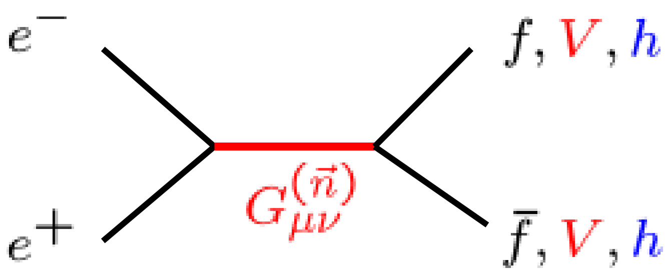


$E_\gamma < 450\text{GeV}$

$E_{T,\gamma} > 300\text{GeV}$

$$\sigma \propto \left(\frac{\sqrt{s}}{M_{4+\delta}}\right)^{\delta+2}$$

Virtual KK graviton mediated process



$$\sum_{\vec{n}} \frac{1}{s - \left(m_{KK}^{(\vec{n})}\right)^2} \to \infty \quad (\text{for } \delta \geq 2)$$

Need regularization

Naïve: Cut Off by $m_{KK}^{MAX} \sim M_{4+\delta}$

$$\boxed{\frac{4\pi\lambda}{M_S{}^4}} = -\frac{8\pi}{M_P^2} \sum_{\vec{n}} \frac{1}{s - \left(m_{KK}^{(\vec{n})}\right)^2}; \qquad \lambda = \pm 1$$

$$\mathcal{M} = \frac{4\pi\lambda}{M_S^4} T_{\mu\nu}(p_1, p_2) T^{\mu\nu}(p_3, p_4)$$

Calculations of some processes

$$e^+e^- \to \begin{array}{l} f\bar{f} \\ \gamma\gamma, W^+W^-, ZZ \\ hh \end{array}$$

$$\sum_{spin} |\mathcal{M}|^2 = \sum |\mathcal{M}_{SM} + \mathcal{M}_G|^2 = \sum \left(|\mathcal{M}_{SM}|^2 + \mathcal{M}_{IF} + |\mathcal{M}_G|^2 \right)$$

Interference: $\sum \mathcal{M}_{IF} = \sum \mathcal{M}^\dagger_{SM}\mathcal{M}_G + h.c. \quad \propto \boxed{\lambda}$

Only KK graviton: $\sum |\mathcal{M}_G|^2 \quad \propto \boxed{\lambda^2}$

$$\mathcal{M} = \frac{4\pi\lambda}{M_S^4} T_{\mu\nu}(p_1, p_2) T^{\mu\nu}(p_3, p_4)$$

Results I: total cross section

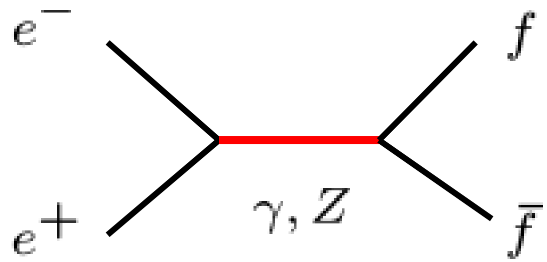
← **new physics evidence**

II: angular dependence of cross section

← **effects due to spin 2 particle exchange**

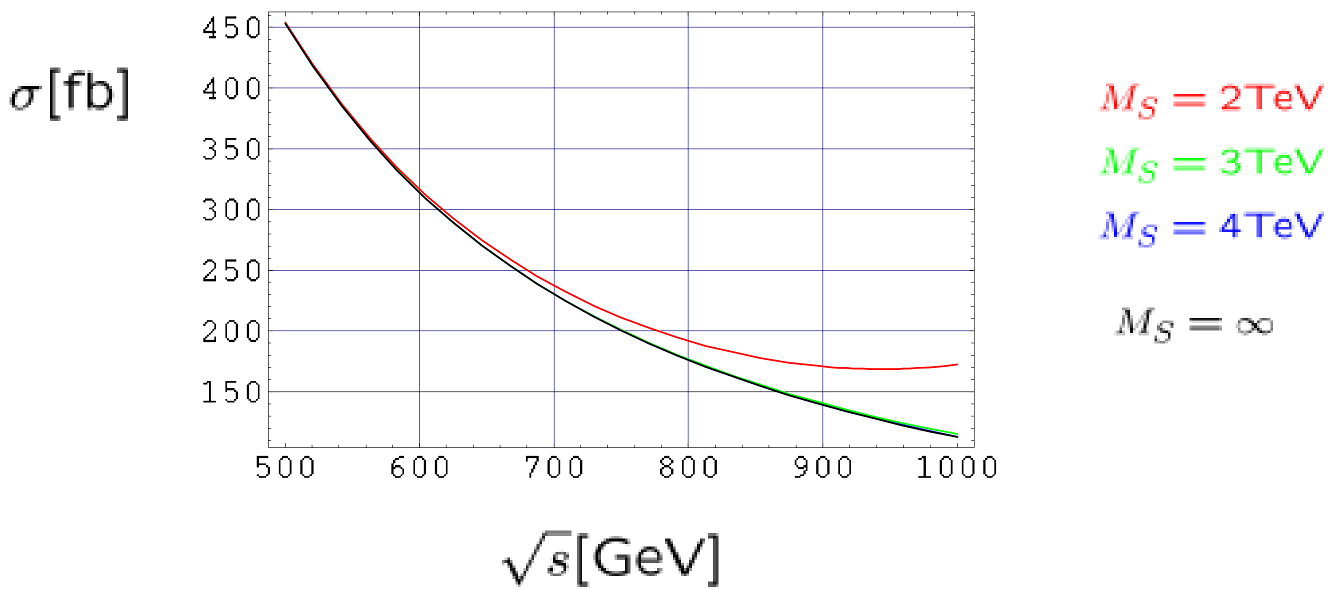
$e^+e^- \to f\bar{f}$ **processes**

SM background



$(f \neq e)$

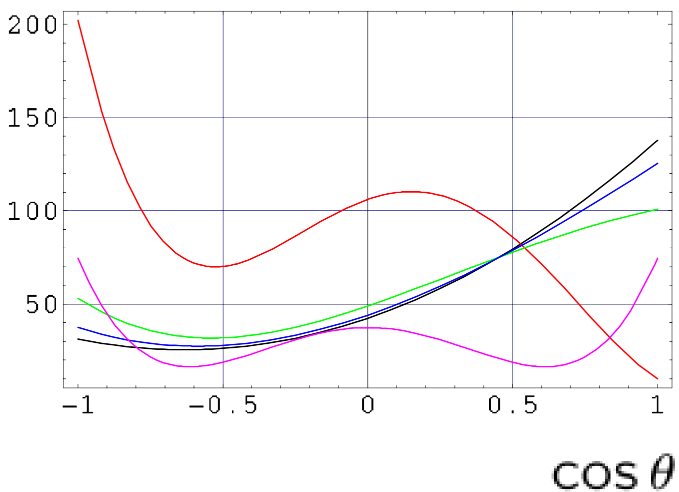
$e^+e^- \to \tau\bar{\tau}$



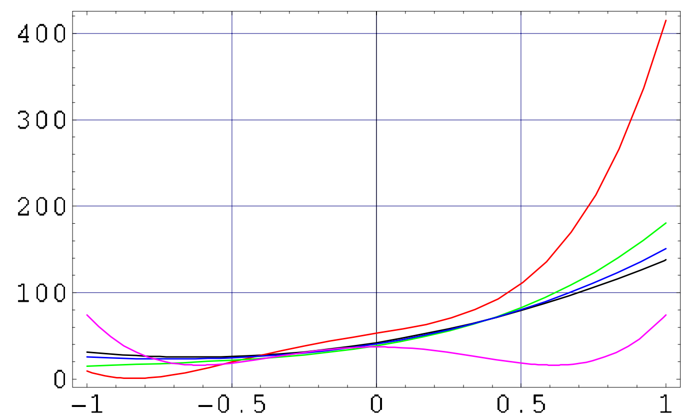
Angular dependence of cross section

$$\frac{d\sigma(e^+e^- \to \tau\bar{\tau})}{d\cos\theta}\,[\mathrm{fb}] \qquad \textbf{at} \quad \sqrt{s} = 1\,\mathrm{TeV}$$

$\lambda = +1$



$\lambda = -1$



$M_S = 2\mathrm{TeV}$

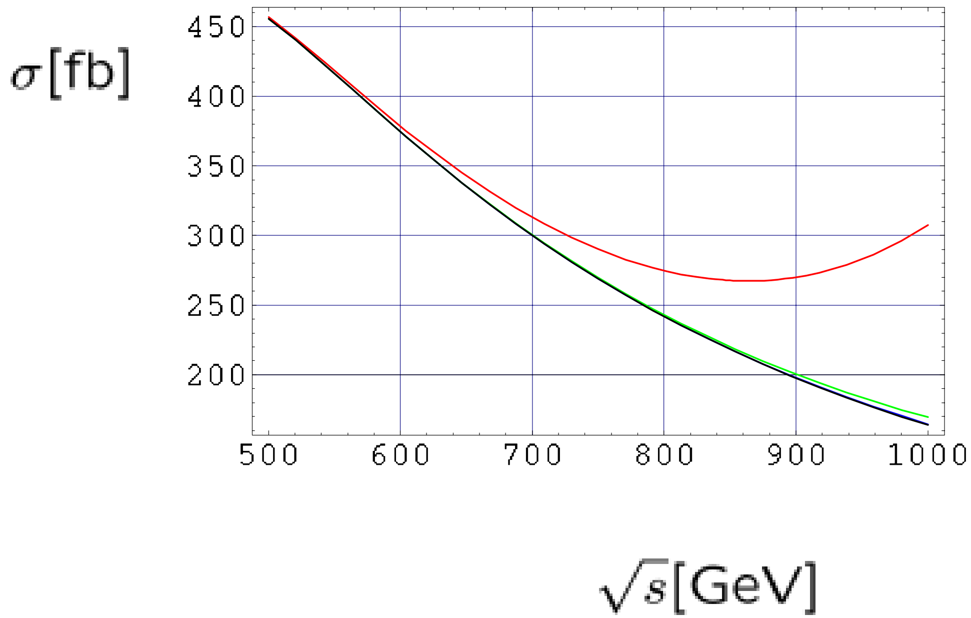
$M_S = 3\mathrm{TeV}$

$M_S = 4\mathrm{TeV}$

$M_S = \infty$

$$e^+e^- \to t\bar{t}$$

$\lambda = +1$



$M_S = 2\text{TeV}$

$M_S = 3\text{TeV}$

$M_S = 4\text{TeV}$

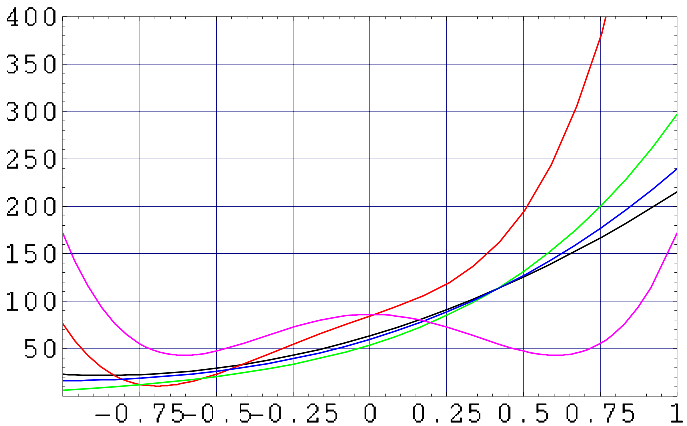
$M_S = \infty$

$m_t = 175\text{GeV}$

Angular dependence of cross section

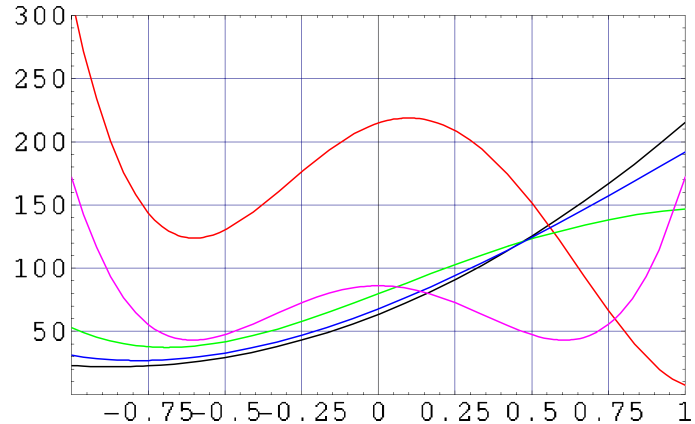
$$\frac{d\sigma(e^+e^- \to tt)}{d\cos\theta}[\text{fb}] \quad \textbf{at} \quad \sqrt{s} = 1\text{TeV}$$

$\lambda = +1$



$\cos\theta$

$\lambda = -1$



$M_S = 2\text{TeV}$

$M_S = 3\text{TeV}$

$M_S = 4\text{TeV}$

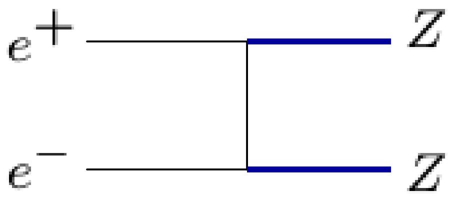
$M_S = \infty$

$m_t = 175\text{GeV}$

$e^+e^- \to VV$ **processes**

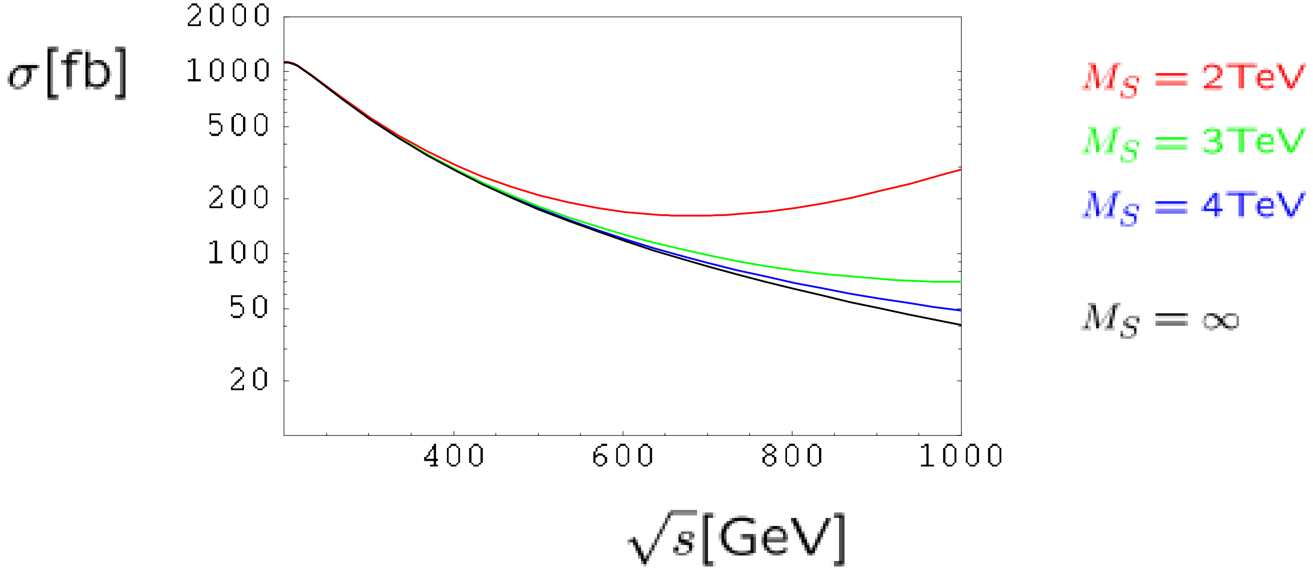
$e^+e^- \to ZZ$

SM background



+ cross

$\lambda = +1$

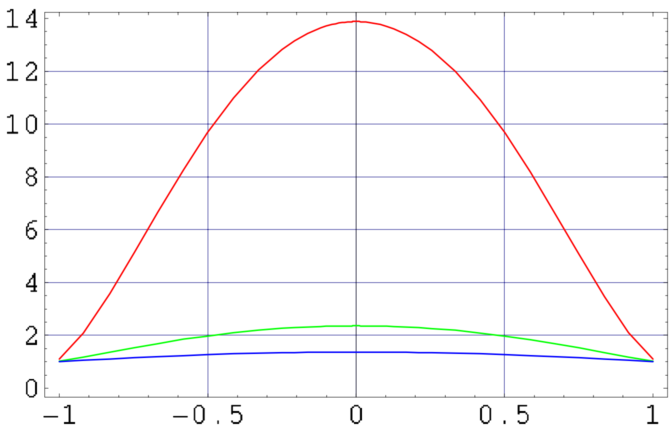


Angular distribution

$$\frac{d\sigma}{d\cos\theta} \Big/ \frac{d\sigma}{d\cos\theta}\Big|_{SM}$$

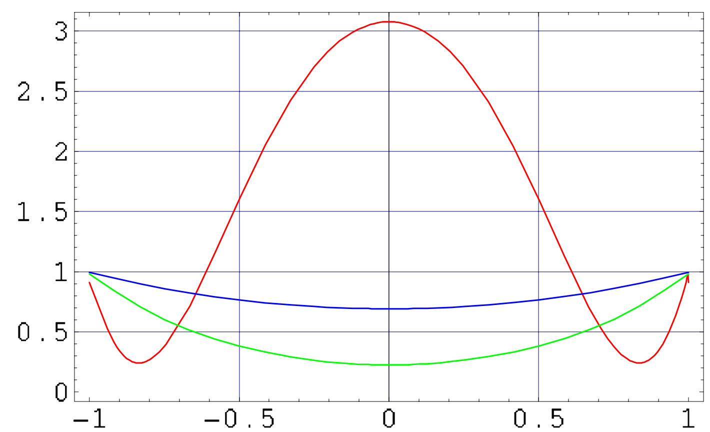
$\sqrt{s} = 1\text{TeV}$

$\lambda = +1$



$\cos[\theta]$

$\lambda = -1$



$M_S = 2\text{TeV}$

$M_S = 3\text{TeV}$

$M_S = 4\text{TeV}$

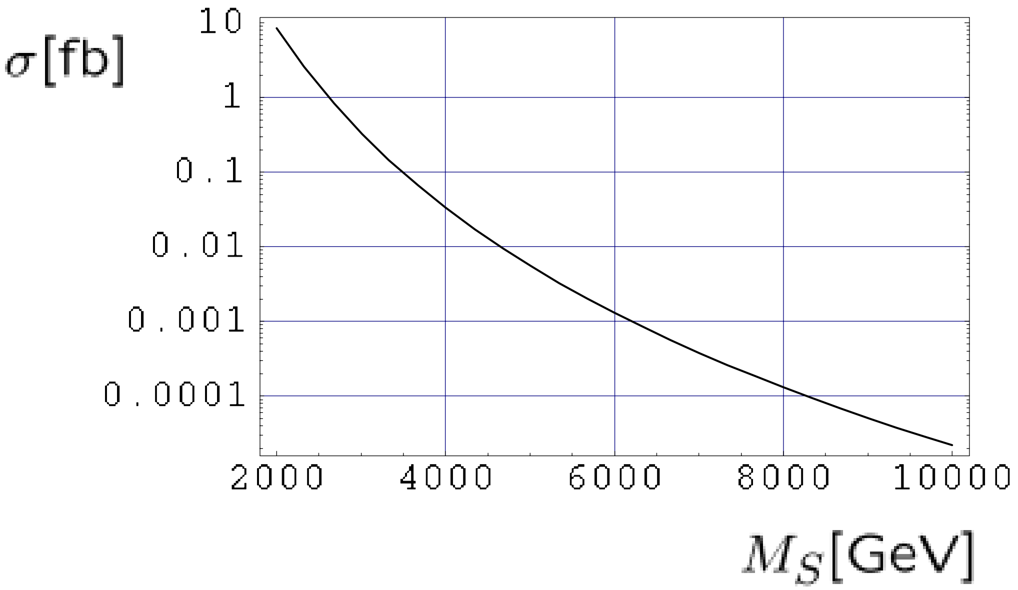
$e^+e^- \to h\bar{h}$ process

KK graviton exchange is dominant

SM background free → very interesting

if this cross section is large enough

$$\sigma(e^+e^- \to hh) = \frac{\pi\lambda^2}{480 M_S^8}\sqrt{1-4\frac{m_h^2}{s}}\left(s^3 - 8m_h^2 s^2 + 16 m_h^4 s\right)$$



$\sqrt{s} = 1$ TeV

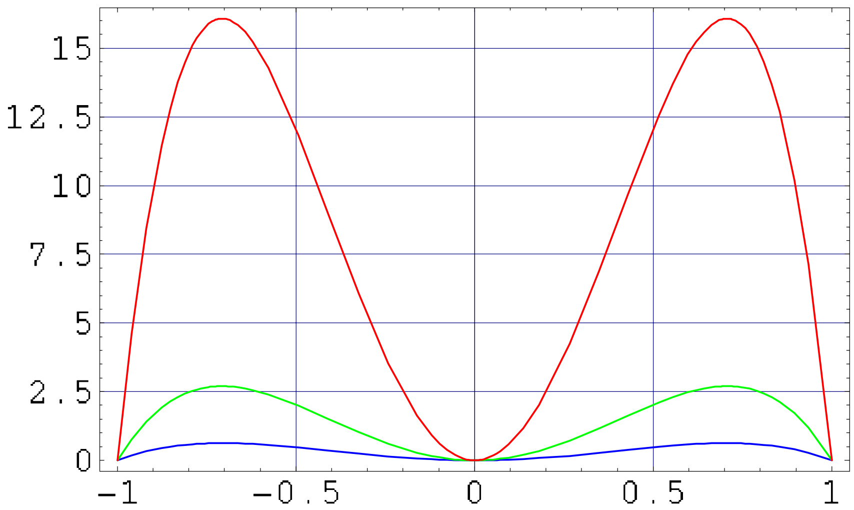
$m_h = 120$GeV

Angular dependence of cross section

$$\frac{d\sigma(e^+e^- \to hh)}{d\cos\theta}\text{[fb]}$$

$$\sqrt{s} = 1\text{TeV}$$

$$m_h = 120\text{GeV}$$



$\cos\theta$

$M_S = 2\text{TeV}$

$M_S = 2.5\text{TeV}$

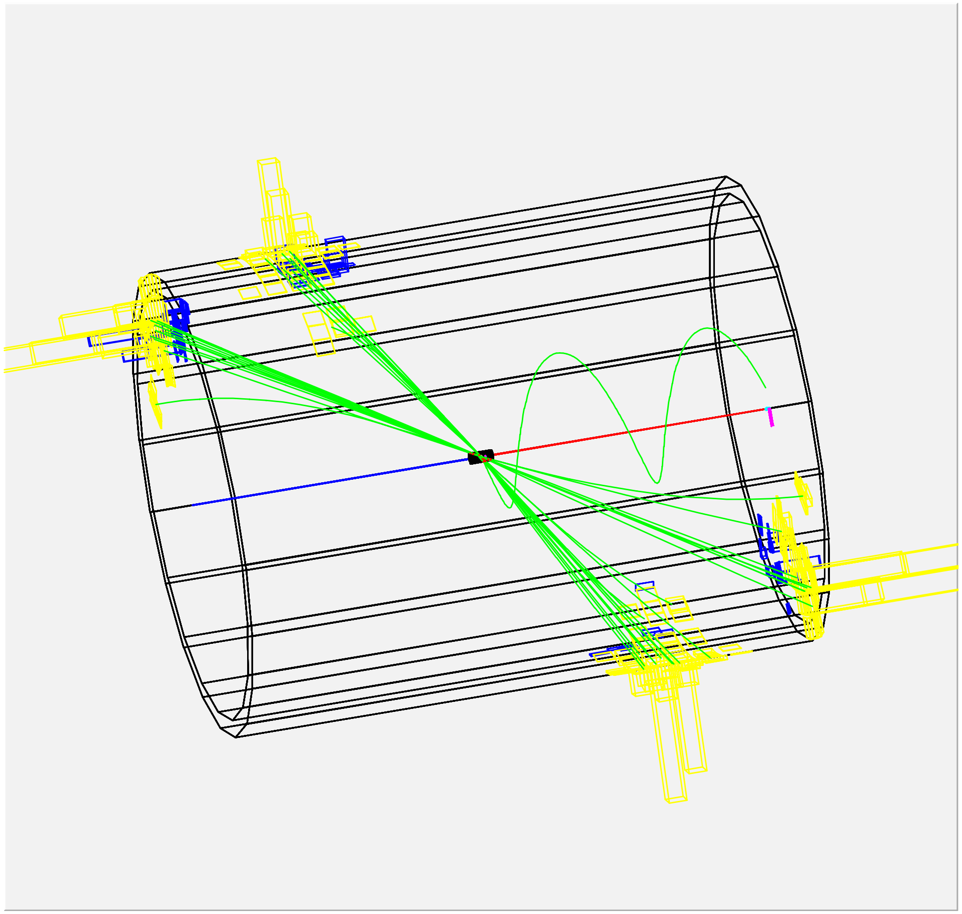
$M_S = 3\text{TeV}$

$$\sum |\mathcal{M}|^2 = \frac{1}{2}\left(\frac{4\pi\lambda}{M_S^4}\right)^2 (t-u)^2(tu - m_h^4)$$

Results of simulation studies (VERY Preliminary!)

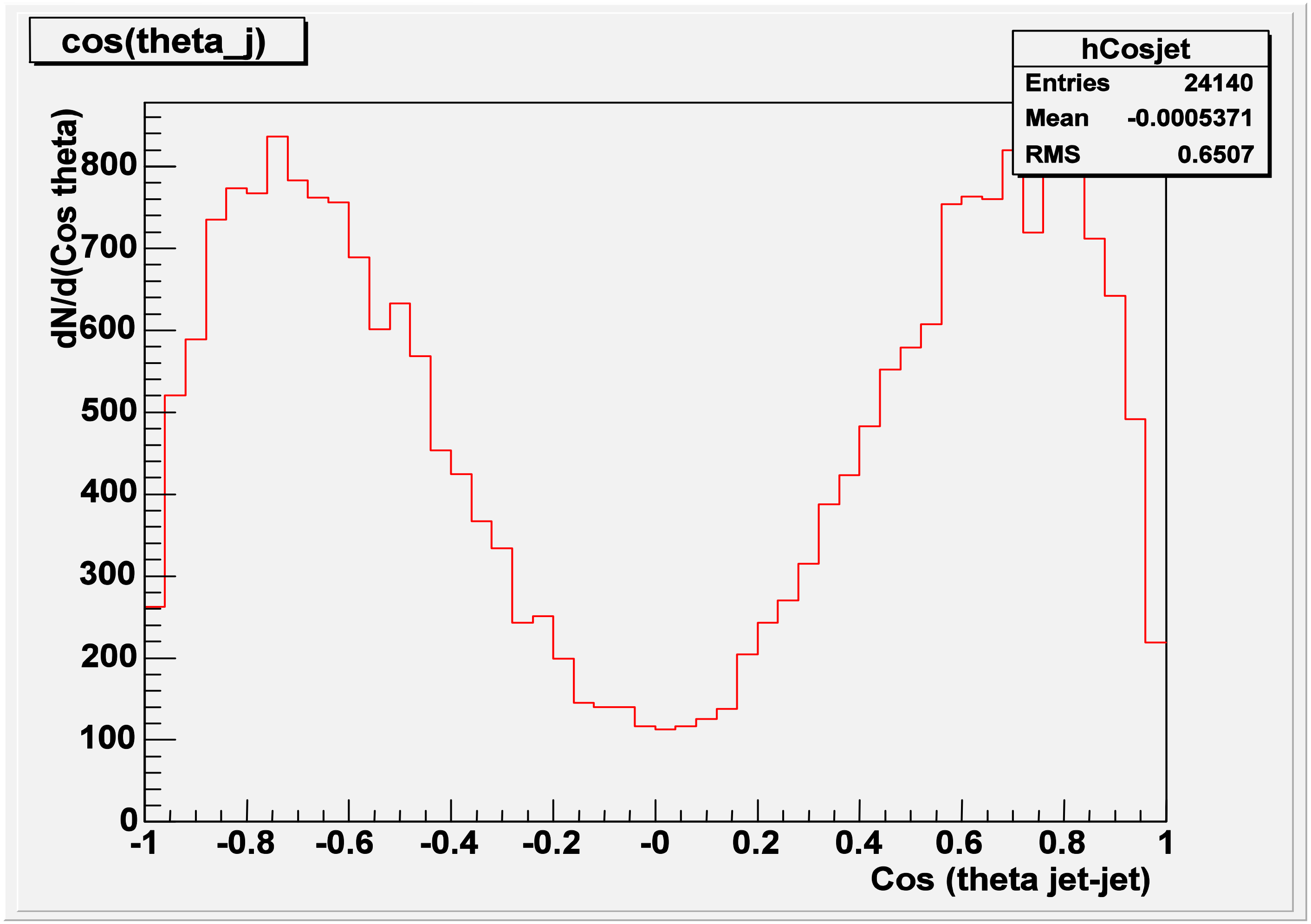
$e^{+}e^{-} \rightarrow hh$ **process**

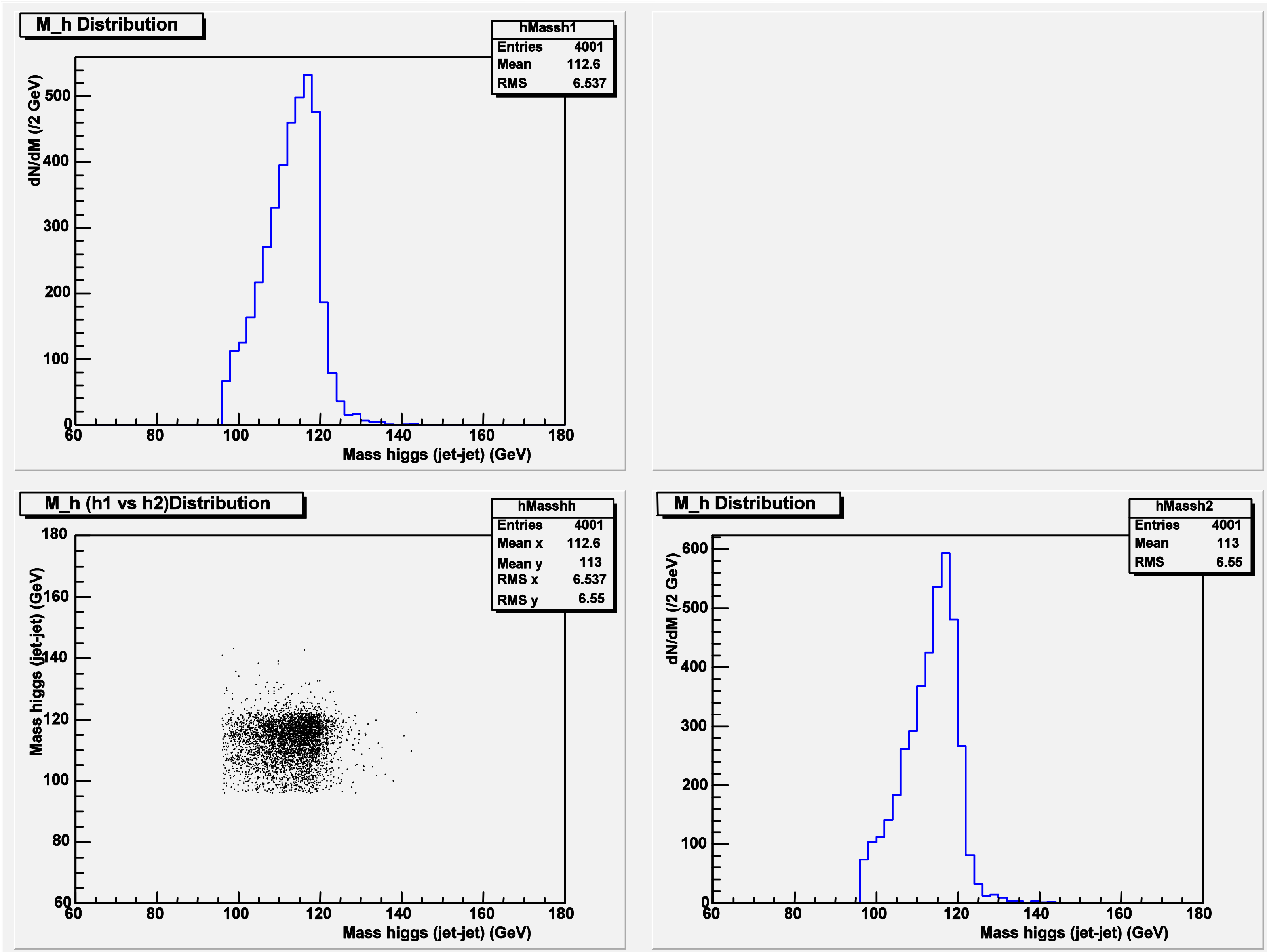
$hh \rightarrow b\bar{b}b\bar{b}$ **4 jet events**



$$\sqrt{s} = 1\text{TeV}$$
$$M_S = 2\text{TeV}$$
$$m_h = 120\text{GeV}$$

Not including SM background




M_h Distribution
hMassh1
Entries 4001
Mean 112.6
RMS 6.537
dN/dM (/2 GeV)
0
100
200
300
400
500
60
80
100
120
140
160
180
Mass higgs (jet-jet) (GeV)
M_h (h1 vs h2)Distribution
hMasshh
Entries 4001
Mean x 112.6
Mean y 113
RMS x 6.537
RMS y 6.55
Mass higgs (jet-jet) (GeV)
60
80
100
120
140
160
180
60
80
100
120
140
160
180
Mass higgs (jet-jet) (GeV)
M_h Distribution
hMassh2
Entries 4001
Mean 113
RMS 6.55
dN/dM (/2 GeV)
0
100
200
300
400
500
600
60
80
100
120
140
160
180
Mass higgs (jet-jet) (GeV)